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CAPITAL BUDGETING DECISIONS AND THE MULTIPLE RATES OF RETURN CONTROVERSY - A REVIEW

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#### Abstract

Capital budgeting as a decision area in finance establishes goals and criteria for investing resources in long term projects. It has a wide application in government as well as in private enterprises. There are various methods of capital budgeting. They are payback period, Accountancy rate of return, Net present value (NPV) Profitability Index (PI) and the Internal rate of return (IRR). Two of the techniques (NPV and IRR), will give the same ranking when two or more independent projects are evaluated. (i.e both will give the same accept- reject decision for independent projects). However, when two mutually exclusive projects are under consideration, a conflict usually occurs in the ranking by either of them. The paper therefore reviewed the various circumstances leading to contradictions between the NPV and IRR and the various bail outs by finance authors. Notwithstanding the fact that the conflict between the NPV and the IRR can be resolved, it was discovered that there are still some grey areas that should be of great concern to authors and this has gone a long way to still confirm the superiority of NPV to IRR as project appraisal techniques.


## KEYWORDS

1. Mutually exclusive projects -These are projects in which when one is taken, the other must be rejected. E.g generation of light by plant or power holding company of Nigeria.
2. Net Present Value (NPV) - This is described as the surplus of the present value of cash receipts over the present value of cash outlay on investment..
3. Internal Rate of Return (IRR)- This is defined as the discount rate which equates the discounted cash inflows from a project to its original cost. It is the discount rate at which a project's NPV equals zero.
4. Non - conventional project -These are projects which reversal of signs in their cash flows. E.g-+-+-+

## INTRODUCTION

Capital budgeting decisions is that decision which establishes goals, and criteria for investing resources in long term projects. It is normally made to actualize the shareholders' wealth maximization objective of a corporate establishment. There are many capital budgeting evaluation criteria to achieve the corporate goal set by the financial manager .These include the traditional methods consisting of the Pay Back Period and the Average Accounting Rate of Return methods. The other methods known as the sophisticated or the discounted cash flow methods consist of the Net Present Value (NPV), Profitability Index (PI) and the Internal Rate of Return (IRR). The latter methods have been adjudged superior based on the
argument that they recognize time value of money and the most popular being the Net Present value (NPV) and the Internal Rate of Return (IRR). In project evaluation process, the Net Present Value (NPV) method and that of Internal Rate of Return (IRR) are seen as twin sisters giving the same investment recommendation. The decision rule of the NPV method is to accept the investment project if its net present value is positive (NPV) $>0$ ) and reject it if the net present value is negative ( $\mathrm{NPV}<0$ ). Positive NPV contributes to the net wealth of the shareholders, which should result in the increased price of a firm's share (Pandey, 2004). On the other hand , the accept - or- reject rule, using the IRR method, is to accept the project if its internal rate of return is higher than the opportunity cost of capital ( $\mathrm{r}>\mathrm{k}$ ). The project shall be rejected if its internal rate of return is lower than the opportunity cost of capital. The decision maker may remain indifferent if the internal rate of return is equal to the opportunity cost of capital and when the NPV is equal to zero (Olowe 1999). According to Van Horne (Ayodele 2002), a number of factors are held responsible for the disagreement between the NPV and the IRR ranking of mutually exclusive projects. These are Timing of cashflows, scale of investment and irregularity of lives of projects. Other sources of bone of contention between the NPV and the IRR (i.e the multiple rates of return and the pump projects problems) form the basis of this paper. An attempt is made to review the various resolutions so far by various authors to assess how adequate or otherwise they are.

## RECONCILIATION OF CONFLICTS BETWEEN NPV AND IRR

1. Timing of cash flows - The problem under this concept arises where and when mutually exclusive projects differ in the way their cash flows are released over their economic lives (Pandey 2004). An example is given below.

| Yr | 0 | 1 | 2 | 3 | 4 | 5 | NPV(A) | IRR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Project <br> A | -1000 | 100 | 20 <br> 0 | 300 | 400 | 1250 | 530.95 | 22.70 <br> $\%$ |
| Project <br> $B$ | -1000 | 200 | 30 <br> 0 | 500 | 500 | 600 | 519 | 25.39 <br> $\%$ |

From above, the two rules favour different projects using $10 \%$ cost of capital despite the two projects have the same outlay and the same time horizon. To resolve the conflict, we assume a common re-investment rate in order to find the terminal sums (TS) of the two projects. If we assume $24 \%$ re-investment rate we have:

$$
\begin{aligned}
& \operatorname{TS}(A)=100(1.24)^{4}+200(1.24)^{3}+--+1250(1.24)^{0}=N 2,825 \\
& \operatorname{NPV} \operatorname{TS}(A)=2,825 /(1.1)^{5}-1000=\mathrm{N} 754
\end{aligned}
$$

Also TS $(B)=200(1.24)^{4}+300(1.24)^{3}+--600(1.24)^{0}=N 3,032.82$
NPV TS(B) $=3032.82 /(1.1)^{5}-1000=N 883$.

