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A STUDY OF MIXED CONVECTION FLOW IN VERTICAL ANNULUS FILLED WITH POROUS MATERIAL HAVING CONSTANT POROSITY

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ABSTRACT

This work analysed the behaviour of fully developed mixed convection flow of an incompressible and viscous fluid in a vertical annulus filled with porous material having constant porosity and saturated with the same fluid. The non-Darcian flow model is utilized for the momentum transfer in the porous domain. The inertia effect is not taken into consideration. The influence of the parameters involved is discussed for isothermal and isoflux heating of the duct surfaces. In addition, criteria for the occurrence of flow reversal are presented.

INTRODUCTION

Convective heat and mass transfer in porous medium has been intensively studied over the past two decades. This is because of its wide applications in geothermal energy engineering, ground-water pollution transport, nuclear waste disposal, chemical reactors engineering, insulation of buildings and pipes, storage of grain coal, and so on. Many studies were carried out. Details on this topic could be found in Nield and Bejan (1999). Darcy (1856) presented the accuracy of these results restricted to specific range of Darcy and Reynolds numbers. Vafai and Tien (1981) have discussed in length the importance of Brickman and Forchheimer terms in the case of forced convention flow over a flat plate and presented the resulting error in heat transfer coefficient when the viscous and inertia terms are negligible. Joshi (1987) considered the fully developed laminar natural convection flows in vertical annuli in which one of the boundaries is isothermal. Verma and Chauhan (1986) have solved unsteady fluid flow through a circular naturally permeable tube surrounded by a porous material. Muralidhar (1989) has extensively studied forced and free convection problems in a vertical porous annulus. While momentum transfer based on Pop and Cheng (1992) investigated Brinkman model in a circular cylinder. Al-Nimr and Darbich (1995) analyzed transient response of free convective fluid in a concentric tube annulus filled with a porous medium. Poulicakos and Kuzmierczak (1987) have studied a theoretical study of fully developed forces convection in a duct partially filled with a porous material. Unsteady developed laminar free convection in vertical parallel plates was numerically investigated by Joshi (1988). Oosthuizen, and Paul (1986) have studied steady laminar natural convection flow in vertical concentric annuli. Vafai (1984) investigated the effects of variable porosity in the presence of a solid boundary and inertial forces in porous media showing that channeling (velocity showing a maximum near the wall) is observed under these conditions which leads to enhancement of heat transfer in some cases.

In addition, White (1991) treats the problem of fully developed laminar flow through a non-porous channel with permeable walls as a classic solution. Wang (1971) derived an analytical solution for Pulsate flow in a channel with permeable parallel walls. Akhilesh, Jha, and Singh (2001) obtained solution of mixed convection flow in a vertical annulus filled with a porous material considering isothermal and isoflux heating at the outer surface of the inner cylinder when the gap between the two cylinders is less, equal and greater than radius of the inner cylinder. Recently Jha (2005) studied free convection flow through an annular porous medium. Kaurangini and Jha (2008) presented results of the studies on combined effects of free and forced convection flow in a vertical annulus filled with porous material having variable porosity when the outer cylinder is isothermally heated and the inner cylinder is isoflux. Kaurangini and Jha (2009) presented the study of mixed convection flow in vertical annulus filled with constant porous material when the outer surface of the inner cylinder is isothermally heated. In the present work, the study of combined effects of free and forced convection flow in a vertical annulus filled with porous material having constant porosity is presented by heating the inner surface of the outer cylinder isothermally or constant heat flux.

MATHEMATICAL ANALYSIS

A steady laminar fully developed mixed convection flow of an incompressible and viscous fluid between a vertical concentric annulus of infinite length filled with porous material having constant porosity was considered. A pressure gradient is taken along z-axis parallel to the axis of the cylinder and r-axis perpendicular to it. In formulating the problem, the following assumptions are made:

1. The fluid obeys Boussinesq approximation.
2. The velocity of the fluid varies in the axial direction only.
3. The heat transfer takes place by conduction only.
4. The pressure gradient is taken into account.

