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DETERMINATION OF SCHWARZSCHILD'S RADIUS OF SOME PLANETARY BODIES IN THE SOLAR SYSTEM USING NEWTONIAN MECHANICS

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ABSTRACT

The need to understand black hole formation, property and absorption of other celestial bodies is an interesting issue in Astrophysics and Astronomy. In this short article we avoided complex mathematical difficulty to obtain Schwarzschild's radius of some planetary bodies, and this radius was computed using Newtonian expression for escape velocity. Results obtained shows that among the planets Jupiter requires a very large amount of gravitational pull to reduce to black hole and was found to be exactly the same with that using Schwarzschild's metric.
Keywords and phrases: Black hole, Schwarzschild's radius, Newtonian expression, escape velocity

INTRODUCTION

It is believed that there are four fundamental forces in nature; Gravitation, Electromagnetism, weak force and strong nuclear forces which hold together the particles that made up atoms. Gravitation is by far the weakest of these forces and as a result is not important in the interactions of atoms or even moderate sized objects but is important only when very large objects or planets in which general relativity predicts the formation of black holes (a region where light cannot escape). Black holes are celestial bodies that are believed to exist in the universe. It is called because it absorbs all the lights that hit the horizon reflecting none. the collapse of any object of heavy mass appears to an outside observer to take an infinite time and the events at distances beyond it radius called Schwarzschild's radius or event horizon are unobservable from outside

(Chifu, 2011). Early studies of gravitation was dated to the work of Newton (1687) which is highly successful and proved accurate in describing the motion of some astrophysical bodies the solar system every day applications but the theory failed in fully description of Big bang theory. According to Newtonian theory, gravitational effects propagate from place to place instantaneously. With the advent of Einstein's special theory of relativity in 1905, a theory uniting the concepts of space and time into that of four dimensional flat Minkowski space-time a problem that is partially in disagreement with Newtonian theory.

Chifu (2011) suggest that, the Schwarzschild's radius can equally be computed from the planetary reduction ratio. Here, we compute the Schwarzschild's radius of some commonly known planetary bodies in the solar system using Newtonian equation to avoid complex mathematical difficulty in describing the position of the planet using four dimensional Minkowski space-time or Euclidean as an altenative to Scwarzschild's metric

METRIC

In general relativity the key concept is the metric. General relativity replaces gravity with curvature of spacetime. The metric tells us to measure distances in space and time.The metric contains all the information about curvature in a simple formula which will allows us to get necessary information around us. Now let us write the expression for the metric in a region far from gravitational fields using spatial coordinate system (r,θ,φ) (Florov and Novikov,1997).

$$ds^2 = -c^2dt^2 + dl^2 \qquad 2.1$$

Where c is the speed of light and dl is the distance in three-dimensional space

$$ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\varphi d\varphi^2) \qquad 2.2$$

Consider a curved spacetime but preserve the condition of spatial spherical symmetry. The space-time itself is not necessarily empty, it may contain matter and physical fields which are also spherically symmetric if we consider their own gravitation and that there exist coordinates in a spherically symmetric spacetime such that its metric is of the form (Florov and Novikov,1997).

$$ds^2 = g_{oo}(x^o,x^1)dx^{o^2} + 2g_{o1}(x^o,x^1)dx^o dx^1 + g_{11}(x^o,x^1)dx^{1^2} + g_{22}(x^o,x^1)(d\theta^2 + \sin^2\theta d\varphi^2) \quad 2.3$$

From the above it is evident that there is a freedom in choosing any of the coordinates such that

$x^o = x^o\left(x^o,x^1\right)$, $x^1 = x^1\left(x^o,x^1\right)$, $\theta=\theta$ and $\varphi=\varphi$

also considering $g_{ij}=0 \quad \forall i \neq j \quad otherwise = 1$

Therefore, the required metric will be of the form:

$$ds^2 = g_{oo}(x^o,x^1)dx^{o^2} + g_{11}(x^o,x^1)dx^{1^2} + g_{22}(x^o,x^1)(d\theta^2 + \sin^2\theta d\varphi^2) \quad 2.4$$

FRIEDMANN, ROBERTSON, WALKER (FRW) METRIC

Consider the entire universe filled uniformly with matter, one can solve the field equations for the metric as a function of time which can be expressed as

$$ds = R(t)\sqrt{dx^2 + dy^2 + dz^2} \quad 2.1.1$$

Where the scale factor $R(\text{r}) = \left(\frac{t}{t_o}\right)^{\frac{1}{2}}$ at early times, $R(t) = \left(\frac{t}{t_o}\right)^{\frac{2}{3}}$ this is the first solution for Einstein field equation with uniformly distributed matter in his General Theory of Gravitation at *t=0* implies *R(0) =0* meaning all the variables above are zero including *S* itself, that is everything in the universe is touching one another under the influence of gravity. This means we can precisely predict the attractive gravitational force between object at a maximum time by telling us the possibility of turning objects to black hole under the certain radius *r*. (Frolov, 1997)

