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STOCK PRICES PREDICTION USING GEOMETRIC BROWNIAN MOTION: ANALYSIS OF THE NIGERIAN STOCK EXCHANGE

Agbam, Azubuike Samuel
Department of Banking and Finance,
Rivers State University, Nkpolu-Oroworukwo, Port Harcourt
Email: azubuikesamuelagbam@yahoo.com

ABSTRACT

In this study, the stochastic price movements of stocks are modeled by a geometric Brownian motion (GBM). The model assumptions of the GBM with drift: continuity, normality and Markov tendency, were investigated using four years (2015 - 2018) of historical closing prices of ten stocks listed on The Nigerian Stock Exchange. The sample for the study is based on the eight sectors of The Nigerian Stock Exchange and most continuously traded stocks. The predicted stocks prices have been compared to actual prices in order to evaluate the validity of the prediction model. On stocks prices prediction using geometric Brownian motion model, the algorithm starts from calculating the value of returns, followed by estimating values of volatility and drift, obtaining the stock prices forecast, calculating the forecast Mean Absolute Percentage Error, calculating the stock expected prices and calculating the confidence level at 95%. The results show that the value of the MAPE is 50% and below for the one to two year holding periods, and above 50% for the three year holding period. The MAPE and directional prediction accuracy method provided support that over short periods the GBM model is accurate. Meaning that the GBM is a reasonable predictive model for one or two years, but for three years, therefore, it is an inaccurate predictor.

Keyword: Stochastic forecasting, Geometric Brownian motion, Stochastic Differential Equation, Stock return, The Nigerian stock Exchange.

INTRODUCTION

The Stock Market is the meeting place of both buyers and sellers (wider domain of trading activities) of stocks. The investors seek to know the future price of their investment and the risk associated with such investment. This motivation is understandable since they demand the certain return of their investment (Bohdalova and Gregus, 2012).

Forecasting is the best method to predict the future of stock price (Omar and Jaffar, 2014). To forecast is to form an expectation of what will happen in the future. Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis

(Fama, 1995). Fundamental analysis intends to determine the value of a stock by focusing on underlying factors that affect a company's actual business and its future prospects (that the price of a stock depends on its intrinsic value and expected return on investment); while the technical analysis studies the price movement of a stock and predicts its future price movement (uses past stock prices and volumes; history will repeat).

There are alternative approaches to the forecasting of stock prices. One very prominent approach is the random walk theory which asserts that market prices of stocks exhibit random walk. The random walk theory is the idea that stocks take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk (Fama, 1970; 1991).

Stock price forecasting is widely accepted as a supporting tool for investment decision and investment risk management. There are many sophisticated techniques employed in fitting the actual and forecasted data. This study explores one of the techniques, namely the Geometric Brownian Motion (GBM) in predicting stock prices. Generally, GBM models in finance refer to the mathematical models used to describe those random movements in stock prices. The GBM model incorporates this idea of random walks in stock prices through its uncertain component, along with the idea that stocks maintain price trends over time as the certain component. Brewer, Feng and Kwan (2012) describe the uncertain component to the GBM models as the product of the stock's volatility and a stochastic process called Weiner process, which incorporates random volatility and a time interval. It assumes that the returns, profits or losses, on the stocks are independent and normally distributed.

The predictive power of the model is evaluated against the criterion: Mean Absolute Percentage Error (MAPE). Data used in this study are drawn from ten (10) companies taken from the different sectors of the Nigerian stock market, in the range of four (4) trading years (3,608 trading days) with the objective of evaluating the forecasting accuracy of using GBM model. This study will be beneficial in that, in trying to answer whether the results from the GBM prediction may help investor predicting prices, hence reducing investment risk in a specific period of time or not, it may help investors as one of the decision tools in modern day investment with big data and growing computing capacity.

The rest of the paper is structured as follows: Section 2 reviews the related literature. Section 3 explains the data and methodology. Section, 4 provides the empirical results and section 5 presents the conclusion and recommendations.

LITERATURE REVIEW

Theoretical Review: The Overview

Robert Brown (1827), the Scottish botanist, while looking through a microscope at particles trapped in cavities in pollen grains, noticed that the grains of pollen suspended in water had a rapid oscillatory motion. Brown published his observations, but was not able to determine the mechanisms that caused this motion. Einstein (1905) published his classic paper in which he explained in precise detail how the motion that Brown had observed was a result of the pollen being moved by individual water molecules. This phenomenon is now known as Brownian motion. The precise definition of Brownian motion is given later in the text but Wiener (1923) gave a formal mathematical theory on the subject and thus it is sometimes referred to as the Wiener process.

In forecasting movement of stock prices, Geometric Brownian Motion (GBM) is a mathematical technique exhibiting the fact of stochastic movement of stock prices. GBM is also being widely used modeling of stock price. Abidin and Jaffar (2012) applied the GBM to forecast future stock prices for the short-term investment in case of 24 Malaysian stocks. It was found that the result of the model was suitable for the short-term investment (a maximum of two week). Recently, Reddy and Clinton (2016) employed GBM in simulating 50 Australian stock prices. Using Capital Asset Pricing Model (CAPM) to estimate expected annual return, and standard deviation of the daily return of stock price for the empiric volatility in the simulation, it was found that over all time horizons the chances of a stock price simulated using GBM moving in the same direction as real stock prices (little greater than 50 percent).

Mathematical Formulation: Stock price modeling

In the real world of financial markets, investors and financial analysts are generally more interested in the gain or loss of the stock over a period of time i.e. the increase or decrease in the price, than in the price self. Therefore modeling the behavior of a stock price can be made through its relative change in the time. We shall therefore give a modeling of the stock return through a stochastic differential equation. The solution to the differential equation will be used to find a mathematical model of the stock price as shown below.

Stock Return

Like the particle being bombarded in the Brownian motion, stock prices deviate from a steady state as a result of being jolted by trades i.e. ask and bid in financial markets. If we consider a stock with price S_t at time t and an expected rate of return ☐, then the return or relative change in its price during the next period of time dt can be decomposed in two parts:

1. A predictable, deterministic and anticipated part that is the expected return from the stock hold during a period of time dt. This return is equal to $\mu S dt$.
2. A stochastic and unexpected part, which reflects the random changes in stock price during the interval of time dt, as response to external effects such as unexpected news on the stock, $\sigma S_t dB_t$.

The certain (predictable, deterministic and anticipated) component represents the return that the stock will earn over a short period of time, also referred to as the drift of the stock. The uncertain (stochastic and unexpected) component is a stochastic process including the stocks volatility and an element of random volatility (Sengupta, 2004). Brewer, Feng and Kwan (2012) showed that only the volatility parameter is present in the Black-Scholes (BS) model, but the drift parameter is not, as the BS model is derived based on the idea of arbitrage-free pricing. For Brownian motion simulations both the drift and volatility parameter are required, and a higher drift value tends to result in higher simulated prices over the period being analyzed (Brewer, Feng and Kwan, 2012).
This definition of the daily return leads to the stochastic differential equation followed by the stock price:

$$dS_t = \mu S_t dt + \sigma S_t dB_t \tag{2.1}$$

or

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t \tag{2.2}$$

The stochastic differential equation (2.1) is the Brownian motion with drift followed by the stock price S_t. The equation (2.2) is the instantaneous rate of return on S_t. The return on S_t in the period of time dt follows an Ito's process. It can also be written for every interval of time of length dt between two consecutive instants, as:

$$\frac{dS_t}{S_t} = d(InS_t) = In(S_t) - In(S_{t-1}) = In[\frac{S_t}{S_{t-1}}] \tag{2.3}$$

$$In\left[\frac{S_t}{S_{t-1}}\right] = udt + \sigma dB_t \tag{2.4}$$

That is:

$$S_t = InS_{t-1} + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\epsilon(\sqrt{dt}) \qquad (2\ 5)$$