Under these assumptions, we considered the flow of fluid through a vertical annulus filled with porous material having constant permeability, by heating the inner surface of the outer cylinder isothermally or constant heat flux causes the temperature gradient in side the system. The governing equations of the problem and its boundary conditions are in equations (1) and (2) respectively.

$$\gamma_{eff}\left[\frac{d^2u'}{dr'^2}+\frac{1}{r'}\frac{du'}{dr'}\right]-\gamma_f\frac{u'}{k}+g\beta\left(T'-T'_0\right)-\frac{\partial p'}{\partial z'}\frac{1}{\rho}=0$$

$$\frac{d^2T'}{dr'^2}+\frac{1}{r'}\frac{dT'}{dr'}=0 \qquad (1)$$

$$u'=0, T'=T'^{'}_2 \; or \; \frac{dT'}{dr'}=-\frac{q}{k^*} \quad \text{at } r'=a$$

$$u'=0, T'=T'_0 \quad \text{at } r'=b \qquad (2)$$

In order to have the problem in non-dimensional form, the following parameters are defined.

$$u=\frac{u'}{u_0}, R=\frac{r'}{a}, \quad T=\frac{T^1-T_0}{T_1^1-T_0^1}, \gamma=\frac{\gamma_{eff}}{\gamma_f}, \mathrm{Da}=\frac{a^2}{k^*}, \quad Gr=\frac{g\beta\Delta Ta^3}{\gamma_f^2}, \mathrm{Re}=\frac{u_0 a}{\gamma_f}, P=\frac{\partial P'}{\partial Z'}\frac{a^2}{u_0}$$

The isothermal heating and isoflux heating conditions $\mathrm{T}=1$ or $\frac{\mathrm{dT}}{\mathrm{dR}}=-1$ could be unified as $\mathrm{A}\frac{\mathrm{dT}}{dR}+BT=C$.

The equations governing flow and heat transfer together with boundary conditions in non-dimensional form are in equation (3) and (4)

$$\gamma\left[\frac{d^2u}{dR^2}+\frac{1}{R}\frac{du}{dR}\right]-\frac{u}{Da}+\frac{GrT}{\mathrm{Re}}+P=0$$

$$\frac{d^2T}{dR^2}+\frac{1}{R}\frac{dT}{dR}=0$$

(3)

$$u=0.\ A\frac{dT}{dR}+BT=C \quad \text{at } R=1$$

$$u=0,\ T=r_t \quad \text{at } R=\lambda \tag{4}$$

Solving the above equations in (3) utilizing the boundary conditions in (4) we have:

$$u=C_6I_0\left(\frac{R}{\sqrt{\gamma Da}}\right)+C_7K_0\left(\frac{R}{\sqrt{\gamma Da}}\right)+\frac{GrDa}{\mathrm{Re}}\left[B_1Log(R)+B_2\right]+PDa \tag{5}$$

$$T=B_1Log(R)+B_2$$

(6)

The pressure gradient is computed as $P=\left[Z_{181}-G\,\mathrm{Re}\,Z_{19}\right]$ using $\frac{1}{2}\left(\lambda^2-1\right)=\int_1^{\lambda}RUdR$ while the skin friction are computed as;

$$\tau_1=\left|Z_1C_6I_1(Z_1)-Z_1C_7K_1(Z_1)+G\,\mathrm{Re}\,DaB_1\right. \tag{8}$$ and

$$\tau_\lambda=\left|Z_1C_6I_1(\lambda Z_1)-Z_1C_7K_1(\lambda Z_1)+G\,\mathrm{Re}\,Da\frac{B_1}{\lambda}\right. \tag{9}$$

$$G\,\mathrm{Re}\,h1=\frac{\left[Z_{23}Z_{181}\right]}{Z_{23}Z_{19}-Z_{22}} \quad \text{and} \quad G\,\mathrm{Re}\,h2=\frac{Z_{29}}{Z_{30}} \tag{10}$$