The decision rule here is that since the NPVTS(B)>NPVTS(A), Project B should be accepted being more profitable than project $A$. This now lend support to the IRR ranking.
2.

Scale of investment - when the cash outlays are of different sizes, the NPV and IRR methods tend to give contradictory ranking to the projects that are mutually exclusive.
Assume that the minimum acceptable discount rate for projects accepted by a firm is $10 \%$ and that it is now considering two mutually exclusive projects B and C with the following cash flow patterns and computed NPVs and IRRs.

| Project | Initial <br> outlay | Cash <br> flows | NPV | Ranking | IRR | Ranking |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| B | N5000 | 7000 | N <br> 1364 | 2nd | $40 \%$ | $1^{\text {st }}$ |
| C | N8000 | 11000 | N <br> 2000 | 1st | $37 \%$ | $2^{\text {nd }}$ |

From the above, the NPV favours project $C$ which the IRR rule favours project $B$.
To resolve the conflicts, we subtract the smaller project (B) from the bigger one (C) to get the incremental project D below.

| B | N 5000 | 7000 | 1364 | 2nd | $40 \%$ | 1st |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| C | N8000 | 11000 | 2000 | 1st | $37 \%$ | 2nd |
| D C C-B | N3000 | N4000 | N636 |  | $33 \%$ |  |

From the evaluation above, the incremental project has positive NPV and has an IRR greater than the cost of capital (10\%) So, it is an acceptable project, hence we choose the project that gives birth to project $D$, which is the mother project $C$.
3. Irregularity of Lives - when two mutually exclusive projects have different economic lives, the NPV and IRR will give conflicting results. This can be seen below.

Consider the data below with 10\% cost of capital

| Yr | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| Project E | -1000 | 600 | 600 | - |
| Project F | -1000 | 400 | 400 | 475 |

The NPVS and IRRS computed from the data give:
$N P V_{E}=N 41.32 \quad I R R_{E}=13 \%$
$\mathrm{NPV}_{\mathrm{F}}=\mathrm{N} 51.36 \quad$ IRR $_{\mathrm{F}}=12.8 \%$
Therefore, NPV recommends project F while IRR recommends project E. in resolving the conflict, the two projects cash flows can be replicated to the same shortest duration (which is six years) and be re-evaluated as shown below.

Project E repeated three times

| Yr | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Project E | -1000 | 600 | 600 | - | - | - | - |
|  |  |  | -1000 | 600 | $\begin{aligned} & \hline 600 \\ & -1000 \\ & \hline \end{aligned}$ | $600$ | $600$ |
|  | -1000 | 600 | -400 | 600 | -400 | 600 | 600 |
|  |  |  |  |  |  |  |  |

Project F repeated two times can be shown below

| Yr | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Project F | -1000 | 400 | 400 | 475 | - | - | - |
| $=$ | -1000 | 400 | 400 | -525 | 400 | 400 | 475 |
| NPV $_{\text {A }}(3,2)=-1000+400$ | $+400+-525$ | +400 | +400 | $+\underline{475}$ |  |  |  |

1.1
(1.1) ${ }^{2}$
(1.1) ${ }^{3}$
$(1.1)^{4}$
$(1.1)^{5}(1.1)^{6}$
$=\mathrm{N} 89.50$
From the evaluation, project E is more preferable and should be accepted in line with IRR's decision.

## Multiple Rates of Return Problems

This problem normally arises in cases with non-conventional investments (projects).
This occurs when some of the net cash flows are negative. Conventional projects have only change in sign while all other cash flows carry positive signs (e.g. -+++). Non conventional projects have reversal of signs in their cash flows (e.g-+-+-+-+) (Van Horne, 1989). For instance, to find the IRR of the following cash flows -100, 270, -180 for years zero, one and two respectively, we calculate as follows.

$$
\begin{equation*}
N P V=-100+\frac{270-180}{1+k \quad(1+k)^{2}} \tag{1}
\end{equation*}
$$

Remember, IRR is that rate of interest that equates the NPV to zero.
Let $1+\mathrm{k}=\mathrm{x}$.
When $\mathrm{k}=\mathrm{IRR}$
NPV $=-100+\frac{270}{x}-\frac{180}{x^{2}}=0$
Multiplying through by $x^{2}$ we have

$$
\begin{gather*}
N P V=-100 x^{2}+270 x-180=0 \\
N P V=-10 x^{2}+27 x-18=0 \tag{3}
\end{gather*}
$$

