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STUDY ON DIMENSIONAL ANALYSIS USING THE FLOW OF WATER THROUGH TWO GEOMETRICALLY SIMILAR TANKS AT DIFFERENT TEMPERATURES

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ABSTRACT

Dimensional analysis is a way of simplifying a physical problem by appealing to dimensional similarity to reduce the number of relevant variables. A multi-parameter problem often has difficulties in achieving complete similarity. Dimensional analysis gives a useful technique of decreasing difficult physical problems to the easiest, simplest and most economical method before obtaining a measurable solution. This study worked on getting the pi terms or groups involved in two geometrically similar tanks allowing the fluid (water) to be completely drained from them using two temperature values of 26.5℃ and 43.5℃ respectively for each tank using n relevant variables and m independent dimensions which was reduced to a relationship between $n - m$ non-dimensional parameters $\pi_1, \pi_2, \ldots, \pi_{n-m}$. As the values of pi 1 (π_1) terms of the big tank are decreasing, the values of the pi 4 (π_4) terms are increasing in all cases.

Keywords: Dimensional analysis, Similar tanks, temperature, pi-terms, Measurable, Technique.

INTRODUCTION

Dimensional analysis is a means of simplifying a physical problem by appealing to dimensional homogeneity to reduce the number of relevant variables. It is particularly useful for; presenting and interpreting experimental data, checking equations, establishing the relative importance of particular physical phenomena, attacking problems not amenable to a direct theoretical solution and physical modelling. In dimensional analysis, it can be said that the mathematical relations in any theory must be equidimensional (Texocotitla et al., 2020). It is in this context that the concept of dimension is introduced, which allows to manage the axiom of similarity using a more grounded mathematical formalism (White, 2011 & Texocotitla et al., 2020).

The Buckingham pi (π) theorem is a significant theorem in dimensional analysis, it offers a way of figuring sets of dimensionless parameters from the given variables, even if the system of the equation is still not known (M. Bahrami). A multi-parameter problem often has difficulties in achieving complete similarity. Dimensional analysis gives a useful technique of decreasing difficult physical problems to the easiest, simplest and most economical method before obtaining a measurable solution. Science starts with the observation and accurate account of things and events, and is the initial step upon which the dimensional analysis stands (Sonin, 2001).

In Engineering and Science, dimensional analysis is the analysis of the association existing amid varying physical quantities by recognizing their fundamental or base quantities (such as mass, length, electric charge and time) and units of measure (such as kilometre vs. miles or kilogram vs. pounds) and following these dimensions as comparisons or calculations are made. Dimensional analysis also called the factor-label method or the unit factor method is an extensively used procedure for the easiest conversion of units from one dimensional unit to another within the metric or SI system than others using the rules of algebra. Dimensional Analysis (also called Factor-Label Method or the Unit Factor Method) is a problem-solving method that uses the fact that any number or expression can be multiplied by one without changing its value (Kent's Chemistry; Goldberg 2006; and Ogden 1999).

Commensurable physical quantities are of the same kind and have same dimension, and can be directly compared to each other, even if they are initially expressed in different units of measure; examples are as in yards and metres, seconds and years, and pounds and kilograms. Incommensurable physical quantities are not of the same kind and thus have different dimensions which make them difficult to be compared directly with each other, irrespective of the original units they are expressed in; examples metres and kilograms, seconds and kilograms and metres and seconds. The most basic rule of dimensionless analysis is that of dimensional homogeneity (Cimbala & Cengel 2006). The rule implies that in a physically meaningful expression, only quantities of the same dimension can be added, subtracted, or compared. Thus, dimensional analysis may be used as a sanity check of physical equations: the two sides of any equation must be commensurable or have the same dimension.