$$Q=2\pi\left[G\,\mathrm{Re}\,Z_{17}+PZ_{18}\right]$$

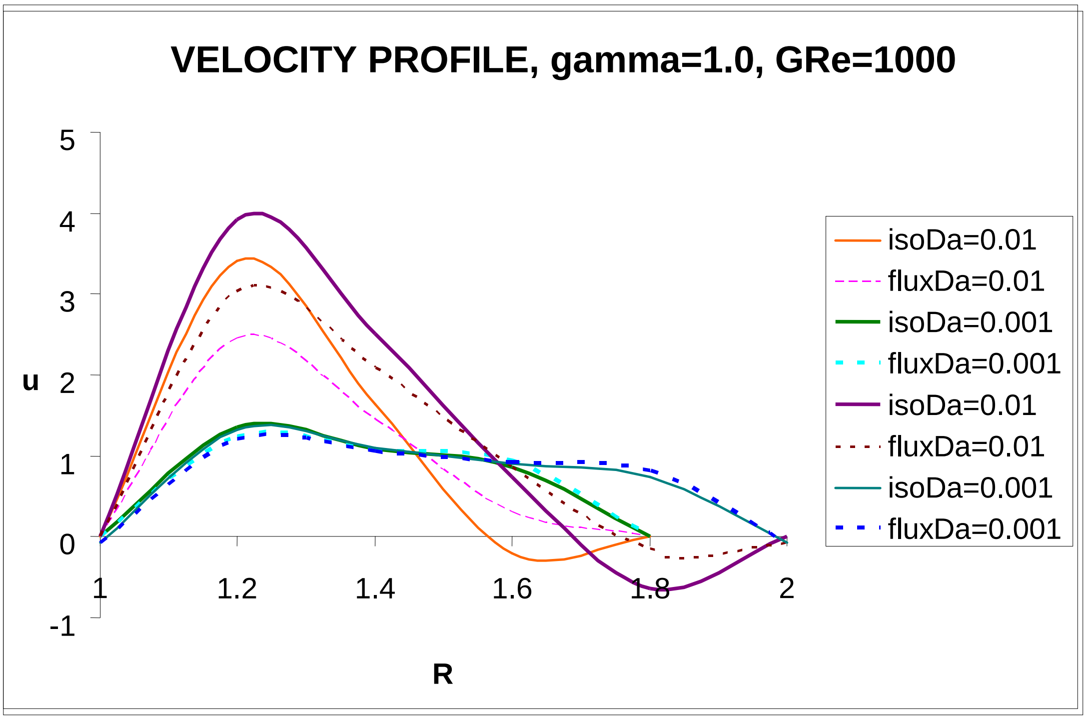
(11)

