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THEORY OF GENERALIZED INTEGRATED AUTOREGRESSIVE BILINEAR TIME SERIES MODELLING

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ABSTRACT
The theory of generalized integrated autoregressive bilinear time series models which are capable of achieving stationary for all nonlinear series are proposed in this paper. These models are denoted by GBL (p, d, 0, r, s). The sufficient conditions for stationary of this bilinear time series models are derived. An algorithm for selecting the best order of the model is proposed. The parameters of the proposed models are estimated using robust nonlinear least squares method and statistical properties of the derived estimates are investigated. The bilinear models are fitted to Wolfer sunspot numbers and stationary conditions are satisfied.
Keywords: Non-linear Least Squares, Parameters, Wolfer sunspot numbers, Algorithm and Stationary

INTRODUCTION
The bilinear time series models have attracted considerable attention during the last years. They have found a variety of applications including those in economy, biology, ecology, software interfailure, signal processing etc Ojo (2010).

An overview of various models and their application can be found by Granger and Anderson (1978), Pham and Tran (1981), Subba Rao (1981), Gabr and Subba Rao (1981), Rao et al. (1983), Liu (1992), Gonclaves et al. (2000), Shangodoyin and Ojo (2003), Wang and Wei (2004), Boonchai and Eivind (2005), Bibi (2006), Doukhan et al. (2006), Drost et al. (2007), Usoro and Omekara (2008), Ojo (2009). The bilinear modes studied by the above authors could not achieve stationarity for all nonlinear series. Rao et al. (1983) gave a set of sufficient conditions for the existence of a strictly stationary stochastic process conforming to the following bilinear model:

$$X_t = \sum_{i=1}^{p} a_j X_{t-j} + \sum_{i=1}^{p}\sum_{j=1}^{q} b_{ij} X_{t-i} e_{t-j} + e_t$$, denoted as BL(p, 0, p, q)

where p is the order of the autoregressive component, and p, q is the order of the nonlinear component. $a_1, a_2, ..., a_p$ are the parameters of the autoregressive component and $b_{11}, ..., b_{pq}$ are the parameters of the nonlinear component.

In this paper, we extend the work of Rao et al. (1983) to the proposed generalized autoregressive integrated bilinear models which are capable of achieving stationary for all nonlinear series; this is an important improvement over other bilinear time series models.

PROPOSED GENERALIZED INTEGRATED AUTOREGRESSIVE BILINEAR TIME SERIES MODELS

We define generalized integrated autoregressive bilinear (GBL) time series models as follows:

$\psi(B)X_t = \phi(B)\nabla^d X_t + \sum_{k=1}^{r}\sum_{l=1}^{s} b_{kl}X_{t-k}e_{t-l} + e_t$, denoted as BL (p, d, 0, r, s)

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 .... - \phi_p B^p$ and

$$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + b_{11}X_{t-1}e_{t-1} + ... + b_{rs}X_{t-r}e_{t-s} + e_t \quad (2.1)$$

$\phi_1,...,\phi_p$ are the parameters of the autoregressive component; $b_{11},.........,b_{rs}$ are the parameters of the nonlinear component; $\phi(B)$ is the autoregressive operator and $\psi(B)X_t = \phi(B)\nabla^d$ is called the generalized autoregressive operator.

The Vector Form of BL (p, d, 0, r, s)

It is convenient to study the properties of a process when the model is in the state space form because of the Markovian nature of the model Akaike (1974).

Let

$$\underset{p\times p}{\Psi} = \begin{pmatrix} -\psi_1 & -\psi_2 & -\psi_3 & ..... & -\psi_{p+d-1} & -\psi_{p+d} \\ 1 & 0 & 0 & ..... & 0 & 0 \\ 0 & 1 & 0 & .... & 0 & 0 \\ 0 & 0 & 0 & ..... & 1 & 0 \end{pmatrix}$$

$$\underset{r\times r}{B_i} = \begin{pmatrix} b_{1i} & b_{2i} & b_{3i} & .... & b_{ri} \\ 0 & 0 & 0 & .... & 0 \\ 0 & 0 & 0 & .... & 0 \end{pmatrix} \quad (i = 1,........, s)$$

and vectors $\underset{1\times p}{C^T} = (1,0,0,.....,0)$ and let $\underset{1\times p}{X^T} = (X_t, X_{t-1},......., X_{t-p+d})$, (Here T stands for the transpose of a matrix) t = ...-1, 0, 1,..... With this notation, we can write the model (2.1) in the vector form as:

$$\mathbf{X}_t = \Psi\mathbf{X}_{t-1} + \sum_{i=1}^{s} B_i \mathbf{X}_{t-1}e_{t-1} + \mathbf{C}e_t$$

STATIONARY AND CONVERGENCE OF GENERALIZED INTEGRATED AUTOREGRESSIVE BILINEAR MODELS

In this section, we give a sufficient condition for the existence of strictly stationary process and convergence conforming to the bilinear model (2.1). This we do through the following theorem.