SCHWARZSCHILD'S METRIC

The FRW metric is valid where matter is approximately to be uniformly distributed. When measuring the size of massive object like planets, the matter is concentrated in a central source; therefore FRW is not the solution of Einstein field's equation (Hooft, 2009)

Solving Einstein's field equations for the region outside a spherical object of mass *M*, gives the Schwarzschild's metric

$$ds^2 = -(1-\frac{2GM}{c^2 r})dt^2 + (1-\frac{2GM}{rc^2})\,\mathrm{dr}^2 + \mathrm{r}^2\mathrm{d}\,\theta^2 + r\sin^2\theta d\varphi^2 \qquad 2.2.1$$

Where the position vector *r* is in spherical coordinates

At $dt^2 = 0, \mathrm{dr}^2 \to \infty$. The metric has a singularity if $\frac{2GM}{rc^2} = 1$. This happens when an object is squeezed to certain radius r_{sh}

$$r_{sh} = \frac{2GM}{c^2} \qquad 2.2.2$$

The above equation can also be expressed in terms of planetary mass ratio as

$$r_{sh} = 3(km)\frac{M}{M_{\oplus}} \qquad 2.2.3$$

Where r_{sh} is the Schwarzschild's radius of a planet of mass *M* and M_O is the mass of the central sun

NEWTONIAN ESCAPE VELOCITY

Consider a planet of mass M with it corresponding radius R. then the gravitational potential energy of it surface is

$$V = -\frac{GMm}{R} \qquad 3.1$$

Where *m* is a point mass of distance *R* from *M*, Since V goes to zero at infinity, the escape velocity is found by setting potential energy to be equal to kinetic energy and solving for *v*, we obtained the expression for the escape velocity for any particle of mass *M* within any Surface is then given by

$$v = \sqrt{\frac{2GM}{R}} \qquad 3.2$$

Where M is the mass of the particle with distance r, from the centre of the surface, if the particles of light of mass M fail to escape from the surface ,when $v=c$

$$c = \sqrt{\frac{2GM}{R}} \qquad 3.3$$

The above equation can be rearranged to give the Schwarzschild's radius r_{sh} which for a given mass must be compressed to form black hole

$$R_{sh} = \frac{2GM}{c^2} \qquad 3.4$$

Where R_{sh} is called Schwarzschild's radius, Substituting the value of G = $6.67x10^{-11}Nm^2Kg^{-2}$ and
$c=3.0x10^8ms^{-1}$ and M is the mass of the object
Then

$$R_{sh}= 1.48x10^{-27} M \text{ (metre/kilogram)} \qquad 3.5$$

RESULTS AND DISCUSSION

The Schwarzschild's radius for a planet is computed directly from the above equation (9) by direct substitution with the value of M

Table1: Scwarzschild's radius of some planetary bodies

S/No.	Class of the Planet	Name of the planet	M(kg)	R_{sh} (m)	r_{sh} (m)
1	Major	Mercury	3.30E+23	0.000488	0.000488
2		Venus	4.87E+24	0.007208	0.007208
3		Earth	5.98E+24	0.00885	0.00885
4		Mars	6.42E+23	0.00095	0.00095
5		Jupiter	1.90E+27	2.812	2.812
6		Saturn	5.69E+26	0.84212	0.84212

7		Uranus	8.68E+25	0.128464	0.128464
8		Neptune	1.02E+26	0.15096	0.15096
9	Dwarf	Pluto	1.29E+22	1.91E-05	1.91E-05
10		Ceres	9.45E+20	1.4E-06	1.4E-06

DISCUSSIONS

Among the two classes of planets the major planets has a Schwarzschild's radius ≥0.49mm while dwarf planets has a radius ≥ 0.019mm. Jupiter has a Schwarzschild's radius of 2812mm which means enormous gravitational pull is required to compress into a black hole followed by Saturn with a radius of 842.12mm, Neptune 151mm and Uranus 128.5mm. Our earth has a Schwarzschild's radius of 8.85mm which is close to that of the planet Venus of radius 7.21mm. Other major planets include Mercury and Mars has less radius compare to other planets which means less gravitational pull is required to form a black hole. The dwarf planets consisting of Ceres and Pluto with an approximate radius 0.0014mm, 0.0191mm respectively indicate a chance of black hole formation with much less gravitational pull

CONCLUSION

The results obtained show that the Schwarzschild's radius can equally be computed accurately using Newtonian approach and provides accurate results but as the mass tends towards to a minimum value, the classical description of the object fails, then the Schwarzschild's metric become much significance

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