The values of the constants are in appendix

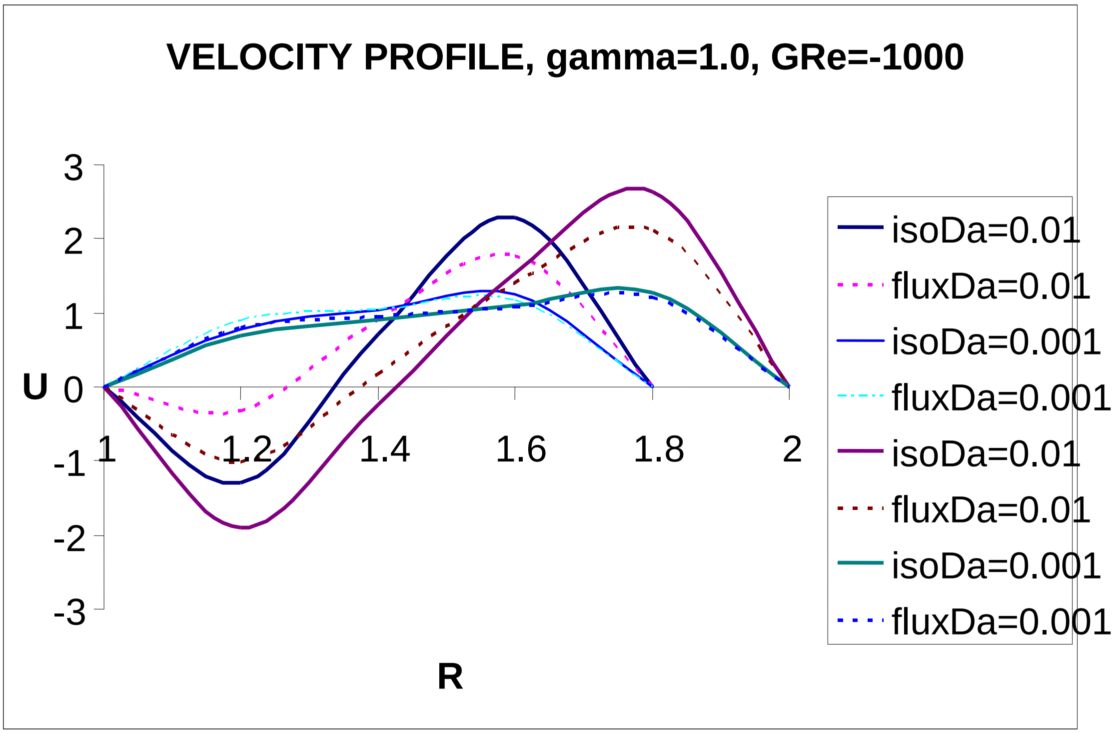
RESULT AND DISCUSSIONS

The velocity distributions of the fluid in the channel formed by two coaxial circular cylinders, as function of Da and λ are illustrated in fig. 1(a&b) for different values of GRe (1000,- 1000) when the gap between the cylinder is less or equal to the radius of inner cylinder ($\lambda \le 2$) , when the gap between the cylinder is greater than the radius of the inner cylinder ($\lambda > 2$) for γ =1 are depicted in Fig 1(c&d).

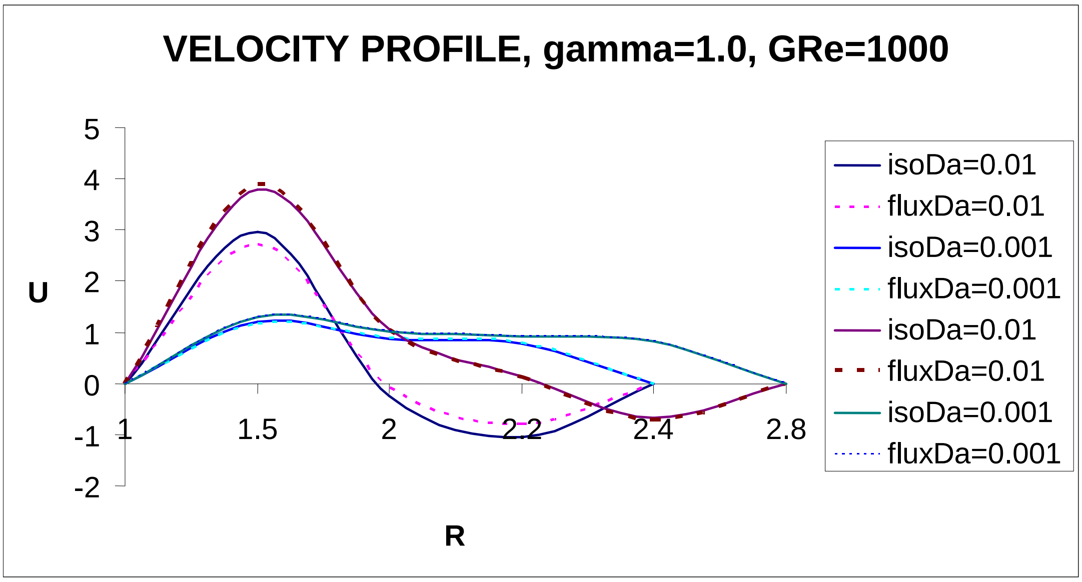
It is observed that for both cases of heating of the inner surface of the outer cylinder, the velocity of the fluid increase with increase in the gap between the cylinders. It is also observed that the reverse flow which is due to the value of GRe manifested in both cases of heating when Da=0.01 for both gaps between the cylinders and values of GRe. Comparison of the velocity distribution curves shows that the velocity is more for isothermal heating of the inner surface of the outer cylinder for Da=0.01 when the gap between the cylinders is less or equal to the radius of the inner cylinder ($\lambda \le 2$). While reverse trends occurs for Da=0.01 when the gap between the cylinders is greater than radius of the inner cylinder. Similarly, the velocity distribution curves show that the velocity is more for isoflux heating for both values of Da (0.01,0.001) and GRe (1000, -1000) when the gap between the cylinders is greater than the radius of the inner cylinder (λ>2) than isothermal heating. It is also observed that an increase in Darcy number Da helps to enhance the fluid flow. This can be explained by the fact that for large Darcy number, the permeability of the medium is large and because of less resistance is offered by the medium on the flow field. It is revealed that, the reversal flow manifested at the inner surface of the outer cylinder for GRe = 1000 and the reverse is the case for GRe = -1000 both gaps between the cylinders respectively.