Equation (2 5) has the solution given by:

$$S_t = S_{t-1}e^{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\epsilon(\sqrt{dt})} \tag{2.6}$$

Equation (2.6) is thus referred to as the geometric Brownian motion model of the future stock price S_t from the initial value S_0. Thus, for the time period t, when $dt = t$, equation (2.6) will therefore become:

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\varepsilon(\sqrt{t})} \tag{2.7}$$

Stochastic Differential Equation (SDE) and Geometric Brownian Motion (GBM)

In financial engineering, it is common to model a continuous time price process described by the Ito-Doob type stochastic differential equation (Elliott and Kopp, 1998; Etheridge, 2002; Ladde and Sambandham, 2004; Ladde. & Ladde, 2007a; and Ladde and

Ladde (2007b). A general stochastic differential equation takes the form:

$$S_t = \mu(S_t\ t)dt + \sigma(S_t;t)dW_t;\ S_{t0} = S_0; \qquad (2.8)$$

Here, $t = t_0$; W_t is a Brownian motion, and $S_t > 0$, h is the price process (Etheridge, 2002). In our previous study, Ladde and Wu (2009), initiated the usage of a classical modeling approach (Ladde & Ladde (2007b) to develop modified Geometric Brownian motion models for the price movement of individual stocks. GBM model is linear stochastic model, since the drift and volatility terms in the above equation are linear in terms of S (see Ladde & Wu, 2010).

Geometric Brownian Motion, (GBM) is a continuous-time stochastic process that satisfies the Stochastic Differential Equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dB_t$$

What the SDE essentially means in this study is that each increment in time results in the price of the stock moving with a drift ($\mu S_t dt$) and a shock ($\sigma S_t dB_t$). The drift can be seen as the general direction of the stock's price whereas the shock is a random amount of volatility that acts on the stock's price. The shock is what will create the curve's noise or jaggedness. To get the formula for the GBM we must find a solution to the SDE. It turns out that the SDE has an analytic solution under Ito's interpretation unlike other SDE's, defined as:

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t}$$

This is what in this study is used to model the prices of a stock. Geometric Brownian motion is defined as

$$S(t) = S_0 e^{X(t)},$$

Where $X(t) = \sigma W(t) + \mu t$ is a Brownian motion with deviation, $S(0) = S_0 > 0$.

Taking logarithm of the above equation, we have:

$$X(t) = In\left(\frac{S(t)}{S_0}\right) = In\big(S(t)\big) - In(S_0) \rightarrow In\big(S(t)\big) = In(S_0) + X(t).$$

Thus, $In(S(t))$ has normal distribution with mean $In(S_0) + \mu t$ And variance $\sigma^2 t$, and for each t value, $S(t)$ has lognormal distribution (Sigman, 2006).

If a Geometric Brownian motion is defined with differential equation $= rSdt + rSdW$, $S(0 = S_0)$, then Geometric Brownian motion is equal to :

$$S(t) = S_0 \exp(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W(t)$$

As Geometric motion has normal log distribution with parameters $In(S_0) + rt - \frac{1}{2}\sigma^2 t$ and $\sigma^2 t$, the mean and variance of geometric Brownian motion are given by

$$E\big(S(t)\big) = S_0 \exp(rt)$$
$$Var\big(S(t)\big) = S_0^2 e^{2rt}(e^{\sigma^2 t} - 1)$$

If the main issue is Geometric Brownian motion $S(t) = S_0 \exp(\mu t + \sigma W t)$, then its random differential equation formula is as follows:
$dS = \left(\mu + \frac{1}{2}\sigma^2\right)S(t)dt + \sigma S(t)dW, S(0) = S_0$

As Geometric Brownian motion has normal log distribution with parameters $In(S_0) + \mu t$ and $\sigma^2 t$, the mean of GBM is equal to $S_0 \exp(\mu t + \frac{1}{2}\sigma^2 t)$, and its variance is as per the following formular:

$$var\big(S(t)\big) = S_0^2 e^{2\mu t + \sigma^2 t}(e^{\sigma^2 t} - 1)$$

Technical Definition of Stochastic Differential Equation

Any variable whose value changes in an uncertain way is said to follow a stochastic process. Stochastic processes describe the probabilistic evolution of the value of a variable through time. A stochastic process S_t is said to follow a Geometric Brownian motion if it satisfies the following Stochastic Differential Equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \qquad (2.9)$$

where W_t is a Wiener process or Brownian motion, and μ (the percentage drift) and σ (the percentage volatility) are constants. The former is used to model deterministic trends, while the later term is often used to model a set of unpredictable events occurring during this motion.

Solving the Stochastic Differential Equation (SDE)

For any arbitrary initial value S_0 the above SDE has the analytical solution (under Ito's interpretation).

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right). \quad (2.10)$$

The derivation requires the use of Ito's calculus, applying Ito's formula leads to:

$$d(InS_t) = (InS_t)' dS_t + \frac{1}{2}(InS_t)' dS_t \quad (2.11)$$

$$\frac{dS_t}{S_t} - \frac{1}{2}\frac{1}{S_t^2} dS_t dS_t \quad (2.12)$$

Where $dS_t dS_t$ is the quadratic variation of the Stochastic Differential Equation.

$$dS_t dS_t = \sigma^2 S_t^2 dt + 2\sigma S_t^2 \mu dW_t dt + \mu^2 S_t^2 dt^2 \quad (2.13)$$

Where $dt \to 0$ dt converges to 0 faster than $d\mu t$, since $dW_t^2 = 0(dt)$.

So the above infinitesimal can be simplified by:

$$dS_t dS_t = \sigma^2 S_t^2 dt \quad (2.14)$$

Taking the exponents and multiplying both sides by S_0 gives the solution claimed above.

Properties

The above solution S_t (for any value of *t*) is a log-normally distributed random variable with expected value and variance given by:

$$E(S_t) = S_0 e^{\mu t} \tag{2.15}$$

$$Var(S_t) = S_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right). \tag{2.16}$$

They can be derived using the fact that

$$Z_t = \exp(\sigma W_t - \frac{1}{2}\sigma_t^2) \tag{2.17}$$

Is a martingale, and that

$$E[\exp(2\sigma W_t - \sigma_t^2)\,|F] = e^{\sigma^{2(t-s)}} \exp(2\sigma W_s - \sigma_s^2), \tag{2.18}$$

$$\forall 0 \leq s < t$$

The probability density function of S_t is:

$$fs\,t(s,\mu,\sigma,t) = \frac{1}{\sqrt{2\pi}} \frac{1}{s\sigma\sqrt{t}} \exp(\frac{InS - InS_0 - \left(\mu - \frac{1}{2}\sigma^2\right)}{2\sigma_t^2}) \tag{2.19}$$

When deriving further properties of GBM, use can be made of the SDE of which GBM is the solution given above can be used.

$$log(S_t) = \log(S_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma W_t)) \tag{2.20}$$

$$= \log(S_t) + \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t \tag{2.21}$$

Taking the expectation yields the same result as above

$$Elog(S_t) = \log(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)t \tag{2.22}$$

The Process for a Stock Price

It is tempting to suggest that a stock price follows a generalized Wiener process; that is, that it has a constant expected drift rate and a constant variance rate. However, this model fails to capture a key aspect of the stock prices. This is, that the expected percentage return required by investors from a stock is independent of the stock's price. If investors require a 14 percent per annum expected return when the stock price is N10, then, all things being equal, they will also require a 14 percent per annum expected return when it is N50.