THEOREM

Let $\{e_t, t \in Z\}$ be a sequence of independent identically distributed random variables defined on a probability space (Ω, F, P) such that E e_t = 0 and $Ee_t^2 = \sigma^2 < \infty$. Let Ψ , B_1, B_2,....,B_q be q+1 matrices each of order p x p and

$$\Gamma_1 = \Psi \otimes \Psi + \sigma^2 (B_1 \otimes B_1),$$
$$\Gamma_j = \sigma^2 [B_j \otimes (\Psi^{j-1} B_1 + \Psi^{j-2} B_2 + ... + \Psi B_{j-1}) + (\Psi^{j-1} B_1 + \Psi^{j-2} B_2 + ... + \Psi B_{j-1}) \otimes B_j + (B_j \otimes B_j)], \quad \text{j = 2, 3,......s.}$$

Suppose all the eigenvalues of the matrix

$$\underset{p^2 q \times p^2 q}{L} = \begin{pmatrix} \Gamma_1 & \Gamma_2 & ...... & \Gamma_{q-1} & \Gamma_q \\ I_{p^2} & 0 & .... & 0 & 0 \\ 0 & I_{p^2} & & 0 & 0 \\ 0 & 0 & & I_{p^2} & 0 \end{pmatrix}$$

have moduli less than unity, i.e, $\rho(L) = \lambda < 1$. Let $\underset{px1}{C}$ be a given column vector. Then there exists a vector valued strictly stationary process $\{X_t, t \in Z\}$ conforming to the vector form of generalized bilinear model $\mathbf{X}_t = \Psi \mathbf{X}_{t-1} + \sum_{j=1}^{s} \mathbf{B}_j \mathbf{X}_{t-j} e_{t-j} + \mathbf{C} e_t$ for every t in Z.

PROOF

The proof of the theorem for the sake of simplicity is carried out in the following steps.

Step 1

Let the process $\{\mathbf{S}_{n,t,} n, t \in Z\}$ be defined as follows:
$S_{n,t} = Ce_t + (\Psi + B_1 e_{t-1})\mathbf{S}_{n-t,t-1} + B_2 \mathbf{S}_{n-2,t-2} e_{t-2} + ... + B_s \mathbf{S}_{n-s,t-s} e_{t-s}$, if n>0 for every t in Z.

We show that $\lim_{n\to\infty} \mathbf{S}_{n,t}$ exists almost surely for every t in Z. If $\mathbf{X}_t$ is the almost sure limit of $\{\mathbf{S}_{n,t,} n \geq 1\}$ for every t in Z, then it is obvious that the process $\{\mathbf{X}_t, t \in Z\}$ conforms to the bilinear model (2.1). It is also easy to check that for every fixed n in Z, $\{\mathbf{S}_{n,t,} t \in Z\}$ is a strictly stationary process.

Step 2

Let $\mathbf{s}_{n,t} = \mathbf{S}_{n,t} - \mathbf{S}_{n-1,t,} t \in Z.$ We show that $E|(\mathbf{s}_{n,t})_i| \leq K\lambda^{n/2}$ for every $n \geq 0$ and i = 1, 2, ….,p, where K is a positive constant. Since $\lambda < 1$, this then implies that $\{\mathbf{S}_{n,t,} n \geq 1\}$ converges almost surely for every t in Z. (If $\{a_{n,} n \geq 1\}$ is a sequence of real numbers satisfying $|a_n - a_{n-1}| \leq K\lambda^n$ for every $n \geq 2$ for some positive constant K and $\lambda < 1$, then it is easy to show that $\{a_{n,} n \geq 1\}$ is a Cauchy sequence of real numbers.)