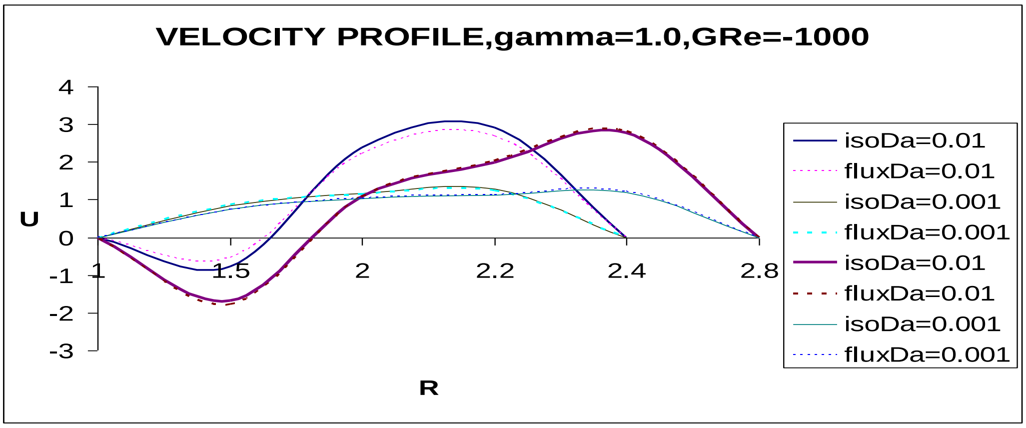
(a) Velocity profile when $\gamma = 1, \lambda \leq 2$ and GRe=1000



(b) Velocity profile when $\gamma = 1, \lambda \leq 2$ and GRe=-1000



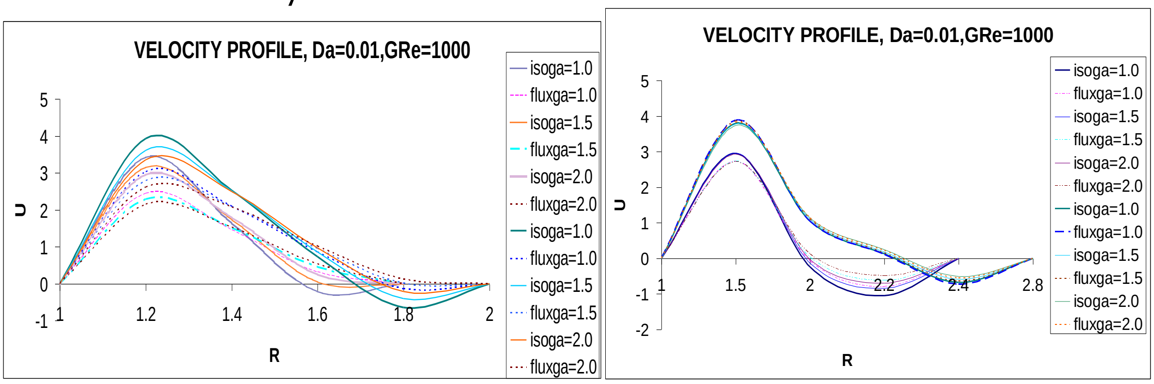
(c) Velocity profile when $\gamma = 1, \lambda$ >2 and GRe=1000



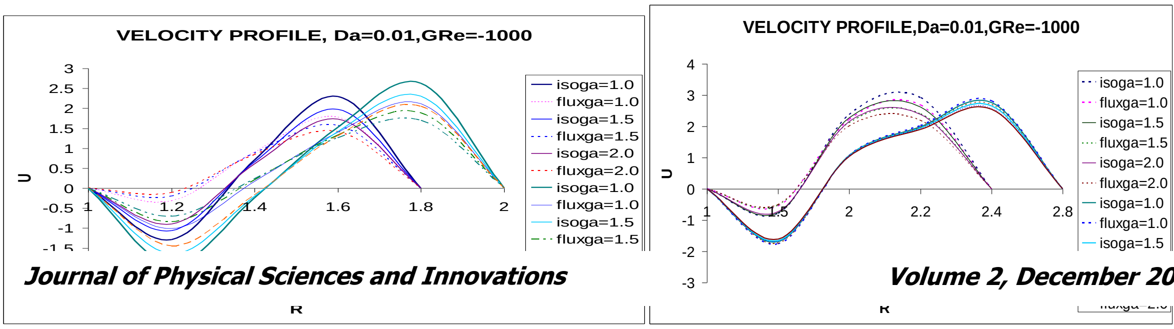
(d) Velocity profile when $\gamma = 1, \lambda$ **>** 2 and GRe=-1000