Clearly, the assumption of constant expected drift rate is inappropriate and needs to be replaced by the assumption that the expected return (i.e., expected drift divided by the stock price) is constant. If S is the stock price at time t, then the expected drift rate in S should be assumed to be μS for some constant parameter μ. This means that in a short interval of time, Δt, the expected increase in S is $\mu S \Delta t$. The parameter μ is the expected rate of return on the stock.

If the coefficient of dz is zero, so that there is no uncertainty, then this model implies that:

$\Delta S = \mu S \, \Delta t$ in the limit, as $\Delta t \rightarrow 0$, so that:

$$dS = \mu S dt$$

or

$$\frac{dS}{S} = \mu dt$$

Integrating between time 0 and time T, we get

$$S_T = S_0 e^{\mu T} \tag{2.23}$$

Where S_0 and S_T are the stock price at time 0 and time T. Because we are using observations at intervals of τ measured in years, the estimate of the annualized volatility will be $\bar{\sigma} = \frac{s}{\sqrt{\tau}}$. This equation shows that, when there is no uncertainty, the stock price grows at a continuously compounded rate of μ per unit of time.

In practice, there is uncertainty. A reasonable assumption is that the variability of the return in the short period of time, Δt, is the same regardless of the stock price. In other words, an investor is just as uncertain about the return when the stock price is N50 as when it is N10. This suggests that the standard deviation of the change in a short period of time Δt should be proportional to the stock price and leads to the model:

$$dS = \mu S dt + \sigma S dt \tag{2.24}$$

Or

$$\frac{dS}{S} = \mu\, dt + \sigma\, dt \tag{2.25}$$

The variable μ is the stock's expected rate of return. The variable σ^2 is referred to as its variance rate.

Model for Price Development

We need only two assumptions concerning efficient market.

1. All the past history of the price development is reflected in the present price.
2. The response of the market to any new price to information is immediate.

Assumption (1) resembles a Markov property.

Let $\Delta t > 0$ and denote $\Delta P = P_{t+\Delta t} - P_t, P = P_t$ for a moment. P_0 having a starting price. in the model it is supposed that the return, $\Delta P/_P$ in our case, can be decomposed into a deterministic and a stochastic part in the following way:

$$\frac{\Delta P}{P} = \mu \Delta t + \sigma \Delta W \tag{2.26}$$

Here the first term $\mu \Delta t$ is the deterministic part, μ is called the drift or the trend coefficient while the second part is a stochastic term with so called volatility, standard error or diffusion σ and $\Delta W = W(t + \Delta t) - W(t)$ standing for the increment of a standard Wiener process. In more general models, both μ and σ may be also functions of $P$and t. Recall that the Wiener process $\{W(t), t \geq 0\}$ is a stochastic process with continuous trajectories such that $W(0) = 0$ with probability 1, for s, t positive the distribution of $W(t) - W(s)$

is normal N(0, |t-s|), and for any $0 < t_0 < t_1 < . . . < t_n < \infty$ the random variable $W(t_0)$, $W(t_1) - W(t_0)$. . . , $W(t_n) - W(t_{n-1})$ (the increments) are independent. Since the distribution of ΔW is $N(0, \Delta t)$, equation (2.25) may be written in the form:

$$\Delta P = \mu P \Delta t + \sigma P \varepsilon \sqrt{\Delta t} \qquad (2.27)$$

where ε is an $N(0,1)$ random variable so that the return $\Delta P/P$ possesses the normal distribution $N(\mu\Delta t, \sigma^2 \Delta t$. This model is useful for discrete modeling and simulation. Finally, for $\Delta t \to 0$, we obtain the stochastic differential equation (SDE).

$$\frac{dP}{P} = \mu dt + \sigma dW. \qquad (2.28)$$

This equation describes the geometric Brownian motion.

Expectation of a Geometric Brownian motion

In order to find the expected asset price, a Geometric Brownian Motion has been used, which expresses the change in stock price using a constant drift μ and volatility σ as a stochastic differential equation (SDE) according to Bjork (2009):

$$\{dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$
$$S(0) = s\} \qquad (2.29)$$

By integrating both sides of the SDE and using the initial condition, the solution to this equation is given by:

$$S(t) = s + \mu \int_0^t S(u)du + \sigma \int_0^t S(u)dW(u) \qquad (2.30)$$

Taking the expectation of both sides yield:

$$E[S(t)] = s + \mu \int_0^t E[S(u)]du + \sigma E[\int_0^t S(u)dW(u)] \qquad (2.31)$$

The expectation of the stochastic integral is simply zero. Substituting $E[S(t)] = m(t)$ and using the initial condition $m(0) = s$, we can express the equation as an ordinary differential equation, according to:

$$\{m'(t) = \mu m(t)$$
$$m(0) = s$$

Clearly, this simple ODE has the solution $m(t) = se^{\mu t}$. Therefore, the expectation of the stock price at time t is:

$$E[S(t)] = se^{\mu t} \qquad (2.32)$$

To find the solution $S(t)$ to the SDE, we can use the substitution $Z(t) = logS(t)$, since the corresponding deterministic linear equation is an exponential function of time. Ito's formula yields:

$$dZ = \frac{1}{S}dS + \frac{1}{2}\left(-\frac{1}{S^2}\right)(dS)^2 \quad (2.32)$$

$$= \frac{1}{S}(\mu Sdt + \sigma SdW) + \frac{1}{2}(-\frac{1}{S^2})\sigma^2 S^2 dt$$

$$= \left(\mu dt + \sigma dW(t)\right) - \frac{1}{2}\sigma^2 dt \quad (2.33)$$

So we have the following equation for $dZ(t)$:

$$\{dZ(t) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW(t) \quad (2.34)$$

$$\{Z(0) = \log s$$

By integrating both sides and substituting back $S(t)$ yields the solution:

$$S(t) = s\,\exp(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)) \quad (2.35)$$

Equivalently, we can express this equation as:

$$\log S(t) - \log s = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t) \quad (2.36)$$

2.10 The expected value of the stock

The expected stock price $E(S_t)$ is thus defined and denoted as:

$$E(S_t) = \exp[InS_{0+}\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma^2 t]$$

$$= \exp(InS_0)\exp\left(\mu + \frac{\sigma^2}{2}\right)t$$

So

$$E(S_t) = S_0\exp\left(\mu + \frac{\sigma^2}{2}\right)t \quad (2.37)$$

Equation (2.37) holds since,

$InS_t - N$(InS_0+(μ-σ^2/2)t,σ√t (2.38)

Stock Price Expected Value and Simulation Model

Let $S(t)$ denote the (random) price of the stock at time $t \geq 0$. Then, $S(t)$ has a normal distribution if $y = In\,S(t)$ is normally distributed. Suppose that $y \sim N(\mu, \sigma^2)$, then the pdf of $y$is given as:

$$f(y)\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-1}{2}(\frac{y-\mu}{\sigma})^2}, \quad y \in (-\infty, \infty)$$
$$\mu \, \epsilon(-\infty, \infty), \ \sigma > 0 \qquad (2.39)$$

We obtain the lognormal probability density function (pdf) of the stock price S_t considering the fact for equal probabilities under the normal and lognormal pdfs, their respective increment area should be equal.

$$f(y)dy = g(S_t)dS_t$$
$$\Rightarrow g(S_t) = \frac{f(g)dy}{dS_t} \qquad (2.40)$$

Substituting $y = InS_t$ & $dy = \frac{dS_t}{S_t}$ in equation (2.40) defines the pdf of S_t :

$$g(S_t) = \frac{1}{S_t \, \sigma\sqrt{2\pi}}e^{\frac{-1}{2}(\frac{InS_t-\mu}{\sigma})^2} \qquad (2.41)$$

Distribution Assumption

A Geometric Brownian Motion assumes the logarithmic change of the stock price to be a normally distributed random variable according to:

$$r_i = \log\left(\frac{S(t_i)}{S(t_{i-1}}\right) = \mu + \epsilon_i, \qquad \epsilon_i \sim Normal(0, \sigma^2) \qquad (2.42)$$

This assumption can also be tested against historical data. A modified distribution can be used to yield a better fit to the distribution of returns, as suggested by Dhesi, Shakeel and Xiao (2016).