Step 3

First, we settle the question of integrability of the $\mathbf{s}_{n,t}$'s. Note that

$$\begin{aligned} \mathbf{s}_{n,t} &= \mathbf{S}_{n,t} - \mathbf{S}_{n-1,t} \\ &= (\Psi + B_1 e_{t-1})\mathbf{s}_{n-1,t-1} + B_2 \mathbf{s}_{n-2,t-2} e_{t-2} + ... + B_s \mathbf{s}_{n-s,t-s} e_{t-s} \\ &= Q_n(e_{t-1}, e_{t-2}, ..., e_{t-n})\mathbf{s}_{0,t-n} = Q_n(e_{t-1}, e_{t-2}, ..., e_{t-n}) C e_{t-n}, \end{aligned}$$

where $Q_n(e_{t-1}, e_{t-2}, \ldots, e_{t-n})$ is a matrix of order p x p and each entry of this matrix is a polynomial in $e_{t-1}, e_{t-2}, \ldots, e_{t-n}$ in which the power index of each e_{t-j} is either 0 or 1. Consequently, every entry in $Q_n(e_{t-1}, e_{t-2}, \ldots, e_{t-n})$ and hence in $\mathbf{s}_{n,t}$ is integrable. It is clear that distribution of $\mathbf{s}_{n,t}$ does not depend on t.

Step 4

It is convenient to deal with the following processes. Define

$\mathbf{s}^*_{n,t} = Q_n(e_{t-1}, e_{t-2}, \ldots, e_{t-n})C,$ if n > 0 for every t in Z. Equivalently, $\mathbf{s}_{n,t} = \mathbf{s}^*_{n,t} e_{t-n}$, n, $t \in Z$.

From the remark made regarding the $Q_n(.)'s$ in step 3, it is obvious that every entry in $\mathbf{s}^*_{n,t}$ is square integrable. Further, it is easy to check that $\mathbf{s}^*_{n,t}{}'s$ satisfy the following equation.

$$\mathbf{s}^*_{n,t} = (\Psi + B_1 e_{t-1})\mathbf{s}^*_{n-1,t-1} + B_2 \mathbf{s}^*_{n-2,t-2} e_{t-2} + \ldots + B_s \mathbf{s}^*_{n-s,t-s} e_{t-s} \qquad (2.3)$$

for every n, t in Z. Also, the distribution of $\mathbf{s}^*_{n,t}$ does not depend on t, since the $e_t{}'s$ are independently identically distributed. Since $\mathbf{s}_{n,t} = \mathbf{s}^*_{n,t} e_{t-n}$ for all n and t in Z.

$$E\left|(\mathbf{s}_{n,t})_i\right| = E\left|(\mathbf{s}^*_{n,t})_i\right|\left|e_{t-n}\right| \le (E((\mathbf{s}^*_{n,t})_i)^2)^{1/2}(Ee^2_{t-n})^{1/2} \le \sigma(E((\mathbf{s}^*_{n,t})_i)^2)^{1/2}$$

for every i = 1, 2, ……,p. It suffices to obtain an upper bound for $E((\mathbf{s}^*_{n,t})_i)^2$ for every i = 1,2,…,p and n, t in Z. For this we evaluate $E(\mathbf{s}^*_{n,t} \otimes \mathbf{s}^*_{n,t}) = M_n$, say

Step 5

Let $D_1 = (\Psi + B_1 e_{t-1})\mathbf{s}^*_{n-1,t-1}$ and $D_i = B_i \mathbf{s}^*_{n-1,t-1} e_{t-i}$ for i=2,3,…,s.

$$\begin{aligned}\mathbf{s}^*_{n,t} \otimes \mathbf{s}^*_{n,t} &= (\sum_{i=1}^{s} D_i) \otimes (\sum_{i=1}^{s} D_i) = \{D_1 \otimes D_1\} + \{D_1 \otimes D_2 + D_2 \otimes D_2 + D_2 \otimes D_1\} \\ &+ \{D_1 \otimes D_3 + D_2 \otimes D_3 + D_3 \otimes D_3 + D_3 \otimes D_2 + D_3 \otimes D_1\} + \ldots\ldots \\ &+ \left\{\sum_{i=1}^{s-1} D_i \otimes D_s + D_s \otimes D_s + \sum_{i=1}^{s-1} D_s \otimes D_i\right\}. \end{aligned} \qquad (2.4)$$

We evaluate the expectation of each expression within each set of brackets {} in (2.4)

Step 6

We write $D_1 \otimes D_1 = ((\Psi + B_1 e_{t-1}) \otimes (\Psi + B_1 e_{t-1}))(\mathbf{s}^*_{n-1,t-1} \otimes \mathbf{s}^*_{n-1,t-1})$.