FIGURE 1

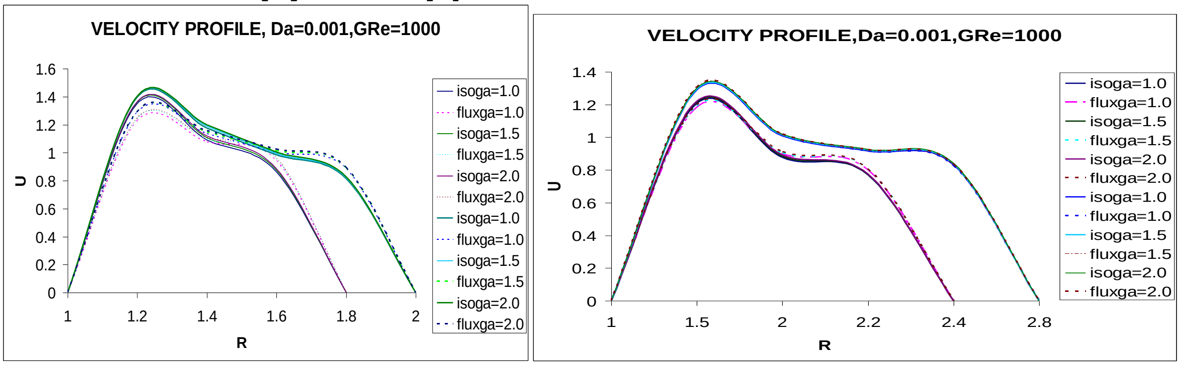
The effect of ratio of viscosities γ on the velocity field is depicted in fig. 2 (a). It is for Darcy number 10^{-2} for the value of GRe=1000, when the gap between the cylinders is less or equal to the radius of the inner cylinder and fig.2 (b) when the gap between the cylinders is greater than the radius of the inner cylinder. It is also depicted in fig.2 (c & d) for Darcy number 10^{-3} for both gaps between cylinders and GRe. Comparison of these figures indicated that the velocity profiles have been more influenced by γ for higher value of the Darcy number. It is noted that, for Darcy number 0.01 and for both gap between cylinders and for all the value of GRe, velocity has increasing tendency with in increase in ratio of viscosities for both cases of heating of the inner surface of the outer cylinder. This means an increase in the ratio of viscosities implies the effective viscosity of the porous medium is greater than the viscosity of fluid for large Darcy number. It is also noted that velocity remains constant with increase in ratio in viscosities for Da=0.001.This clearly means that the impact of viscous term on the flow behavior is insignificant for small values of Darcy number.



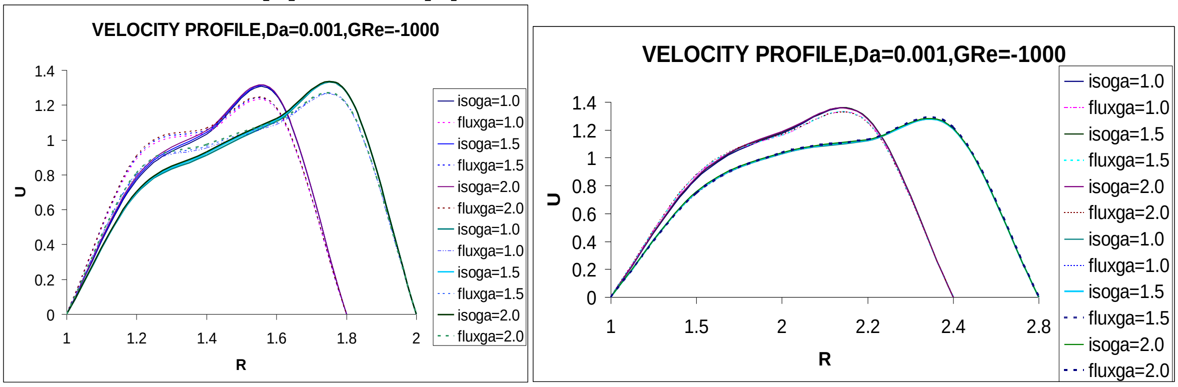
(a) Velocity profile when $\alpha = 0.01$**and GRe=1000**



(b) Velocity profile when $\alpha = 0.01$ **and GRe=-1000**



(c) Velocity profile when $\alpha = 0.001$ **and GRe=1000**



(d) Velocity profile when $\alpha = 0.001$ **and GRe=-1000**

FIGURE 2

The numerical values of skin frictions τ_1, τ_λ , pressure gradient p, and mass flow rate for both values of GRe depicted in Table 1 for isothermal and isoflux cases respectively. The numerical value of the skin friction is more for isothermal condition than isoflux condition. The pressure gradient decreases because of increase in the gap between the cylinders. The mass flow rate increases with increase in the gap between the cylinders.