Dimensional analysis is most often used in physics and chemistry and also finds applications in mathematics, finance, economics, accounting and fluid mechanics. In fluid mechanics, dimensional analysis is performed in order to obtain dimensionless pi terms or groups. Using suitable pi terms or groups, it is possible to develop a similar set of pi terms for a model that has the same dimensional relationship (Waite & Fine, 2007). The pi terms provide a short cut to developing a model that represents a certain prototype. Common dimensionless groups in fluid mechanics include Reynold number (Re), Froude number (Fr), Euler number (Eu), Mach number (Ma) etc. However, in some disciplines as it is the case with Economics, the concept of dimension and their respective principles are practically unknown. Very few researches have emphasized the implications of Dimensional Analysis in the economic discipline; among them are (Grudzewski & Roslanowska, 2013; Texocotitla et al., 2020).

This study worked on getting the pi terms or groups involved in two geometrically similar tanks allowing the fluid (water) to be completely drained from them using two temperature values of 26.5°C and 43.5°C respectively for each tank.

MATERIALS AND METHOD

The equipment used are two geometrically similar cylindrical tanks, stopwatch, ruler and a thermometer.

Appropriate measurements to show that the two tanks are geometrically similar were performed. It was ensured that the bigger tank is twice the size of the small tank (i.e. twice the height, twice the diameter and twice the diameter of its drain hole).
The large tank was filled with cold water of temperature 26.5^0C, of water depth (h) = 0.25m and at time (t) = 0.00s, this was recorded and at this material time, the drain hole was closed. Thus h = h (t) was obtained. The ranges in time when t = 0 and h = H (full tank i.e. initial depth of water) to the final time when h = 0 m (i.e. tank completely drained) was noted and recorded.

This same procedure/process with the same temperature of cold water was performed using the small tank, and to ensure geometric similarity, the initial water level in the small tank was kept at one-half of the one used in the large tank. Furthermore, the experiment was conducted again with the two tanks using warm water at temperature of 43.5^{0}C and four sets of h (t) values were obtained and recorded.

It was assumed that the depth (h) of water in the tank is a function of its initial depth (H). The diameter of the tank (D) = 3m, the diameter of the drain hole (d) at the bottom of the tank = 0.025m, the acceleration due to gravity is given as g, the fluid density represented as ρ, and viscosity = μ. Dimensionless parameters with H, g and ρ used as repeating variables were developed with t as the dependent parameter. Dimensionless time, $tg^{\frac{1}{2}} / H^{\frac{1}{2}}$, as a function of the dimensionless depth $\frac{h}{H}$ was calculated for each of the four conditions tested.

Tables 1 and 2 show the values obtained when the bigger and the smaller tanks were run at a temperature of 26.5°C while tables 3 and 4 show the values obtained when the bigger and the smaller tanks were run at a temperature of 43.5°C respectively. The same values of time (t) and their corresponding depths (h) were used for the two temperature values for each of the tanks as shown in figure 1 below.

Table 1: Big tank with T = 26.5°C

S/No.	h(m)	t(s)	$\pi_1 = {}^{h}/_{H}$	$\pi_4 = t g^{1/2}/H^{1/2}$
1	0.25	0.00	1.00	0.00
2	0.20	1.01	0.80	6.33
3	0.15	1.76	0.60	11.02
4	0.10	1.92	0.40	12.03
5	0.00	2.36	0.00	14.78

Table 2: Small tank with T = 26.5°C

S/No.	h(m)	t(s)	$\pi_1 = {}^{h}/_{H}$	$\pi_4 = t g^{1/2}/H^{1/2}$
1	0.25	0.00	1.00	0.00
2	0.20	1.01	0.75	0.65
3	0.15	1.76	0.50	0.99
4	0.10	1.92	0.25	1.50
5	0.00	2.36	0.00	2.11

Table 3: Big tank with T = 43.5°C

S/No.	h(m)	t(s)	$\pi_1 = {}^{h}/_{H}$	$\pi_4 = t g^{1/2}/H^{1/2}$
1	0.25	0.00	1.00	0.00
2	0.20	1.01	0.80	6.33
3	0.15	1.76	0.60	11.02
4	0.10	1.92	0.40	12.03
5	0.00	2.36	0.00	14.78