Since $\mathbf{s}^*_{n-1,t-1}$ is a function of $e_{t-2}, e_{t-3}, \ldots, e_{t-n}, \mathbf{s}^*_{n-1,t-1}$ and e_{t-1} are independently distributed. So,

$E(D_1 \otimes D_1) = ((\Psi \otimes \Psi + \sigma^2(B_1 \otimes B_1)) = \Gamma_1 M_{n-1}.$

Step 7

Expanding $\mathbf{s}^*_{n-1,t-1}$, we obtain

$$\begin{aligned} D_1 \otimes D_2 &= ((\Psi + B_1 e_{t-1})(\Psi + B_1 e_{t-2}) \otimes B_2 e_{t-2})(\mathbf{s}^*_{n-2,t-2} \otimes \mathbf{s}^*_{n-2,t-2}) \\ &+ ((\Psi + B_1 e_{t-1}) \otimes B_2 e_{t-2})(B_2 \mathbf{s}^*_{n-3,t-3} e_{t-3} \otimes \mathbf{s}^*_{n-2,t-2}) + \ldots + \\ &((\Psi + B_1 e_{t-1}) \otimes B_2 e_{t-2})(B_s \mathbf{s}^*_{n-1-s,t-1-s} e_{t-1-s} \otimes \mathbf{s}^*_{n-2,t-2}). \end{aligned}$$

Therefore, $E(D_1 \otimes D_2) = \sigma^2((\Psi B_1) \otimes B_2) M_{n-2}$

In a similar fashion, we can show that $E(D_2 \otimes D_1) = \sigma^2(B_2 \otimes (\Psi B_1)) M_{n-2}$

and $E(D_2 \otimes D_2) = \sigma^2 (B_2 \otimes B_2) M_{n-2}$. Consequently, the expected value of the entire expression in the second set of such brackets is $\sigma^2 (B_2 \otimes (\Psi B_1) + (\Psi B_1) \otimes B_2 + B_2 \otimes B_2) M_{n-2} = \Gamma_2 M_{n-2}$.

Step 8: Pursuing ideas similar to those used in step 7, we can show that the expected value of the entire expression in the third set of such brackets in (2.4) is

$$\sigma^2 (B_3 \otimes (\Psi^2 B_1 + \Psi B_2) + (\Psi^2 B_1 + \Psi B_2) \otimes B_3 + B_3 \otimes B_3) = \Gamma_3 M_{n-3}.$$

Step 9: The expectations of other expressions can be evaluated analogously. Finally, we obtain

$$M_n = E(\mathbf{s}^*_{n,t} \otimes \mathbf{s}^*_{n,t}) = \sum_{i=1}^{s} \Gamma_i M_{n-i} \text{ for all n.}$$

Step 10

Since $M_n = E(\mathbf{s}^*_{n,t} \otimes \mathbf{s}^*_{n,t})$, we have

$$E((\mathbf{s}^*_{n,t})_i)^2 \le K^{'} \lambda^n$$

where $\rho(L) = \lambda < 1$ and $K^{'}$ is a positive constant.

DESCRIPTION OF ALGORITHM FOR FITTING GENERALIZED INTEGRATED AUTOREGRESSIVE BILINEAR MODELS

For the sake of simplicity, we will break the algorithm down into the following steps.

Step 1

Fit various order of autoregressive model of the form

$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + e_t$

Step 2

Choose the model for which Akaike Information Criterion (AIC) is minimum among various order fitted in step 1.

Step 3

Fit possible subsets of chosen model in step 2 using $2^q - 1$ subsets approach Hagan and Oyetunji (1980).

Step 4

Choose the model for which AIC is minimum among the fitted models in step 3 to have the best subset model.

Step 5

Fit various order of the generalized bilinear model of the form $X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + b_{11} X_{t-1} e_{t-1} + ... + b_{rs} X_{t-r} e_{t-s} + e_t$

and choose the model for which AIC is minimum

Step 6

The model with the minimum AIC is the best generalized bilinear model.