TABLE 1 Numerical Values of Skin Friction, pressure gradient and mass flux

		ISOTHERMAL				ISOFLUX			
Da	γ	τ_1	τ_λ	P	Q	τ_1	τ_λ	P	Q
0.01	1.0	-35.37770	-39.25449	500.93040	7.03717	-15.52460	-27.95777	344.728	7.03717
	1.5	-26.51730	-29.94459	498.68350		-11.02700	-21.76121	346.019	
	2.0	-21.39022	-24.40847	496.10750		-8.43325	-18.07841	346.882	
0.001	1.0	16.41713	-44.75930	1458.16700		23.68490	-40.00533	1294.065	
	1.5	14.09964	-36.62882	1471.14700		19.94464	-32.84386	1307.699	
	2.0	12.72571	-31.78696	1482.35700		17.72322	-28.58020	1319.486	
0.01	1.0	-39.77547	-39.89415	483.28430	9.42478	-23.85043	-31.10942	370.457	9.42478
	1.5	-30.48918	-30.86931	481.18080		-17.95470	-24.30473	370.301	
	2.0	-25.01785	-25.48226	479.01550		-14.48077	-20.24603	369.971	
0.001	1.0	15.15756	-44.04629	1427.00200		20.79781	-40.58256	1309.538	
	1.5	12.92180	-36.00077	1436.21500		17.47349	-33.22900	1319.153	
	2.0	11.59319	-31.20707	1444.11900		15.49628	-28.84806	1327.410	

The numerical values of critical GRe at the outer surface of inner cylinder (GReh1) and at the inner surface of the outer cylinder (GReh2) are presented in Table 2 for some typical values of non-dimensional parameters. From this Table, it is clear that, GReh1 and GReh2 are inversely proportional to the Darcy number for both cases of isothermal and isoflux heating for a fixed λ. Hence, we conclude that, Darcy number exert a destabilizing influence on the flow formation. Furthermore, it can be observed that the effect of λ on GReh2 as Darcy number decreases in case of isothermal heating for either gap between the cylinders less than or greater than the radius of the inner cylinder. While the value of GReh2 decreases with increase of λ for isoflux heating. Lastly, we conclude that, the occurrence of flow reversal at the outer surface of inner cylinder can be checked by increasing the gap between the cylinders. These critical values of GRe are responsible for the occurrence of flow reversal at the outer surface of the inner cylinder and at the inner surface of the outer cylinder.

TABLE 2 The values of the critical GRe

		When $\lambda \le 2$			when · >2	
CASE	Da	·	Greh1	Greh2	Greh1	Greh2
Isothermal	0.010	1.8	-265.4464	432.3828	-200.7198	366.9798
	0.001		-1931.1470	2881.0440	-1691.6210	3004.8340
	0.010	2.0	-233.5831	393.5128	-183.5639	362.9459
	0.001		-1824.6340	2902.0730	-1608.4620	3132.8360
Isoflux	0.010	1.8	-451.6035	735.6119	-229.2713	419.1808
	0.001		-3285.4560	4901.5120	-1932.2470	3432.2560
	0.010	2.0	-336.9893	567.7190	-178.2833	352.5049
	0.001		-2632.3900	4186.8050	-1562.1910	3042.7130

CONCLUSION

The fully developed flow characteristics including phenomena of flow reversal of the mixed convection in a vertical annulus filled with porous material are investigated theoretically. The most important outcome of the numerical calculations, is that behavior of the flow formation of the fluid as a result of mixed convection between the cylinders filled with porous material and saturated with the same fluid can be controlled by applying a suitable mode of heating process and also by changing the gap between the cylinders. The occurrence of the reversal flow is found to be strongly dependent on the value of $\frac{Gr}{\mathrm{Re}}$=GRe