Table 4: Small tank with T = 43.5°C

S/No.	h(m)	t(s)	$\pi_1 = {}^{h}/_{H}$	$\pi_4 = t g^{1/2}/H^{1/2}$
1	0.25	0.00	1.00	0.00
2	0.20	1.01	0.75	1.08
3	0.15	1.76	0.50	1.25
4	0.10	1.92	0.25	1.26
5	0.00	2.36	0.00	16.29

From the tables 1 to 4, the values of $\pi_1 = \frac{h}{H}$, and $\pi_2 = \frac{tg^{1/2}}{H^{1/2}}$ were calculated as shown above. The graph of the depth (h) on the Y-axis and time (t) on the X-axis was plotted on a single graph for each of the four sets of data as shown in figure 1 below.

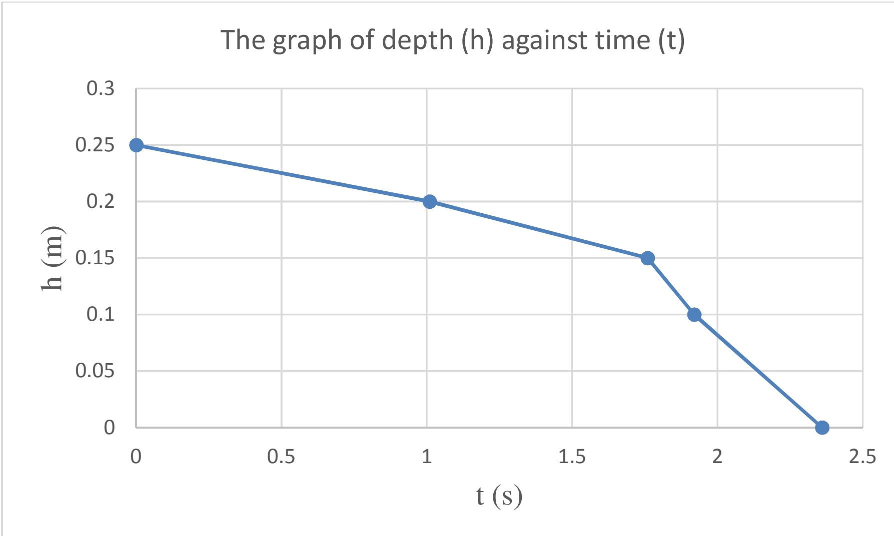

Figure 1: graph of the depth (h) on the Y-axis and time (t)

Table 5: List of parameters and their MLT units

Parameters	Unit	MLT
H	m	L
h	m	L
D	m	L
d	m	L
t	s	T
ρ	kg/m^3	ML^{-3}
μ	kg/ms	$ML^{-1}T^{-1}$
g	ms^{-2}	LT^{-2}

Independent dimensions (most influential parameter) are:
Geometry = H
Kinematic = g
Dynamic = ρ

Hence with the above n relevant variables and m independent dimensions. This can be reduced to a relationship between n – m non-dimensional parameters $\pi_1, \pi_2, \ldots, \pi_{n-m}$.
$8 - 3 = 5\ \pi$ terms.
Thus;
$\pi_1 = h \times H^A \times g^B \times \rho^C$................. (1)
$\pi_2 = D \times H^A \times g^B \times \rho^C$................ (2)
$\pi_3 = d \times H^A \times g^B \times \rho^C$................ (3)

$\pi_4 = t \times H^A \times g^B \times \rho^C$................. (4)
$\pi_5 = \mu \times H^A \times g^B \times \rho^C$............... (5)

By consideration:
Diameter of tank (D) and drain hole diameter (d) in tank are considered as constant throughout the experiment.
Since there is no water compression, viscosity (μ) of fluid is also constant throughout the experiment.
π_2, π_3, and π_5 are considered to be negligible, as such not shown in tables above.

