APPLICATIONS OF GAMMA MODEL IN AGE DURING PREGNANCY AND MALARIA INFECTIONS AMONG PREGNANT WOMEN

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ABSTRACT
The study was carried out to determine the probability distribution of the age at pregnancy and malaria infection in south western Nigeria. The samples studied consisted of eighty-eight (88) and seventy-seven (78) pregnant women who were already infested of malaria parasites at University of Lagos Teaching Hospital (LUTH), Idi-Araba and Obafemi Awolowo University Teaching Hospital (OAUTH), Ile-Ife, respectively. Some descriptive measures were used to examine the main properties of the distributions for each location and combine. The frequency polygons were obtained to determine the underlying distribution. The distributions were negatively skewed; a departure from normality. Beta distribution was estimated and one of the parameters was found to be out of bound (LUTH: α = -1404.27, β = 1358.13, OAUTH: α = -1342.85, β = 1299.95, COMBINE: α = -1204.26, β = 1156.63). The parameters of the Exponential distributions were not significant. Gamma model was fitted for each location and combine. The estimates were significant at 95% level of significance. Parameters' tests reveal that there is no significant difference between the parameters from the two locations. This implies the samples from the two locations could have come from the same population.
Key Words: Probability, Malaria, Pregnancy, Gamma, Beta, exponential, Model

INTRODUCTION
Malaria is a life threatening parasitic disease. Malaria infection during pregnancy is a major public health issue in tropical and subtropical regions throughout the world [1]. Ninety percent of the global malaria occurs in Africa [1, 2]. The consequence of malaria on pregnant women cannot be over stressed.

Pregnant women and the unborn children are vulnerable to malaria which is a major cause of prenatal mortality, low birth weight and maternal anaemia [3].

In the past, work has been done on the Age-Structure gametocyte allocation which links immunity to epidemiology in malaria parasit In the study, a model was proposed and found that mosquito infection rates correlate positively with individual gametocyte density [4]. Another study inferred that malaria disease is dangerous especially an infection with *Plasmodium falciparum* is more dangerous during pregnancy. Some probability models on age during a particular event were fitted in the literature. This includes probability distributions related to waiting time for consecutive conceptions or birth intervals, and the distributions of the actual age at marriage have been quite popular among researchers in mathematical demography [5]. Considerable attention has not been given to the age at risk of malaria infection among pregnant women. A distribution for age at first marriage in a cohort determined by a three parameter exponential risk function defined by

$$\eta(x) = \beta \ell^{-\gamma \ell^{-\delta x}}$$

for x>0 and β , γ, δ are positive constants.

The distribution of age at first marriage was used as the distribution of the convolution of a number of exponentially distributed components. Their distributions as the convolution of a normal distribution of age of entry into a marriage and a few as three exponentially distributed delays received remarkable agreement between theoretical and observed values[6]. The lognormal probability model of the age at marriage among those who eventually marry was also proposed [7].

Gamma Model

There are two ways of parameterzing the gamma model that are common in the literature. In addition, different authors use different symbols for shape and scale parameters. Below are some two ways of parameterizing gamma model with **γ = α,** the shape parameter, and **b = 1/β = β,** the scale parameter.
In this research work the parameters α, β is considered because the EXCEL uses **α, β** while the Dataplot uses **γ, β**.
For a random variable **X**, the PDF of gamma model is given below as:
$f(X, \alpha, \beta) = \beta^{\alpha}/\Gamma(\alpha)\, X^{\alpha-1} e^{-\beta X}$, or
$f(X, \alpha, \beta) = 1/(\beta^{\alpha}\, \Gamma(\alpha))\, X^{\alpha-1} e^{-X/\beta}$, with parameters α, β and
$f(X, \gamma, \beta) = 1/(\beta^{\gamma}\Gamma\gamma)\, X^{\gamma-1}e^{-X/\beta}$, with parameters γ, β.
The CDF is as follows:
$F(X) = {}_0\int^{\infty} f(X)\, dx$.
RELIABILITY: $R(X) = 1- F(X)$
FAILURE RATE: $h(X) = F(X) / R(X)$.
MEAN: α / β or $\alpha\beta$ or $\gamma\beta$
VARIANCE: α / β^2 or $\alpha\beta^2$, or $\gamma\beta^2$.
When **α=1**, the gamma reduces to an exponential model with **β =λ**

The aim of the study is to determine the probability distribution of the age at risk of malaria infection among pregnant women with the applications of Gamma model.