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NOMENCLATURE

$u'=$ Dimensional Velocity
$r'=$ Dimensional Radial Coordinate
$g=$ Acceleration due to gravity
$\rho=$ Density of the fluid
$\beta=$ Volumetric Dimensional Thermal Coefficient
$T_0^1=$ Initial Dimensional Temperature
$T_1^1=$ Final Dimensional Temperature at the inner wall
$T_2^1=$ Final Dimensional Temperature at the outer wall
$\frac{\delta p'}{\delta z'}=$ Dimensional Pressure Gradient
$a=$ Radius of the Inner Cylinder
$b=$ Radius of the Outer Cylinder
$u_0=$ Average Velocity
$\gamma_f=$ Kinematics Viscosity of the Fluid
$\gamma_{eff}=$ Effective Kinematics Viscosity of the fluid saturated Porous Domain
K= Permeability of the Porous Material
$\mu=$ Dynamic Viscosity of the Fluid
τ_1=Skin friction at the outer surface of the inner cylinder
τ_λ= Skin friction at the inner surface of the outer cylinder
Gr=Grashof number
Re=Reynolds number
Da= Darcy number
T= Dimensionless temperature
P=Dimensionless pressure gradient
GReh1= critical values of GRe1
GReh2=Critical values of GRe

APPENDIX

$$B_1 = \frac{C - Br_t}{A - BLog(\lambda)} \quad , B_2 = \frac{Ar_t - CLog(\lambda)}{A - BLog(\lambda)}$$

$$Z_8 = I_0(Z_1)K_0(Z_2) - K_0(Z_1)I_0(Z_2)$$

$$Z_9 = Z_3K_0(Z_2) - Z_4K_0(Z_1)$$

$$Z_{10} = K_0(Z_2) - K_0(Z_1)$$

$$Z_1 = \frac{1}{\sqrt{\gamma Da}}; Z_2 = \frac{\lambda}{\sqrt{\gamma Da}}$$

$$Z_{11} = \frac{\lambda^2 Log(\lambda)}{2} - \frac{1}{4}\left(\lambda^2 - 1\right)$$

$$Z_{12} = B_2 \frac{\left(\lambda^2 - 1\right)}{2}$$

$$Z_3 = B_2 Da; Z_4 = Da\left[B_1 Log(\lambda) + B_2\right]$$

$$Z_5 = K_0(Z_1)I_0(Z_2) - K_0(Z_2)I_0(Z_1)$$

$$Z_6 = Z_3I_0(Z_2) - Z_4I_0(Z_1)$$

$$Z_7 = I_0(Z_2) - I_0(Z_1)$$

$$Z_{13} = \frac{\left[I_1(\lambda Z_1) - I_1(Z_1)\right]}{Z_1}$$

$$Z_{14} = \frac{\left[K_1(\lambda Z) - K_1(Z_1)\right]}{Z_1}$$

$$Z_{15} = \frac{1}{2} Da\left(\lambda^2 - 1\right)$$

$$C_7 = -\frac{\left[G \operatorname{Re} Z_6 + PDaZ_7\right]}{Z_5}$$

$$Z_{16} = Da\left[B_1 Z_{11} + Z_{12}\right]$$

$$C_6 = -\frac{\left[G \operatorname{Re} Z_9 + PDaZ_{10}\right]}{Z_8}$$

$$Z_{17} = \left[Z_{16} - \frac{Z_9 Z_{14}}{Z_8} + \frac{Z_6 Z_{14}}{Z_5}\right]$$

$$Z_{18} = \left[Da\frac{Z_7 Z_{14}}{Z_5} + Z_{15} - \frac{DaZ_{10}Z_{13}}{Z_8}\right]$$

$$Z_{181} = \frac{\frac{1}{2}\left(\lambda^2 - 1\right)}{Z_{18}}$$

$$Z_{19} = \frac{Z_{17}}{Z_{18}}$$

$$Z_{22} = Z_{21}Z_6 - Z_{20}Z_9 + B_1 Da$$

$$Z_{23} = Z_7 Z_{21} Da - Z_{10} Da Z_{20}$$

$$Z_{27} = Z_{26}Z_6 - Z_{25}Z_9$$

$$Z_{28} = Da\left[Z_7 Z_{26} - Z_{10}Z_{25}\right]$$

$$Z_{24} = \frac{DaB_1}{\lambda}$$

$$Z_{25} = \frac{Z_1 I_1(\lambda Z_1)}{Z_8}$$

$$Z_{29} = Z_{28}Z_{18}$$

$$Z_{30} = Z_{28}Z_{19} - Z_{27} - Z_{24}$$