Solving for x with quadratic formula

$$
\begin{aligned}
& \quad X=1.50 \text { or } 1.20 \\
& \text { But } x=1+k \\
& 1+\mathrm{k}=1.50 \text { or } 1.20 \\
& \text { IRR }=\mathrm{k}=1.50-1 \text { or } 1.20-1 \\
& 0.50 \text { or } 0.20 \\
& 50 \% \text { or } 20 \%
\end{aligned}
$$

From the result above, it shows that IRR is not consistent by giving two results under which the project would be accepted because both $50 \%$ and $20 \%$ are above the cost of capital (10\%) which gives negative NPV (-3.3) as a unique value. To corroborate Bello (1998), we will be left confused to make investment recommendation at any cost of fund that is used. Comparing the cost of fund in-between the two rates with the lower rate of $20 \%$. IRR will recommend to reject the project whereas if the same cost of fund is compared with the higher rate of $50 \%$ it will recommend to accept the project. This is termed inconsistency. Viewing consistency from another angle is that if a project is acceptable at a cost of fund say $x$ per cent, the same project should be acceptable at any cost of fund below $x$ per cent or if a project is not acceptable at a cost of fund, such a project should not be acceptable at any cost higher than that cost, otherwise the acceptance would not be consistent (Koutsoyiannis, 1982 and Van Horne, 1989). However, from our analysis above, the problem of multiple rates of return can be resolved with the use of a modified rate by discounting each negative cash flows as the firms cost of capital. That is, we discount -180 to time 1, and to 270 . The aim of this is to obtain a revised stream of cash flow which contains no negative cash flows.

Using cost of capital of $15 \%$ we have:

| Year | 0 | 1 | 2 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | -100 | 270 | -180 |  |
| $+(\underline{180})$ |  |  |  |  |

The result now has become a conventional type, hence we can now determine the rate. $-100+113.48$

$$
1+k=0
$$

$100(1+k)=113.48$
$1+k=\underline{113.48}$
$100=1,1348$

$$
\begin{aligned}
K & =0.1348 \\
& =13.48 \%
\end{aligned}
$$

Since $13.48<15 \%$ opportunity cost of capital, the project would be rejected in line with the decision of the NPV which gives negative value.

## The "Pump project"

The pump project is a classical example of multiple rates of return project often cited in literature to back the argument of superiority of NPV technique over the IRR in nonconventional projects' evaluation. Here it is assumed that an oil company is trying to decide whether or not to install a high-speed pump in a well which is already in operation. The pump will cost N 1600 to install. During its first year operation it will produce N10,000 more oil than the old pump which is currently in use but during the second year, the high speed pump produces $\mathrm{N} 10,000$ less oil because the well has been depleted. Using $10 \%$ cost of capital, the NPV rejects the acceptance of the high-speed pump project by giving -773.55 as a unique solution while the project will be accepted by IRR since both IRRs of the investment ( $25 \%$ or $400 \%$ ) exceed the opportunity cost of capital of $10 \%$. The IRR calculation leads to multiple rates.

The illustration is given below:

| Yr | 0 | 1 | 2 | NPV | IRR |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | -1600 | 10000 | -10000 | -77355 | $25 \%$ | OR |
|  |  |  |  |  | $40 \%$ |  |



From the NPV profile, the conflict between NPV and IRRs established conspicuously at a cost of fund below $25 \%$, say $10 \%$. While NPV rejects the projects at that rate, the IRR accepts it. This conflict seems to have been resolved by Teichroes et al (Bello,1998) suggestion that negative and positive terms in the cash stream of a project should be interpreted differently as investment and financing (source of funds) respectively, thereby allowing for the application of different borrowing and lending rates instead of a single rate in the evaluation process of a conventional project. Hence, every non-conventional project is to be seen as a mixed project of borrowing and lending. The negative terms are outflows to the project (i.e the firm lending money to the project), an investment on the part of the firm and should therefore be discounted at the firm's lending rate (the project's borrowing rate) which should be the IRR. The positive terms on the other hand are inflows to the firm (i.e the firm borrowing money from the project), a source of fund to the firm and should therefore be discounted at the firms borrowing rate (project's lending rate) which should be market rate of interest because the alternative source of borrowing open to the firm is the capital market. The application of different rates to the inflows and outflows tends to resolve the multiple rates problem and the conflict between the two criteria.

From the foregoing, the net compound value (NCV) can be calculated thus:

= -N936--------------------------------------------------------14

From the result, it shows that both NPV and NCV give the same recommendation to reject the project in question. To support Bello (1998), the IRR can be calculated thus: Period zero is an outflow of N1600 and so should attract the first lending rate as: -1600 (1+IRR)--------eq(5)

The second period is an inflow of $\mathrm{N} 10,000$ and should attract the firms borrowing rate (the market rate of interest(1). However there is need to determine the actual amount the firm borrowed or sourced from the project by subtracting the first period amount being borrowed.