ESTIMATION OF THE PARAMETERS OF GENERALIZED BILINEAR MODELS PROPOSED

The joint density function of $(e_m, e_{m+1},....,e_n)$ where m = max (r, s) is given by

$$\frac{1}{(2\pi\sigma_e^2)^{(n-m+1)/2}}\exp\left(\frac{1}{-2\sigma_e^2}\sum_m^n e_t^2\right)$$

Since the Jacobian of the transformation from $(e_m, e_{m+1},....,e_n)$ to $(X_m, X_{m+1},....,X_n)$ is unity, the likelihood function of $(X_m, X_{m+1},....,X_n)$ is the same as the joint density function of $(e_m, e_{m+1},....,e_n)$. Maximising the likelihood function is the same as minimizing the function $Q(G)$, where

$$Q(G) = \sum_{i=m}^{n} e_{t,}^2 \qquad (2.5)$$

with respect to the parameter $G' = (\psi_1,....,\psi_p; B_{11},....,B_{rs})$

Then the partial derivatives of Q(G) are given by

$$\frac{dQ(G)}{dG_i} = 2\sum_{t=m}^{n} e_t \frac{de_t}{dG_i} \qquad \text{(i = 1, 2,.....,R)} \qquad (2.6)$$

$$\frac{d^2Q(G)}{dG_i dG_j} = 2\left(\sum_{t=m}^{n} e_t \frac{de_t}{dG_i}\frac{de_t}{dG_j} + \sum_{t=m}^{n} e_t \frac{d^2e_t}{dG_i dG_j}\right)$$

where these partial derivatives of e(t) satisfy the recursive equations

$$\frac{de_t}{d\psi_i} + \sum_{j=1}^{s} W_t(t)\frac{de_{t-j}}{d\psi_i} = \begin{cases} 1, & \text{if } i = 0 \\ X_{t-i}, & \text{if } i = 1, 2, \ldots, p \end{cases}$$

(2.7)

$$\frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{de_{t-j}}{dB_{kmi}} = -X_{t-k}e_{t-m} \quad \text{(k=1,2,...,r ; m}_i\text{ =1,2,...,s)} \qquad (2.8)$$

$$\frac{d^2e_t}{d\psi_i d\psi_i'} + \sum_{j=1}^{s} W_j(t)\frac{d^2e_{t-j}}{d\psi_i d\psi_i'} = 0 \quad \text{(i, i' = 0, 1, 2, ..., p)} \qquad (2.9)$$

$$\frac{d^2e_t}{d\psi_i dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{d^2e_{t-j}}{dB_{kmi}d\phi_i} + X_{t-k}\frac{d^2e_{t-mi}}{d\psi_i} = 0$$

(i=0,1,2,...,p ; k_i =1,2,...,r; m_i=1,2,...,s) (2.10)

$$\frac{d^2e_t}{dB_{kmi}dB_{kmi}'} + \sum_{j=1}^{s} W_j(t)\frac{d^2e_{t-j}}{dB_{kmi}dB_{kmi}'} + X_{t-k}'\frac{d^2e_{t-mi}}{dB_{kmi}} = -X_{t-k}\frac{de_{t-m}}{dB_{kmi}'}$$

(k, k' =1,2,...,r ; m_i m_i' = 1,2,...,s) (2.11)

$$W_j(t) = \sum_{j=1}^{s} B_{ij}X_{t-j}$$

We assume e_t = 0 (t = 1, 2, ..., m-1) and also

$\frac{de_t}{dG_i} = 0, \frac{d^2 e_t}{dG_i dG_j} = 0$, (i, j = 1, 2, …, R; t = 1, 2, …, m-1)

From $e_t = 0$ (t = 1, 2, …, m-1), $\frac{de_t}{dG_i} = 0, \frac{d^2 e_t}{dG_i dG_j} = 0$, and $\frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{de_{t-j}}{dB_{kmi}} = -X_{t-k} e_{t-m}$ (k=1,2,…,r ; m_i =1,2,…,s), it follows that the second order derivatives with respect to ψ_i (i = 0, 1, 2, …, p) are zero. For a given set of values $\{\phi_i\}$ and $\{B_{ij}\}$ one can evaluate the first and second order derivatives using the recursive equations, 2.7, 2.8 and 2.11. Now let

$$\mathbf{V}'(\mathbf{G}) = \frac{dQ(\mathbf{G})}{d\mathbf{G}_1}, \frac{dQ(\mathbf{G})}{d\mathbf{G}_2}, \ldots\ldots, \frac{dQ(\mathbf{G})}{d\mathbf{G}_k}$$