MATERIALS AND METHODOLOGY

125 blood samples each were collected from pregnant women attending ante-natal clinic of University of Lagos Teaching Hospital (LUTH) and Obafemi Awolowo University Teaching Hospital (OAUTH) out of which eighty-eight were positive at LUTH and seventy-eight were positive at OAUITH. The positive individuals were distributed along age and subjected to statistical methods. Frequency polygons were constructed for each location and combine to determine the underlying distribution using GenStat Disc3 edition (a standard statistical package for analysing and fitting of models). The parameters of gamma models were estimated and fitted to the distributions. Likewise, the parameters of beta and the exponential models were carried out.

ESTIMATING THE PARAMETERS OF GAMMA MODEL

In Statistics, the method of moments is a method of estimation of population parameters by equating sample moments with unobservable population moments and then solving those equations to be estimated. For a gamma model with probability density function
$f(x)= [\ \beta^{\alpha} / \Gamma(\alpha)]\, X^{\alpha-1} e^{-\beta X}$

The first moment, i.e., the expected value, of a random variable with this probability model is $E(X) = \alpha / \beta$

And the second moment, i.e., the expected value of its square, is

$E(X^2) = [\alpha (\alpha + 1)] / \beta^2$

These are the population moments.

The first and the second sample moments m_1 and m_2 are respectively

$m_1 = (X_1+X_2+ . . . X_n) / n$

And

$m_2 = (X1+X2+ . . . Xn) / n.$

Equating the population moments with the sample moments, we get

$\alpha / \beta = m_1$

And

$[\alpha (\alpha + 1)] / \beta^2 = m_2$

Solving the above equations for α and β, we get

$\beta = m_1 / (m_2 - m_1^2)$

$\alpha = m_1^2 / (m_2 - m_1^2)$

We then use the two quantities as estimates, based on the sample, of the two unobservable population parameters α and β.

GENERALIZED METHOD OF MOMENTS (GMM)

The generalized method of moments is a very general statistical method for obtaining estimates of parameters of statistical models. It is a generalization, developed by Lars Peter Hansen of the method of moments [8].

The term **GMM** is very popular among econometricians but is hardly used at all outside of economics, where the slightly more general term estimating equations is preferred. The method is closely related to the classical theory of minimum chi-square estimation.

The idea of generalized method of moments is to use moment conditions that can be found from the problem with little effort. We assumed that the data are a stochastic process

(X_1, X_2, . . .). In mathematical language, we start out with a (vector value) function f that depends on the parameter, $\boldsymbol{\theta = \theta_0}$, i.e.

$E[f(Y_i,\ \theta_0) = 0.$

To turn this function into a parameter estimate, we minimize the associated chi-square Statistic

$\theta = \arg_{min} (\Sigma f (Y_i, \theta))^T A (\Sigma f (Y_i, \theta))$

where superscript ***T*** is used for transpose, and ***A*** is a positive definite matrix. ***A*** may be known a priori or estimated from sample.

The parameterized gamma model is given as

$$f(X,\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} X^{\alpha-1} \ell^{-x\beta} \quad x>0,\ \alpha,\beta>0$$

Where X is the age at pregnancy, β is the scale parameter and α, the shape parameter.

The following parameters were estimated using the GenStat disc3 edition for each location and combine:

The parameterized beta1 model

Beta Type 1 (Finite Range)

f(x) = x**(α-1).(1-x)**(β-1)/Beta(p,q)

The parameterized exponential model is stated below

$f(X,\beta)=\beta\ell^{-\beta x}$, x>0, β>0

The tests were insignificant at 95% significance level.

RESULTS AND DISCUSSION

This study presents the probability distribution of the age at pregnancy and malaria parasites infection at Lagos University Teaching Hospital, Idi-Araba Lagos and Obafemi Awolowo University Teaching Hospital, Ile-Ife In the year 2008. The frequency polygon for each location and combined are presented in figures 1, 2, and 3. The distributions were negatively skewed, a departure from normality. The gamma distribution fits into the data. The parameters' tests for each location and combine were significant at α= 0.05 (Table 1). Also one of the parameters of the beta model was out of bound (Table 2). Therefore the data could not have being generated by a beta distribution. Likewise, the parameters estimated for the exponential distributions were also not significant at α= 0.05.(Table 3). Therefore, it can be concluded that the age during pregnancy and malaria parasites infection can be generated by gamma distribution.