Geometric Brownian Motion Formation

Let denote the stock price process by$S(t)$, the return on the stock by σ, then the return of the stock price is:

$$Return = \frac{change \ in \ price}{original \ price} \qquad (2.43)$$

Consider a small subsequent time interval $(t, t + \Delta t)$ during which $(S(t))$ becomes $(S(t + \Delta t) = S(t + \Delta S(t))$. The return on the stock price between time t and $t + \Delta t$ is given by:

$$Return = \frac{S(t+\Delta t) - S(t)}{S(t)} = \frac{\Delta S(t)}{S(t)} \qquad (2.44)$$

This is the return of the stock price. We model this return of the stock's price as akin to a two asset portfolio consisting of a non-risky asset such as a bond and a risky asset such as a derivative. If the return of the non-risky asset is μ then in a small time interval dt the return would be μdt. The return of the risky asset however is uncertain and this uncertainty or randomness is captured by the Brownian motion $B(t)$. In an infinitesimal time interval Δt the return of the asset is given by:

$$\frac{\Delta S(t)}{S(t)} = \mu\Delta t + \sigma dB(t) \tag{2.45}$$

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dB(t)$$

Or

$$dS(t) = \mu S(t)dt + \sigma S(t)dB(t) \tag{2.46}$$

where σ is the standard deviation of the stock return and depends on the random change in the asset price to external effects such as unexpected news. Equation (2.45) is referred to as the Geometric Brownian motion. In integral form:

$$S(t) = \mu \int_0^t S(u)du + \sigma \int_0^t S(u)dB(u) \tag{2.47}$$

$S(t)$) is said to be an Ito's process.

Derivation of Geometric Brownian Motion

Time series analysis is a popular data mining task for making the proper decisions based on relevant information in the financial sector today. Different types of artificial intelligence models can be seen in the literature. However, most of these models have been shown the poor realist approach to volatility modeling. As such Geometric Brownian motion approaches are generally used for predicting short-term and long term predictions in finance today (Omar and Jaffar, 2011; Abidin and Jaffar, 2012; and Breckling, 1989).

The closing price represents the most closed up-to-date valuation of stocks or indices until trading commences again on the next trading day. The volatility of the closing prices provides an uncertainty to invest capitals to shocks over the time. The daily closing price values can be converted into daily return (Seneviratne and Jianguo, 2013) as follows:

$$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} \quad (2.48)$$

Where: R_i denotes the market return index and S_t and S_{t-1} represents the asset values of the t^{th} day and their previous day respectively. Moreover, if the price on two consecutive days was not available, it would be assumed that the price remained unchanged and hence return is zero. The positive values of the return indices indicate the profit while the negative values represent the loss of investments. Besides that, higher rate of returns gave higher profit gaining.
According to the definition, the mean of return distribution of drift μ can be defined as follows ((Omar and Jaffar, 2011; Abidin and Jaffar, 2012; and Black and Scholes, 1973):

$$\mu = \ddot{R} = \frac{1}{M}\sum_{t=1}^{M} R_\iota \quad (2.49)$$

Where: number of returns in the sample defined by *M*. The volatility of the function is better indicator for measuring the functions of shares up and down. It can be used for measuring the risk level of the function. The volatility or sample standard deviation is given by equation (2.50) below:

$$\sigma = r = \sqrt{\frac{1}{(M-1)\delta t}\sum_{t=1}^{M}(R_i - \acute{R}\,)^2} \quad (2.50)$$

Solution to Geometric Brownian Motion

A unique solution to Equation (2.46) is obtained as follows:

Let $f(t = InS(t,)$, then $f'(t) = \frac{1}{s(t)}$ and $f''\big(S(t)\big) = -\frac{1}{s(t)^2}$

Hence

By Ito formula for the process

$$d\big(InS(t)\big) = f'dS(t) + \frac{1}{2}f''\sigma^2(t)dt$$

$$d\big(InS(t)\big) = \frac{1}{S(t)}dS(t) + \frac{1}{2}\left(-\frac{1}{S(t)^2}\right)\sigma^2(t)S(t)^2dt$$
$$d\big(InS(t)\big) = \big(\mu S(t)\mu t + \sigma S(t)dB(t)\big) - \frac{1}{2}\sigma^2dt$$
$$d\big(InS(t)\big) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dB(t)$$

Integrating both sides gives

$$InS(t) = InS(0) + \left(\mu - \frac{1}{2}\sigma^2\right)t + dB(t)$$

$$In(\frac{s(t)}{s(0)} = \frac{\sigma^2}{2} + \sigma B(t)$$

$$\frac{S(t)}{S(0)} = e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B(t)}$$

$$S(t) = S(0)e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B(t)}$$
$$S(t) = S(0)esp\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma dB(t)\} \qquad (2.51)$$

METHODOLOGY

Population and Sample Design

Population

As at May 31, 2018, The Nigerian Stock Exchange has 169 listed companies with a total market capitalization of over ₦13 trillion. All listings are included in The Nigerian Stock Exchange All- Shares index. In terms of market capitalization, The Nigerian Stock Exchange is the largest stock exchange in Africa.

The Nigerian Stock Exchange has a total of eight (8) sectors. The population of this study is made up of all the eight (8) sectors of The Nigerian Stock Exchange. Most listed companies have been reclassified for better grouping with the eight (8) sectors. And most of the old sectors have now been merged under one sector while others have been modified. These changes will enable more accurate market-related analyses at the local and global levels, including analysis of local economic sector performance. The changes will also facilitate the development of new investment instruments, such as indices, etc. Each sector is clearly defined and further divided into sub-sectors, which describe the nature of business activities.

The eight (8) sectors in The Nigerian Stock Exchange include the following:: 1. Financials. The financial sector consists of Banks, Investment funds, Insurance companies, among others. 2. Basic Materials. The utilities sector consists of electric, gas and water companies as well as integrated providers 3. Consumer Goods, 4. Consumer Services, 5. Energy (Oil & Gas) 6. Healthcare, 7. Industrials, and 8. Technology

Sample Size

The sample size is ten (10) stocks, and the sampling technique is modified judgmental sampling technique. In order to measure the daily returns of the Nigerian Stock Market (NSM), the daily index of the stock market proxied by NSM 10 share index were observed. Each of the 10 stocks is chosen from the eight (8) sectors of the Nigerian Stock Exchange. The remaining two (2) were taken from the financial and construction sectors. This is because the financial sector has the highest number (63) of quoted stocks, while construction has the most frequently traded stocks. The sample size is limited to ten (10) stocks because of the nature of the data. The data is so large (daily), and high frequency.