RESULTS AND DISCUSSION

From the eight (8) relevant variables and three (3) independent dimensions (H, g and ρ) that were considered in the flow of water through the two geometrically similar tanks, we got;
From Equation (1),

$\pi_1 = h \times H^A \times g^B \times \rho^C$
$M^0L^0T^0 = [L] \times [L]^A \times [LT^{-2}]^B \times [ML^{-3}]^C$
$M^0 = M^C \quad \rightarrow \ C = 0$
$L^0 = L^{1+A+B-3C} \quad \rightarrow \ A = -(1 + B)$
$T^0 = T^{-2B} \quad \rightarrow \ B = 0 \,; A = -1$

Therefore, $\pi_1 = \frac{h}{H}$

From Equation (2);

$\pi_2 = D \times H^A \times g^B \times \rho^C$
$M^0L^0T^0 = [L] \times [L]^A \times [LT^{-2}]^B \times [ML^{-3}]^C$
$M^0 = M^C \quad \rightarrow \ C = 0$
$T^0 = T^{-2B} \quad \rightarrow \ B = 0$
$L^0 = L^{1+A+B-3C} \quad \rightarrow \ A = -1$

therefore, $\pi_2 = \frac{D}{H}$

From Equation (3);

$\pi_3 = d \times H^A \times g^B \times \rho^C$
$M^0L^0T^0 = [L] \times [L]^A \times [LT^{-2}]^B \times [ML^{-3}]^C$
$M^0 = M^C \quad \rightarrow \ C = 0$
$T^0 = T^{-2B-3C} \quad \rightarrow \ B = 0$

$$L^0 = L^{1+A+B-3C} \quad \rightarrow \; A = -1$$

therefore, $\pi_3 = \frac{d}{H}$

From Equation (4);

$$\pi_4 = t \times H^A \times g^B \times \rho^C$$
$$M^0L^0T^0 = [L] \times [L]^A \times [LT^{-2}]^B \times [ML^{-3}]^C$$
$$M^0 = M^C \quad \rightarrow \; C = 0$$
$$T^0 = T^{1-2B} \quad \rightarrow \; B = 1/2$$
$$L^0 = L^{A+B-3C} \quad \rightarrow \; A = -1/2$$

therefore, $\pi_4 = t \cdot \sqrt{\frac{g}{H}}$

And from Equation (5);

$$\pi_5 = \mu \times H^A \times g^B \times \rho^C$$
$$M^0L^0T^0 = [ML^{-1}T^{-1}] \times [L]^A \times [LT^{-2}]^B \times [ML^{-3}]^C$$
$$M^0 = M^{1+C} \quad \rightarrow \; C = -1$$
$$T^0 = T^{-1-2B} \quad \rightarrow \; B = -1/2$$
$$L^0 = L^{-1+A+B-3C} \quad \rightarrow \; A = -3/2$$

therefore, $\pi_5 = \frac{\mu}{H^{3/2}}$

The graph of the dimensionless water depth, $\frac{h}{H}$, as a function of dimensionless time, $tg^{\frac{1}{2}} / \; H^{\frac{1}{2}}$ for each of the four sets of data was plotted as shown below. Figures 2 and 4 for the bigger tank at 26.5°C and 43.5°C provided the same dimensionless water depth values and also the same dimensionless time values irrespective of the temperature difference.

