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EFFICIENCY OF ORDINARY LEAST SQUARES IN REGRESSION MODELS WITH AUTOCORRELATED DISTURBANCES IN A CLASSICAL LINEAR STATISTICAL MODEL.

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ABSTRACT

The illustrative values of the asymptotic efficiency for the selected values of ρ and λ shows that, when ρ and λ are both positive, it is clear, that ρ is the dominant parameter. Efficiency declines from 90 percent to about, 10 percent as ρ rises from 0.2 to 0.9, with variations in λ having relatively minor effect. The diagonal entries are equal to those in the first row of table (2) since if $\lambda = 0$ or if $\rho = \lambda$, the efficiency measure simplified to $(1 - \rho^2)/(1 + \rho^2)$. Looking at the left hand side of the table, the efficiencies are symmetrical across the first row where the x_t series are random. The remaining rows show that λ now exerts a much stronger effect and that the combination of a positive λ and negative ρ can moderate the dramatic declines in efficiency shown in the right – hand side of the table. These calculations are of course, only illustrative, but, they indicate the possibility of a serious loss in efficiency if Ordinary Least Squares is applied in the context of autocorrelated disturbances.

Keywords: *Efficiency, Ordinary Least Squares, Autocorrelated disturbances.*

INTRODUCTION

One of the drawbacks of the least squares, when the error terms are autocorrelated is that there is loss in efficiency of ordinary least squares. It has been known that the efficiency of Ordinary Least Squares in a linear regression containing an autocorrelated disturbance term depends essentially on the structure of the matrix of observation on the independent variables. If this matrix is allowed to vary arbitrarily, the efficiency of the least squares relative to the Gauss-Markov with known, autocorrelation coefficient can be made arbitrarily close to zero. But if the column of observations on the K, independent variables are linearly dependent on a set of K characteristic vectors of the variance co-variance matrix, the efficiency might no be zero.

The consequences of applying Ordinary Least Squares to a relationship with autocorrelated disturbances are qualitatively similar to those already derived for heteroscedatic case, namely, unbiased but inefficient estimation and invalid inference procedures. It is possible to illustrate some of these factors quantitatively for certain simple case. Considering the model

$$Y_t = \beta X_t + \mu_t (1)$$

with $\mu_t = \rho\mu_{t-1} + \epsilon_t$

where $E(\epsilon) = 0 \; and E(\epsilon\epsilon^1) = \sigma_\epsilon^2 I$

If OLS is applied to equation (1), we know from (2) and (3)

$$\in (UU')$$
$$= \sigma \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ . & . & . & \cdots & . \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{bmatrix} \quad (2)$$

Which is the variance matrix for a disturbance following a first – order autoregressive scheme and

$$\begin{aligned} var(b) &= E\{(b-\beta)(b-\beta)^1\} \\ &= E\{(X^1X)^{-1}X^1\mu\mu^1X(X^1X)^{-1}\} \\ &= \sigma_\mu^2(X^1X)^{-1}X\Omega X\{(X^1X)^{-1}\} \quad (3) \end{aligned}$$

where Ω = equation (2)

and as may be verified by multiplying out

$$\Omega^{-1}$$
$$= \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \cdots & 0 & 0 & 0 \\ . & . & . & \cdots & . & . & . \\ 0 & 0 & 0 & \cdots & -\rho & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \cdots & 0 & \rho & 1 \end{bmatrix} \quad (4)$$

METHODOLOGY

By substituting equation (2) into equation (3) for model in equation (1) gives var (OLSb)

$$= \frac{\sigma_\mu^2}{\sum_{t=1}^{n} X_t^2} \left[1 + 2\rho \frac{\sum_{t=1}^{n-1} X_t X_{t-1}}{\sum_{t=1}^{n} X_t^2} + 2\rho^2 \frac{\sum_{t=1}^{n-2} X_t X_{t+2}}{\sum_{t=1}^{n} X_t^2} + \cdots + 2\rho^{n-1} \frac{X_1 X_n}{\sum_{t=1}^{n} X_t^2} \right] \quad (5)$$