The ten (10) stocks include: OKOMU PALM OIL, J. BERGER, UPDC REAL ESTATE INVESTMENT TRUST, DANGOTE SUGAR, GUARANTEE TRUST BANK, AXAMANSARD INSURANCE (MANSARD), NEIMETH, CHAMS, DANGOTE CEMENT and SEPLAT.

Data Collection

Data was collected for 10 companies listed on the Nigerian Stock Exchange, daily stock price data was obtained from the exchange database over the period four (4). The start date for the simulation was 1st January, 2015, which was chosen to avoid any effects of seasonality in stock prices. The stock prices (open, high, low and close) of the companies are used as gathered data.

Model Specification

Application of geometric Brownian motion (GBM)

The GBM method is used to forecast stock prices of the selected companies using the Monte
Carlo simulation in the EXCEL software to determine the accuracy and effectiveness. For the GBM method, the procedures are as follows:

1. The data are tested for normality using the computer software.
2. The daily drift, daily volatility and the average drift are determined using the formula shown below:

$$Daily\ Rate\ of\ Return = \frac{Annual\ Rate\ of\ Return}{No.of\ trading\ days\ in\ the\ year} \quad (3.1)$$

$$Daily\ Volatility = \frac{Annual\ Volatility}{\sqrt{No.of\ trading\ days\ in\ a\ year}} \quad (3.2)$$

$$Average\ Drift = Daily\ Rate\ of\ Return - 0.5\ \text{x}\ Daily\ Daily\ Volatility^2 \quad (3.3)$$

3. The value of the random number generated from probability distribution, ε, is determined using the EXCEL function of NORM.S.INV (RAND). This function gives a random number from the normal distribution table.
4. Once all variables are known, the future stock value is determined using the Geometric Brownian motion formula as shown below:

$$S(t) = S(0)e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\varepsilon(t)} \quad (3.4)$$

Where:
$S(t) =$ future stock value
$S(0) =$ initial stock value
μ = daily drift
σ = daily volatility
ε = value from probability distribution

Analytical Layout of Geometric Brownian motion

Statistical Layout of Geometric Brownian motion

Let Ω be the set of all possible outcomes of any random experiment and the continuous time random process X_t, defined on the filtered probability space $(\Omega, F, \{F_t\}_{t\epsilon T}, P)$.

where, F is the $\sigma - algebra$ of event,

$\{F_t\}_{t\epsilon T}$ denotes the information generated by the process X_t over the time interval [0,T].

P is the probability measure.

Definition: A random variable 'X' have the lognormal distribution with parameters μ and σ if log (X) is normally distributed. i.e.,

$$\log(X) \sim N(\mu, \sigma^2)$$

Definition: A real valued random process $W_t = W(t, w)$ on the time interval [0,∞] is Brownian Motion or Wiener Process if it satisfies following conditions (Karlin & Taylor, 2012 and Ross, 1996).

1. Continuity: $W_0 = 0$
2. Normality: for $0 \leq s < t \leq T, W_t - W_s \sim N(0, t - s)$
3. Markov Property: For $0 \leq s' < t' < s < t \leq T, W_t - W_s$ $independent\ of\ W_{t'} - W_{s'}$

A stochastic process S_t is used to follow a Geometric Brownian Motion if it satisfies the following stochastic differential equation.

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where, W_t is a Wiener process (Brownian Motion) and μ & σ are constants.

Normally, it is called the percentage drift and σ is called the percentage volatility. So consider a Brownian Motion trajectory that satisfies this differential equation. The right hand side term $\mu S_t dt$ controls the trends of this trajectory and the term $\sigma S_t dW_t$ controls the random noise effect in the trajectory.

After applying the technique of separation of variable, the equation becomes:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

Taking integration of both side

$$\int \frac{dS_t}{S_t} = \int (\mu dt + \sigma dW_t) dt$$

Since $\frac{dS_t}{S_t}$ relates to derivative of In(S_t) the $It\bar{o}$ calculus becomes:

$$\text{In}\left(\frac{dS_t}{S_t}\right) = \left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$$

Taking the exponential in both sides and plugging the initial condition S_0, the analytical solution of Geometric Brownian Motion is given by:

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$$

The constants μ and σ are able to produce a solution of Geometric Brownian Motion throughout time interval. For given drift and volatility the solution of Geometric Brownian Motion in the form:

$$S_t = S_0 \exp[x(t)]$$

where, $x(t) = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t$

Now the density of Geometric Brownian Motion is given by

$$f(t,x) = \frac{1}{\sigma x (2\pi t)^{1/2}} \exp[-(\log x - log x_0 - \mu t)]/2\sigma^2 t$$

Sample Estimation:

Suppose that a set of input: $t_1, t_2, t_3 \ldots \ldots \ldots \ldots$ and a set of corresponding output: $S_1, S_2, S_3 \ldots \ldots \ldots \ldots$ from S_t and the set of data is in the mle function $L(\theta)$. Since Geometric Brownian Motion is a Markov Chain Process

$$L(\theta) = f_\theta(x_1, x_2, x_3 \ldots \ldots \ldots) = \prod_{i=1}^{n} f_\theta(x_i)$$

Now taking derivative of the right hand side, we get

$$\bar{m} = \sum_{i=1}^{n} \frac{x_i}{n}$$

$$\bar{v} = \sum_{i=1}^{n} \frac{(x_i - m)^2}{n}$$

where $\bar{m}$ and $\bar{v}$ are the mle of m and v respectively and $x_i = \log S(t_i)$-log S$(t_i - 1)$

Mathematical Layout of Geometric Brownian Motion

Suppose X is a continuous random variable follow lognormal distribution, then $v = InX$ is a random variable which is normally distributed with mean μ and variance σ^2. Symbolically:

$$v = InX \sim N(\mu, \sigma^2)$$

The probability density function from variable v becomes:

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}(\frac{v-\mu}{\sigma})^2] \, for - \infty < v, \mu < \infty \, and \, \sigma > 0$$

$$for \, v = InX, dv = d(InX) = \frac{1}{x} dX$$

and

$$h(x) = \frac{f(v)dv}{dx} = \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}(\frac{Inx-\mu}{\sigma})^2] \frac{1}{x} dx$$

thus, probability density function becomes:

$$h(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp[\frac{-1}{2}(\frac{Inx-\mu}{\sigma})^2]; x > 0$$

where μ and σ^2 represents mean and variance of the lognormal variable x.

Now,

$E(x) = \int_{-\infty}^{\infty} xh(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}(\frac{Inx-\mu}{\sigma})^2] \, dx$

(M4.1)

If $y = Inx - \mu, then \, dy = \frac{1}{x} dx \, and \, equation (4.1) becomes$

$$E(e^{y+\mu}) = \int_{-\infty}^{\infty} \frac{1}{\sigma x\sqrt{2\pi}} \exp(y+\mu) \exp[\frac{-1}{2}(\frac{y}{\sigma})^2] \, dy$$

$= e^{\mu} e^{\frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}(\frac{y-\sigma^2}{\sigma})^2] \, dy$

(M3.4.2)

$If \, z = \frac{y-\sigma^2}{\sigma}, then \, dz = \frac{1}{\sigma} dy,$

and equation (M4.2) becomes:

$E[e^{z\sigma+\sigma^{2+\pi}}] = \exp(\mu + \frac{\sigma^2}{2}) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-Z^2}{2}\right] dz$

(M3.4.3)

The integral part of the above equation is a probability density function of standard normal distribution subject to the conditions integral $Inx = -\infty$ becomes $y = -\infty$ and $Inx = +\infty$ becomes $y = +\infty$ in equation (M4.2) and integral $y = -\infty$ becomes $z = -\infty$ and $y = +\infty$ becomes $z = +\infty$ in equation (M4.3).