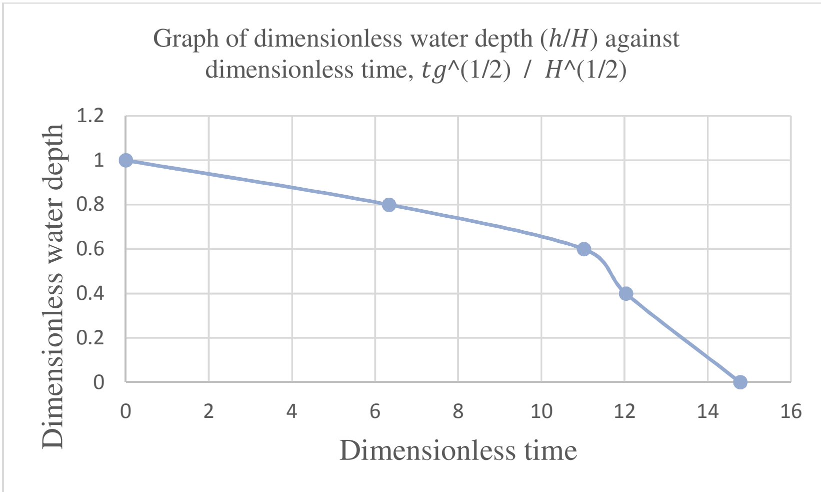

Figure 2: Dimensionless water depth against dimensionless time

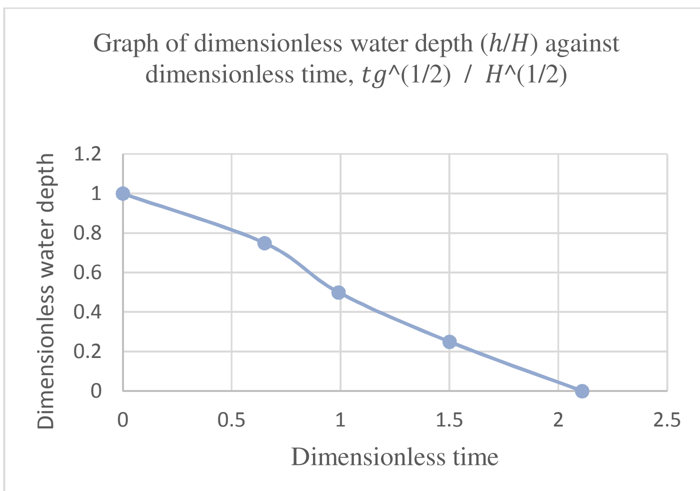

Figure 3: Dimensionless water depth against dimensionless time

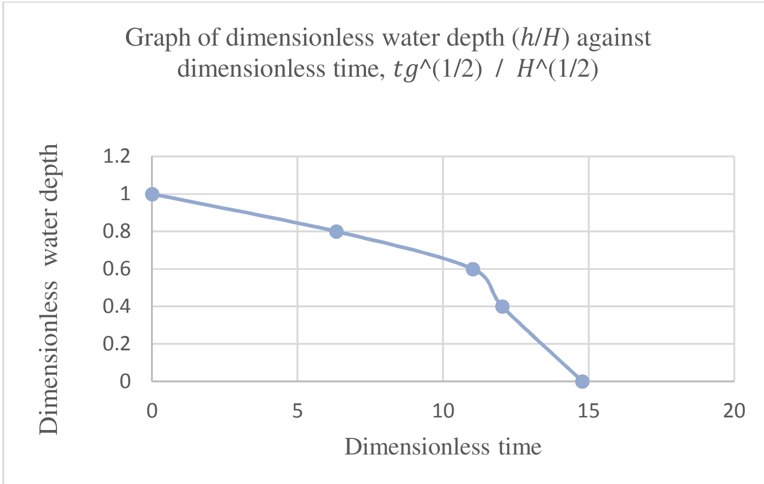

Figure 4: Dimensionless water depth against dimensionless time

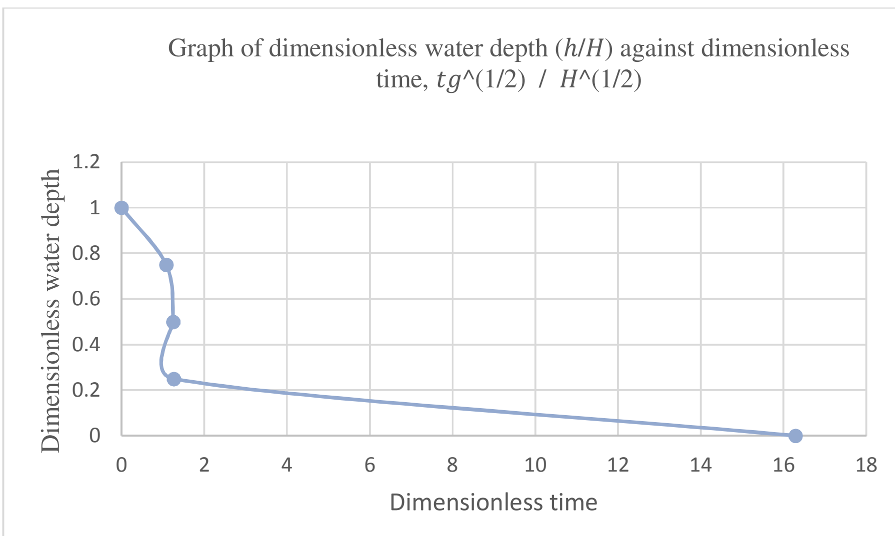

Figure 5: Dimensionless water depth against dimensionless time

Figures 3 and 5 above are the evaluated values of the Dimensionless water depth against dimensionless time. Results show that the π_1 and π_4 terms for the bigger tank are always greater than that of the small tank except where the h/H of the small and big tanks are at 1.00, which gave a 0.00 value for the π_4 terms in each case, as also shown in tables 3 and 4 respectively irrespective of the temperature(s) being used.

CONCLUSION

The results obtained from the two geometrically similar tanks allowing the fluid (water) to be completely drained from the tanks at two different temperature values of 26.5℃ and 43.5℃ respectively and at different time intervals using n relevant variables and m independent dimensions and reduced to a relationship between n – m non-dimensional parameters π_1, π_2,..., π_{n-m} provided a pi 1 (π_1) term of $\frac{h}{H}$ and a pi 4 (π_4) of $t \cdot \sqrt{\frac{g}{H}}$.