and let $\mathbf{H}(\mathbf{G}) = [d^2 Q(\mathbf{G}) / d\mathbf{G}_i d\mathbf{G}_j]$ be a matrix of second partial derivatives as in Krzanowski (1998). Expanding **V**(**G**), near $G = \hat{G}$ in a Taylor series, we obtain $V(\hat{G})_{\hat{G}=G} = 0 = V(G) + H(G)(\hat{G} - G)$ Rewriting this equation we get $\hat{G} - G = -H^{-1}(G)V(G)$, and thus obtain an iterative equation given by $G^{(k+1)} = G^{(k)} - H^{-1}(G^{(k)})V(G^{(k)})$ where $G^{(k)}$ is the set of estimates obtained at the k^{th} stage of iteration. The estimates obtained by the above iterative equations usually converge. For starting the iteration, we need to have good sets of initial values of the parameters. This is done by fitting the best subset of the linear part of the bilinear model.

Numerical Example: The Wolfer Sunspot Data

To present the application of the models proposed, we will use a real time series dataset, the Wolfer sunspot, available in Box et al. (1994). The scientists track solar cycles by counting sunspots – cool planet-sized areas on the Sun where intense magnetic loops poke through the star's visible surface. It was Rudolf Wolf who devised the basic formula for calculating sunspots in 1848; these sunspot counts are still continued.

As the Wolfer sunspot data set represent a non-stationary series, the bilinear models proposed in this paper may be applied. The Wolfer sunspot data set, is considered at different sample sizes of 50, 150 and 250. For the fitted model below we have used the algorithm and the estimation technique in the previous section.

Fitted Model at t=50

$X_t = 0.314548X_{t-1} - 0.458429X_{t-2} - 0.302114X_{t-4} - 0.220568X_{t-5} - 0.386159X_{t-6} - 0.002758X_{t-1}e_{t-1} - 0.020647X_{t-1}e_{t-2} - 0.018189X_{t-1}e_{t-3} + 0.015317X_{t-2}e_{t-1} + e_t$

Fitted Model at t=150

$X_t = 0.412820X_{t-1} - 0.271125X_{t-2} - 0.270908X_{t-3} - 0.339150X_{t-5} - 0.293320X_{t-7} + 0.000325X_{t-1}e_{t-1} - 0.020870X_{t-1}e_{t-2} - 0.002425X_{t-1}e_{t-3} + 0.018075X_{t-2}e_{t-1} + 0.009283X_{t-2}e_{t-2} - 0.008691X_{t-2}e_{t-3} - 0.019234X_{t-3}e_{t-1} - 0.007737X_{t-3}e_{t-2} + e_t$

Fitted Model at t=250

$X_t = - 0.239576X_{t-2} - 0.361665X_{t-3} - 0.238746X_{t-4} - 0.325416X_{t-5} - 0.328627X_{t-6} - 0.209789X_{t-7} - 0.365561X_{t-8} + 0.000633X_{t-1}e_{t-1} - 0.010392X_{t-1}e_{t-2} + 0.007590X_{t-}$

$_1e_{t-3} + 0.005443X_{t-2}e_{t-1} + 0.000716X_{t-2}e_{t-2} - 0.005326X_{t-2}e_{t-3} - 0.013130X_{t-3}e_{t-1} + e_t$

The derived statistics from the above fitted models are given in table1, table 2 and table 3 below.

Table 1. Goodness of fit of generalized integrated autoregressive bilinear models at *t* = 50, t= 150 and t=250. All models are significant at *P*<0.001.

	t=50	t=150	t=250
Residual variance	250.20	193.20	285.50
Akaike information criterion	8.52	8.21	8.55
Bayesian information criterion	8.65	8.36	8.68
R-Square	0.58	0.61	0.55
Adjusted R-Square	0.55	0.59	0.54
F(Statistic)	20.93	31.18	49.29

From table 1, we could see the behavior of the proposed models at different level of sample sizes. The smallest residual variance was recorded at sample size of 150. The proposed model does not encourage working with large sample size.

CONCLUSION

This study focused on new bilinear models that could handle all non-linear series. Bilinear models at different levels of sample sizes were considered using the non-linear real series. Moreover, estimation of parameters has witnessed a unique, consistent and convergent estimator that has prevented the models from exploding, thereby making stationary possible.

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