If ρ were known and GLS was applied to equation (1), then the substitution of equation (4) in the general formula

$$Var(b_*) = \sigma_\mu^2 (X^1 \Omega^{-1} X)^{-1}$$

gives

$$var(GLSb_*) = \frac{\sigma_\mu^2}{\sum_{t=1}^{n} X_t^2} \left(\frac{1-\rho^2}{1+\rho^2 - 2\rho \sum_{t=1}^{n-1} X_t X_{t-1} / \sum_{t=1}^{n} X_t^2} \right) \qquad (6)$$

Comparing equation (6) with equation (5) shows that the efficiency of Ordinary Least Squares is measured by the ratio of the two terms in parenthesis and thus depends not just on ρ, but also on the nature of the x variable. Let us suppose that x follows a stable AR (1) scheme with parameter λ. As the sample size n gets very large, the terms involving x are approximated by the autocorrelation coefficients of x, which are simply the successive powers of λ. Thus the assympotic efficiency of Ordinary Least Squares is given by

$$asyeff(OLSb) = \frac{1-\rho^2}{(1+\rho^2-2\rho\lambda)(1+2\rho\lambda+2\rho^2\lambda^2+..)}$$

$$= \frac{(1-\rho^2)(1-\rho\lambda)}{(1+\rho^2-2\rho\lambda)(1+\rho\lambda)} \qquad (7)$$

APPLICATION

Table I shows illustrative values of assymtotic efficiency for selected values of ρ and λ

Table 1: Asymptotic efficiency of Ordinary Least Squaresb (percent)

	P							
Λ	-0.9	-0.8	-0.5	-0.2	0.2	0.5	0.8	0.9
0.0	10.5	22.0	60.0	92.3	92.3	60.0	22.0	10.5
0.2	12.6	25.4	63.2	92.9	92.3	58.4	19.8	9.1
0.5	18.5	34.4	71.4	94.6	93.5	60.0	18.4	7.9
0.8	35.9	56.2	85.4	97.5	96.6	71.4	22.0	8.4
0.9	52.8	71.8	92.0	98.7	98.1	81.3	29.3	10.5

A second problem with the application of Ordinary Least Squares is that the conventional formula will in this example, estimate $\sigma_{\mu}^{2}/\sum_{i=1}^{n} X_{t}^{2}$, whereas equation 7 shows that this is no longer true variance. As the same size gets vary large, the ratio of the conventional formula to the true variance is given by

$$\tau = \frac{1-\rho\lambda}{1+\rho\lambda}$$

Thus the proportionate bias that the conventional program will impart to the estimation of the true sampling variance of the Ordinary Least Squares b is, in the limit

$$\text{Asympototic proportionate bias} = \frac{-2\rho\lambda}{1+\rho\lambda} \qquad (8)$$

Table 2 shows values of this statistic for selected values of ρ and λ. Again, it is instructive to consider the table in two halves. A, positively autocorrelated disturbance in conjunction with a positively autocorrelated (x) series implies underestimation of the sampling variance by the conventional Ordinary Least Squares formula. If ρ = λ = 0.9, the estimated variance will only be about one-tenth of the correct number, which would cause a serious overestimation of t statistics and significance levels in conventional inference procedures. On the other hands, different signs for ρ and λ, will cause an overestimation of the sampling variance. Casual empiricism indicates that the predominant situation in applied studies is a conjunction of positive autocorrelation in both disturbance and explanatory variance so that underestimation of var (b) is the more likely situation.

Table 2: percentage bias in estimating var(b)

P						
λ	-0.9	-0.5	-0.2	0.2	0.5	0.9
0.0	0	0	0	0	0	0
0.2	43.9	22.2	8.3	-7.7	-18.2	-30.2
0.5	163.6	40.0	22.2	-18.2	-40.0	-62.1
0.9	852.6	163.6	43.9	-30.5	-62.1	-89.3

The comparison involved in table 2 has implicity taken σ_μ^2 as known. In practice it must be estimated from the sample data, and here agin there is a possibility of bias if the disturbance term is autocorrelated.

CONCLUDING REMARKS

When ρ and λ have the same sign, the bias accentuates the bias analysed in table 2. It is clear that autocorrelated disturbances are a potentially serious problem, and it is very important to be able to test for their existence. Such tests are the Durbin – Watson test. The relative efficiency of the ordinary least squares as compared to the Gauss – Markov estimator depends to a great extent on the x – matrix used. The relative efficiency of Ordinary Least Squares increases with increasing correlation for certain cases important in practice and this seems to run contrary to what one should expect on the basis of Monte Carlo studies.

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