Now the expected stock price:

$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma B_t\right]$

(M3.4.4)

is called the Geometric Brownian Motion with drift.

Where,

S_0 = Actual beginning stock price
μ = Mean of lognormal distribution
σ^2 = Variance of lognormal distribution
B_t = Brownian Motion at time 't' with $\mu = 0$ and defined as,
$B_t = \mu t + \sigma W_t$
(M3.4. 5)
where, W_t = Wiener process at time 't'.
Now $E(B_t) = \mu t + E(\sigma W_t) = \mu t \; as \; E(W_t) = 0 \; and$

$$Var(B_t) = E(B_t)^2 - [E(B_t)^2] = \sigma^2 t$$

Hence Brownian Motion with drift is normally distributed with mean μt and variance $\sigma^2 t$. Symbolically:

$$B_t \sim N(\mu t, \sigma^2 t)$$

Thus

$$In\, S_t \sim N(InS_{t-1} + \left(\mu - \frac{1}{2}\sigma^2\right) t, \sigma^2 t)$$

Hence expected stock price at time 't' for future stock is

$$E(S_t) = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t\right] and$$

$$Var(S_t) = S_0^2 \exp(2\mu + \sigma^2 t)\,[\exp(\sigma^2 t) - 1]$$

With 95% confidence interval, S_t becomes:

$$\exp\left[InS_0 + \left(\mu - \frac{1}{2}\sigma^2\right) t - 1.96\sigma\sqrt{t}\right] \le S_t$$
$$\le \exp\left[InS_0 + \left(\mu - \frac{1}{2}\sigma^2\right) t + 1.96\sigma\sqrt{t}\right]$$

Model Assumptions

Generally, the returns can be assumed as a random variable, which has closed enough to a normal distribution with a non-zero mean and standard deviation. So the distribution of asset returns can be defined as follows:

$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} = Mean + Standard\ deviation\ x\ \varphi$
(3.5)

Statistically, it is difficult to measure mean scale of the distribution with the smaller parameter δt which represents the time gap between assets. The drift rate or growth rate μ of the distribution can be assumed as a constant and defined as equation (3.6).

Mean=$\mu\delta t$ (3.6)

The volatility (standard deviation) of the distribution is significant and elusive quantity in the theory of derivatives. The standard deviation of the asset returns over a time step δt is given as equation (3.7), (Chang, Lima and Tabak, 2004).

Standard deviation = $\sigma\delta t^{1/2}$
(3.7)
Putting these scalings explicitly from equations (3.6) and (3.7) into asset return model represents in equation (3.5), then, we have:

$$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} = \mu\delta t + \sigma\delta t^{1/2} \quad (3.8)$$

The term dw_i be a random variable, from normally distribution with mean zero and variance δt. So equation (3.8) can be simplified as follows:

$$dS_t = \mu S_t \delta t + \sigma S_t dw_t \quad (3.9)$$

Integrating equation (3.7) with respect to t;

$$\int_0^t \frac{dS_t}{S_t} = \mu t + \sigma w_i \quad (3.10)$$

Where; let we assume that, $w_0 = 0$. It is clear that, term S_i under the Ito process. So we used Ito's calculation for our further study,

$$d(InS_t) = \frac{dS_t}{S_t} - \frac{1}{2}\mu^2 dt \quad (3.11)$$

Now, substitute equation (3.9) into the equation (3.7) we get;

$$d(InS_t) = \mu dt + \sigma dw_i - \frac{1}{2}\sigma^2 dt \quad (3.12)$$

Integrating both sides with respect to t, we get:

$$S_t = S_0\, esp[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma w_i] \quad (3.13)$$

Where, $w_t = X_t - X_0$. Equation (3.12) indicated continuous stochastic process of Geometric motion that we used for simulation for the forecast of stock market indices.

Model Accuracy Testing

Time series forecasting considered as a technique which can be used for predicting future aspects of many operations. Numerous methods have been carried out by many research works to accomplish their goals. In this study, Mean Absolute Percentage Error (MAPE) is used to compare the prediction accuracy of the model. The accuracy model as defined by Wang, Wang and Zhang (2012) is as follows:

$$\varepsilon_{MAPE} = \frac{1}{M}\sum_{j=1}^{M}\frac{|X_A - X_P|}{X_A} \quad (3.14)$$

Where X_A and X_P represent the actual value and predicted value of the indices respectively. The table below represents the scale of judgment of forecast accuracy regarding error (MAPE) and clearly indicated that minimum values of MAPE make more accuracy for forecasting future prediction (Omar and Jaffar, 2011).

According to Lawrence, Klimberg and Lawrence (2009), there are three measurements of forecasting model which involve time period, t. The measurements are number of period forecast, (n), actual value in time period at time, (t, Y_t) and forecast value at time period $t, (F_t)$. They are widely used to evaluate the forecasting method that considers the effect of the magnitude of the actual values (Abidan and Jaffar, 2012).

DATA ANALYSIS, RESULT AND FINDING

The focus of this study is the prediction of prices of some randomly selected stocks in the Nigerian equity market based on Geometric Brownian Motion (GBM) model. We tested a hypothesis using GMB model. This hypothesis is as follows:
To test whether there is significant difference between simulated and actual prices for individual stocks.

Descriptive Statistics

Table 4.1 reports the descriptive statistics for each stock including the name of the stock, the year, initial stock price, market capitalization at the time of writing, annualized return and annualized volatility.

Table 4.1 Estimated value of drift and Volatility

Company	Year	So	Capitalization	Annualized Return	Annualized Volatility
Okomuoil	2015	30.6	11679481.8	0.201747818	0.002085833
Okomuoil	2016	60.63	12768514.7	0.858167364	0.001451518
Okomuoil	2017	92	30959838.65	1.768125314	0.00225967
Okomuoil	2018	49	27954536.52	-0.90970765	0.003011194
Jberger	2015	43.1	47776120.23	-0.487998721	0.153089828
Jberger	2016	38	4948270.325	-0.000253937	0.001296546
Jberger	2017	30	9983149.129	-0.57234506	0.002245321
Jberger	2018	20.6	9737483.554	-0.731399226	0.004290049
Updcreit	2015	10	990908.3816	0	0
Updcreit	2016	10	708549.5238	0	0
Updcreit	2017	10	867866.9231	0	0
Updcreit	2018	5.4	499356.3333	-0.99999375	0.003363608
Dangsugar	2015	5.52	17681535.24	-0.084191379	0.001701482
Dangsugar	2016	9.3	9629526.42	0.666211326	0.001874139
Dangsugar	2017	17.5	75703292.83	0.82637403	0.001830949
Dangsugar	2018	9.6	16702210.39	-0.519080058	0.002068239
Guaranty	2015	15.99	545595301.9	-0.359589531	0.002028295
Guaranty	2016	33.55	412998572.8	1.18524257	0.001585651
Guaranty	2017	40.1	895998641.3	0.000176975	0.001158893
Guaranty	2018	26	710435945.1	-0.35556623	0.00146611
Mansard	2015	2.3	13084395.31	-0.266938174	0.001657355
Mansard	2016	2.29	954959.179	-0.004347864	0.001932512
Mansard	2017	2.55	5331427.502	0.170818332	0.002197771
Mansard	2018	1.8	8599281.444	-0.207628128	0.003793643
Neimeth	2015	0.86	343222.8042	0.146356816	0.001868705
Neimeth	2016	0.71	232844.9087	-0.195603037	0.00236049
Neimeth	2017	0.52	438604.4852	0	0.002954215
Neimeth	2018	0.51	362793.479	-0.047760146	0.004473222
Chams	2015	0.5	1481231.86	0	0.000162404
Chams	2016	0.5	560262.2438	0	0
Chams	2017	0.37	305429.9333	-0.927934342	0.001203156
Chams	2018	0.23	2615628.584	-0.257141801	0.006
Dangcem	2015	129.83	148378478.3	-0.311793641	0.001393708
Dangcem	2016	205	126932656.5	0.586666955	0.001371768
Dangcem	2017	227	275003312.8	0.123234556	0.001388337
Dangcem	2018	164	248703672.6	-0.318474196	0.001348321
Seplat	2015	170.88	41901472.8	-0.508837709	0.001606948
Seplat	2016	425	146836758.9	1.349137638	0.002183293