This gives 10,000-1600 (1 + IRR) -------------eq (6)
Equation six represent the effective borrowed sum. It is the amount that should be charged the cost of fund (k). Thus we have:(10000-1600(1+IRR) (1+k)-------- eq(7)

The third period is an outflow of 10000 and it ends the transaction between the project and the firm. Therefore, it is best interpreted as the repayment of the principal and interest loan amount borrowed from the project in the second period. Then we have ;

```
(10,000 - 1600(1+IRR) ) (1+k) - 10000 = 0------------------eq(8)
(10,000-1600(1+IRR) (1+k) = 10000---------------------------eq (9).
```

At $10 \%$ cost of fund, the solution becomes:
(10000-1600(1+IRR)(1.1) = 10000 ---------------eq(10)
$11000-1760(1+$ IRR $)=10000$------------------eq(11)
-1760 (1+ IRR ) = -1000 -----------------------------eq(12)
1 + IRR = 0.5682 --------------------------------------eq(13)
IRR $=-0.4318$ or43.18\%
eq(14)

This is the compounding value IRR (CV IRR).
By the calculation the conflict between NPV and IRR is resolved because both methods reject the project at $10 \%$ cost of fund. We can as well use the present value approach to generate what can be called the present value IRR (PVIRR). If the internal rate of return criterion is consistent, all things being equal, both CVIRR and PVIRR are supposed to give the same recommendation on the same project just as the net present value (NPV ) of -773.55 and net compound value (NCV) of -936 giving the same recommendation of rejection of the project.

For PVIRR calculation, we go thus:
$-\underline{1600}+\underline{10,000}-\underline{10,000} \quad=0$--------------eq (15)
$1+$ IRR $(1+0.1)(1+\text { IRR })^{2}$
$-\underline{1600}+\underline{10000}-\underline{10000}=0 \quad----------------------e q(16)$
$-1600+9090.9091-\underline{10000}=0$-----------------eq (17)

$$
\begin{aligned}
& -10,000 \quad=-9090.9091+1600 \text {----------------eq (18) } \\
& (1+\text { IRR })^{2} \\
& =-7490.9091 \\
& (1+\text { IRR })^{2}=1.3350---------------------e q(19) \\
& (1+\text { IRR })=\sqrt{ } 1.3350=1.1554-\cdots--------e q(20) \\
& \text { IRR }=0.1554 \text { or } 15.54 \% \text {-------------eq(21) }
\end{aligned}
$$

The results confirmed that both $15.54 \%$ and $-43.18 \%$ are IRR of the same project calculated at present value method and compound value method respectively. In other words, $15.54 \%$ PVIRR is equivalent to $-43.18 \%$ CVIRR of this project. Contrary to expectation it is observed that the two values gave conflicting recommendations on the projects: the PVIRR accept the project while the CVIRR rejects it at the same cost of fund of 10 per cent.

## DISCUSSION

From the foregoing, we can see that NPV is generally more consistent and applicable to the evaluation of non-conventional projects than IRR. Firstly, the use of higher interest rate ( in situation of multiple rates problem) to discount a year's cash flow back to immediate previous year may not always, be the case as suggested by some authors. If a lower rate of interest than $10 \%$ is used, we may still be faced with the contradictions between IRR and NPV in their project recommendations. Secondly, to render support to Owuala (2000), the assumption implicit in the NPN method is more realistic since a project's worth is better evaluated against the opportunity cost of financing it. And the present values involved in the NPV method can be appropriately measured in today's monetary values summed up. Thus, if faced with two projects $A$ and $B$, then their combined NPV is given as NPV(A +B) $=N P V(A)+$ $N P V(B)$. This is known as the value additivity principle (VAP). IRR is not amenable to this.

Finally, on the use of " pump project" discussed and assessed, the compound value IRR ($43.18 \%$ ) was being compared with the net present value (-773.46). Where as compound value IRR ( $-43.18 \%$ should be compared with net compound value of -936 . Agreed that they both give negative values and will reject the project (i.e the same recommendation) and by implication resolution to the conflict. However, when PVIRR of $15.54 \%$ is compared with NPV of -773.46 , which should have been the case, the conflict still exist at $10 \%$ cost of fund (i.e NPV rejects the project while IRR accepts the same project. Therefore, the latter situation is the ideal in real life phenomenon because investment decisions are taken at time zero and not after the expiration of the life span of a project. It follows then that the focus should be on NPV and PVIRR which is practicable instead of NCV and CVIRR which are only relevant in theory.

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