For the bigger tank at 26.5℃ and 43.5℃, the same values for the dimensionless water depth and dimensionless time at different times were recorded irrespective of the difference in temperature values applied. As the values of pi 1 (π_1) terms of the big tank are decreasing, the values of the pi 4 (π_4) terms are increasing in all cases.

REFERENCES

Ain A. Sonin. (2001). the Physical Basis of Dimensional Analysis (Second Edition).

Cimbala, John; Cengel, Yunus (2006). "Dimensional homogeneity" Essential of Fluid Mechanics; Fundamentals and Applications: McGraw Hill. P. 203 ISBN 9780073138350

Goldberg, David (2006). Fundamentals of Chemistry (5th ed.). McGraw Hill. ISBN 978-0-07-322104-5.

Grudzewski, W. M. & Roslanowska, K. (2013). Application of Dimensional Analysis in Economics. Amsterdam: IOS Press.

Kent's Chemistry page. Dimensional analysis or the Factor Label Method.

M. Bahrami. Dimensional Analysis and Similarity.

Ogden, James (1999). The Handbook of Chemical Engineering. Research and Education Association. ISBN 978-0-87891-982-6.

Texocotitla, M. A., Alvarez-Hernández, M. D., & Alvarez-Hernández, S. E. (2020). Dimensional Analysis in Economics: A Study of the Neoclassical Economic Growth Model. Journal of Interdisciplinary Economics, 32(2), 123–144. https://doi.org/10.1177/0260107919845269

Waite, Lee; Fine, Jerry (2007). Applied Biofluid Mechanics. New York: McGraw-Hill p. 260. ISBN 978-0-07-147217-3

White, F. M. (2011). Fluid Mechanics. 7th ed. New York: McGraw Hill.