Seplat	2017	635	76893252.57	0.835266275	0.001511631
Seplat	2018	490	108699817.8	-0.599309616	9.84921e-05

Source: Author

Table 4.1 exhibits for each period the annualized return in column 5. Average capitalization and initial prices are in columns 4 and 3 respectively. Annualized volatility of the stock obtained by applying the GBM formula is displayed in column 6 of the table. The table provides very important information about the behavior of the stocks. The drift or assumed annual expected return of the stocks differs from one period to another and across companies. For instance, Okomuoil had positive expected annualized return for years 2015 to 2017, but in 2018, the company's expected return was negative. Updcreit has zero annualized return for years 2015 to 2017 and negative return in 2018. All the companies have negative expected annualized return in 2018. J-Berger is the worst performed company among them because it has negative return through the whole period. Seplat has the highest return in 2016 (about 135 percent), while, Okomuoil has the highest in 2017. The most capitalized companies are Dang-Sugar, Dang-Cement, Guaranty and Seplat and the least capitalized ones are Mansard, Neimeth and Chams. Initial market prices are low for Chams, Neimeth, Mansard and Updcreit. However, the lowest volatile company is Updcreit. The behaviour of the stock price is therefore significant in the prediction of future stock prices.

We have been able to estimate the future expected return and study the properties underlying the GBM for the selected stocks, but we have not tested whether the theoretical model (that is, GBM model) truly reflects the actual reality of the stock prices. Thus, we proceed to the test of the proposed hypotheses of the study.

The unit period of time, which we used in this simulation is of one-day length (1 day =1/252 ≈0.004 year). On the average there are 252 trading days in the year. The value of the stock volatility and its drift can be respectively estimated according to the general GBM model. The results are shown in table 4.2

Test of hypothesis

Hypothesis - The Actual Prices of the Individual Stocks are not significantly different from the Stimulated Prices.

This test is an attempt to investigate whether or not there is a significant difference between actual stock prices and those simulated using the GBM model for individual companies. Table 4.2 provides the outputs of this preposition.

Comparing the actual and simulated prices yields different results.

Table 4.3 Actual and Stimulated Prices of Individual Stock

		2015		2016		2017		2018	
S/N	COMPANY	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED
1	OKOMUOIL	30.6	37.44156	60.63	143.00352	92	539.233139	49	19.7287175
2	JBERGER	43.1	26.505397	38	37.988802	30	16.9257434	20.6	9.91203821
3	UPDCREIT	10	10.039273	10	9.9850189	10	10.9123159	5.4	1.9864749
4	DANGSUGAR	5.52	5.0737467	9.3	18.104468	17.5	39.9857689	9.6	5.71211327
5	GUARANTY	15.99	11.157865	33.55	109.75317	40.1	40.1046041	26	18.220356
6	MANARD	2.3	1.7615778	2.29	2.2795416	2.55	3.02526241	1.8	1.46242446
7	NEIMETH	0.86	0.9955201	0.71	0.583913	0.52	0.52004551	0.51	0.53518181
8	CHAMS	0.5	0.6107052	0.5	0.5039048	0.37	0.61070742	0.23	0.3866218
9	DANGCEM	129.83	95.050742	205	368.60188	227	256.782998	164	119.275644
10	SEPLAT	170.88	102.73047	425	1637.8678	635	1463.97478	490	269.103828

Source: Author

As shown in the table, the predicted prices are yielded by the theoretical model (GBM) and in comparison with the actual stock prices; we observe cases of significant differences. For instance, there are significant differences in Okomuoil's actual and stimulated prices for the period 2016, 2017 and 2018. In 2015, the difference is not too obvious (we record about N6.84). J-Berger reveals a difference between the actual and stimulated prices for year 2015, 2017 and 2018, but in 2016, the stimulated price and actual price are approximately at par. There is no difference between the approximate predicted prices and actual prices for Updcreit in period of 2015 to 2017; however, much disparity is noted in 2018 where the actual price is N5.4 and predicted is N1.97. For Dang-Sugar and Guaranty Trust Bank, there is

difference between the actual and predicted price for year 2015 and 2017 respectively, but for other years, much differences are observed. Mansard, Neimeth and Chams do have significant difference between the actual and predicted prices for all the periods under investigation, meaning that the theoretical model significantly reflects the actual reality of the stock prices of these companies. Lastly, Dang-Cement and Seplat yield actual prices that are significantly different from the stimulated prices throughout the sample period.
However, the summary results of test one are presented in table 4.3

Table 4.3 Preposition Summary Results

	Correlation			Mean Absolute Percentage Error		
	12-Month	24-Month	36-Month	12-Month	24-Month	36-Month
Mean	-0.0149	0.000921	0.170405	0.502666	0.462861	0.590441
mean-absolute	0.014902	0.000921	0.170405	0.502666	0.462861	0.590441
Median	-0.01481	5.89E-05	0.366873	0.603449	0.393031	0.514993
median-absolute	0.014812	5.89E-05	0.366873	0.603449	0.393031	0.514993
Range	0.238638	1.788266	1.568857	0.913048	0.80997	0.93167
Min	-0.14158	-0.91565	-0.73448	0.008695	0.047987	0.14347
Max	0.097057	0.872616	0.834376	0.921742	0.857958	1.075141
standard-deviation	0.075004	0.734138	0.536576	0.328501	0.267266	0.336565

Note that the MAPE value between 1% and 10% indicates accurate forecast, 11% and 20% shows good forecast, 21% and 50% deemed reasonable and above 50% is inaccurate forecast. (see, **Abidin & Jaffar, 2014)**
Source: Author

Table 4.3 is the summary results of the individual stocks, which are based on the test of the first preposition. As shown in the table, the mean-correlation value for the shortest holding period (12 months) is negative, but as the holding period of the stocks increases to 24 and 36 months, the mean-correlation value becomes positive. This implies that predicted values of these stocks move in the opposite direction in the short-term holding period, but as the holding period increases, the predicted stock prices on an individual basis converge to real stock prices. This position is also confirmed by the median; which records negative value for short holding period and positive for long holding period. The absolute median and standard deviation are smaller for the short holding period than for the long holding period. This suggests that the correlation between the predicted prices and the actual prices is less volatile in the short holding period than in the long holding period. Alternatively, the relationship between the stimulated value and real value of these stocks is more stable in the one-year holding period than in the two to three year holding periods. The value of the MAPE is 50% and below for the one to two year holding periods, and above 50% for the three year holding period. Meaning that the GBM is a reasonable predictive model for one or two years, but for three years; therefore it is an inaccurate predictor.