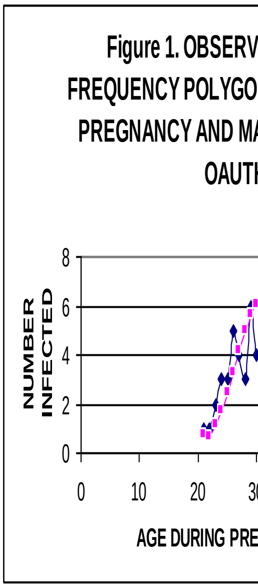

Figure 1. OBSERV FREQUENCY POLYGO PREGNANCY AND MA OAUTH

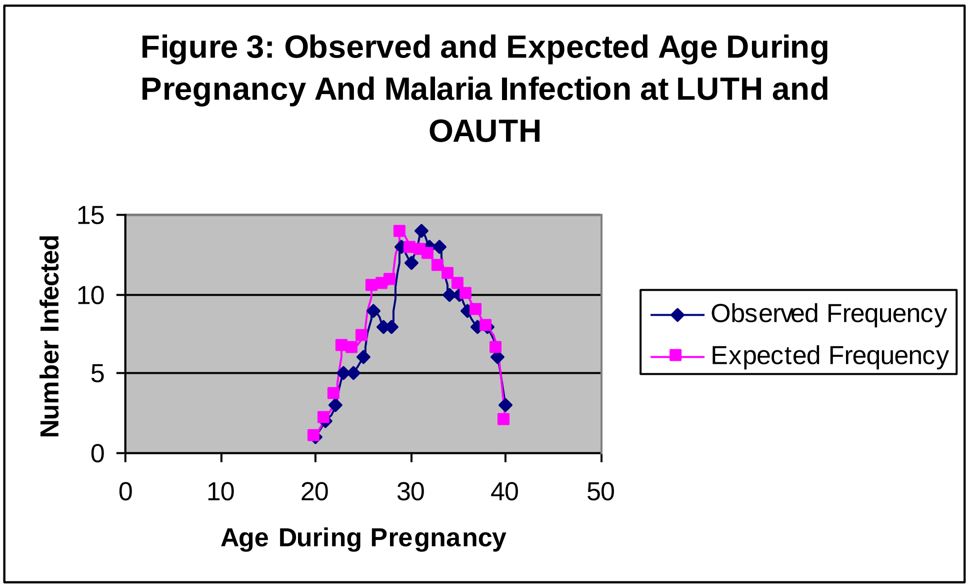

Figure 3: Observed and Expected Age During Pregnancy And Malaria Infection at LUTH and OAUTH

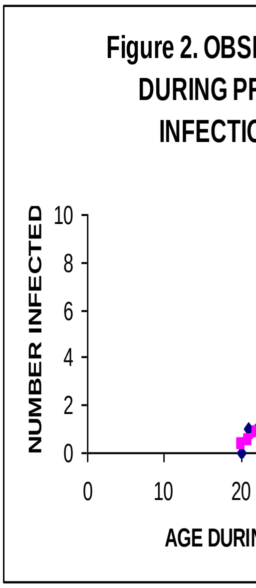

Figure 2. OBS DURING PI INFECTIO

Table 1: Summary of parameter Estimates (Gamma Models)

Table 7 ESTIMATES	**LUTH**	**OAUITH**	**COMBINE**
(α)	44.5636	40.0746	42.0423
Standard error(α)	6.6932	6.3903	4.5964
Confidence interval (α)	31.4449, 57.6823	27.5496, 57.6823	35.0334, 51.0512
(β)	1.4482	1.2691	1.3496
Standard error(β)	0.2187	0.2036	0.1484
Confidence interval(β)	1.0205, 1.8759	0.8700, 1.6682	1.0441, 1.6258
Sample size	88	78	166
Degree of freedom	6	6	10
Deviance	6.65	3.9	11.40

Table 2: Summary of parameter estimates (Beta1 model)

ESTIMATES	**LUTH**	**OAUITH**	**COMBINE**
α	-1404.27	-1342.85	-1404.27
(β)	1358.13	1299.95	1358.63

Table 3: Summary of parameter estimate (Exponential Model)

ESTIMATES	**LUTH**	**OAUITH**	**COMBINE**
B	250.03	-1342.85	-1404.27
Defiance	258	230	393
Degree of freedom	11	11	11

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