DISCUSSION OF FINDINGS

Most of the prior studies have suggested that there is a weak relationship between the two variables. We have reported a negative correlation during short periods of simulation, which becomes positive with longer forecast horizons. Noise or volatility in the market makes simulated stock price and actual stock price to have a negative correlation in the short term, whereas stock prices stabilize to its mean value in the longer run causing a positive correlation between simulated and actual prices.

To give more confident to the researchers, we measured the accuracy of the forecast model by looking at the MAPE value. Table 4.3 shows the forecast price and actual price from 2015 to 2018. The value of the MAPE is 50% and below for the one to two year holding periods, and above 50% for the three year holding period. Meaning that the GBM model is a reasonable predictive model for one or two years, but for three years, it is an inaccurate predictor. It was found that MAPE was lowest over simulation periods of one-holding period and two-holding period, but the error tended to increase when longer horizons were considered.

The absolute median and standard deviation are smaller for the short holding period than for the long holding period. This suggests that the correlation between the predicted prices and the actual prices is less volatile in the short holding period than in the long holding period. Alternatively, the relationship between the stimulated value and real value of these stocks is more stable in the one-year holding period than in the two to three year holding periods.

Summary of findings

We examined whether there is significant difference between actual and predicted stock prices based on individual stocks. The GBM results in this context reveal that in 2015, 70 percent of the selected stocks have actual prices that are approximately equivalent to the simulated prices, while 30 percent have significant differences. Furthermore, 50 percent, 60 percent and 40 percent of the companies show actual prices that are similar to the simulated prices in 2016, 2017 and 2018 respectively, while in these same periods 50 percent, 40 percent and 60 percent reveal actual prices that are far below the predicted prices.

CONCLUSION

Simulations show a slightly high percentage (therefore reasonable forecast) especially in the longer period of evaluated timeframe, in this case three years; that is to say, the longer the evaluating period, the higher the MAPE (the lower the accuracy).

RECOMMENDATIONS

Practically feasible is that the technician whose primary motive is to make gain at the expense of other participants should identify high volatile stocks in any holding period for effective prediction. Regulatory authorities in the market should focus on long-term stability by holding reserves that can cushion off the effects of illiquidity, particularly in the time of extreme turbulence.

REFERENCES

Abidin, S. & Jaffar, M.M. (2012). A review on geometric Brownian motion in forecasting the share prices in Bursa Malaysia. *World Applied Sciences Journal*, **17**, 87-93.

Black, F. & Scholes, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy*, 81(3):637-654.

Bohdalová, M., & Greguš, M. (2012) *Stochastic analysis of the financial markets*. Bratislava:

Comenius University Press.

Breckling, J. (1989). The analysis of directional time series: Applications to wind speed and direction; *Series Lecture Notes in Statistics, Berlin,* Germany: Spring.

Brewer, K., Feng, Y., & Kwan, C. (2012). Geometric Brownian motion, option pricing, and simulation: Some spreadsheet-based exercises in financial modeling. *Spreadsheets in Education, 5*(3), article 4. Retrieved from http://epublications.bond.edu.au/ejsie/

Brown, R. (1827), A brief account of microscopical observations made on the particles contained in the pollen of plants, *London and Edinburgh Philosophical Magazine and Journal of Science, 4,* 161-173

Chang, E. J., Lima, E. J. A. & Tabak, B. M. (2004). Testing for predictability in emerging markets. *Emerging Markets Review 5:* 295-316.

Dhesi, G., Shakeel, M. B. & Xiao, L. (2016). Modified Brownian motion approach to modeling returns distribution. *Wilmott Magazine 82*: 74-77.

Einstein, A. (1905). Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendiernten Teilchen, *Annalen der Physik,* **17**, 549 – 560;

Elliott, R. J. & Kopp, P. E. (1998). *Mathematics of Financial Markets*, Springer Verlag, New York,

Ermogenous, A. (2005). Brownian motion and its application in the stock market. Retrieved from http://academic.udayton.edu/EPUMD/Volume%202_2006/brownian2.pdf - 694.

Etheridge, Alison (2002). *A course in Financial Calculus*, Cambridge, Cambridge, 2002.

Fama, E. F. (1970). Efficient capital market: A review of theory and empirical work. *The Journal of Finance, 25(2):* 383-417. Papers and proceedings of the twenty-eighth Annual Meeting of the American finance Association New York, N. Y. December, 28-30, 1969, Published by Blackwell Publishing for the American Finance Association.

Fama F.E. (1991). Efficient capital market II. *Journal of Finance, 46(5)* 1575 –1617

Fama, F. E. (1995). Random walks in stock market prices. *Financial Analysts Journal, 51(1),* 75- 80. http://dx.doi.org/10.2469/faj.v51.n1.1861

Ladde, G. S. & Sambandham, M. (2004). *Stochastic Versus Deterministic Systems of Differential Equations*, AMarcel Dekker, Inc., New York.

Ladde, G.S. and L. Wu, 2009. Development of modified geometric Brownian motion models by using stock price data and basic statistics nonlinear analysis: *Theory, methods and applications, 71 (12):* 1203-1208.

Ladde, A. G. & Ladde, G. S. (2007a). Dynamic Processes Under Random Environment, *Bulletin of the Marathwada Mathematical Society* 1(8)2:96-123.

Ladde, A. G. & Ladde, G. S. (2007b). *An Introduction to Differential Equations-II: Stochastic Modeling, methods and Analysis,* in press.

Ladde G. S. & Wu, L. (2010). Development of nonlinear stochastic models by using stock price data and basic statistics. Neural, Parallel, and Scientific computations 18: 269-282

Lawrence, K. D; Klimberg, R. K. & Lawrence, S. M. (2009). *Fundamentals of Forecasting Using* Excel, Industrial Press Inc.

Merton, Robert C. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science, 4(1):* 141-183

Omar, A., & Jaffar, M.M. (2011). Comparative analysis of geometric Brownian motion model in forecasting FBMHS and FBMKLCI Index in Bursa Malaysia, IEEE symposium on Business, Engineering and industrial applications, , Langkawi, Malaysia, 157-161.

Omar, A., & Jaffar, M.M. (2014). Forecasting share price of small size companies in bursa Malaysia using geometric Brownian motion. *Journal International Journal of Applied Mathematics & Information Sciences 8(1): 107-112.* Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Selangor, Malaysia.

Reddy, K. & Clinton V. (2016). Simulating stock prices using geometric Brownian motion:

Evidence from Australian companies, *Australasian Accounting, Business and Finance Journal,* 10(3) 23-47

Seneviratne, D. M. K. N. & Jianguo, W. (2013). Analyzing the causal relationship between stock prices and selected macroeconomic variables: Evidence from Sri Lanka, *ZIJBEMR 3(9):* 1-110.

Sengupta, C. (2004). *Financial modeling using Excel and VBA.* New Jersey, United States: John Wiley & Sons.

Sigman, K. (2006). Stationary marked point processes, in Springer Handbook of Engineering statistics, Springerlink.

Wang, J. J; Wang, J. Z: Zhang, A. & and Guo, S. P. (2012). Stock index forecasting based on hybrid model, *Omega, 40:* 758-766.

Wiener, N. (1923) Differential-space, *Journal of Mathematics and Physics Massachusetts Institute of Technology*, **2**, 131-174; 1923.