# STOCHASTIC MODELING OF DYNAMIC PILE CAPACITY USING HILEY, JANBU AND GATES FORMULAE

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## ABSTRACT

The reliability assessment of the load carrying capacities of piles based on dynamic approach using Hiley, Janbu and Gates formulae is reported in this paper, this has become necessary because pile capacities determined from dynamic formulae have shown poor correlations and wide scatter when statistically compared with static load test results. In practice, uncertainties are common phenomena in engineering, therefore all the interrelated variables in the load carrying capacities of piles should be treated as random variables. Assuming practical probability density functions, the concept of the First-Order Reliability Method (FORM) as a powerful tool for estimating nominal probability level of failure associated with uncertainties is therefore invoked for estimating the implied reliability levels associated with the formulae of Hiley, Janbu and Gates. The results show that there is a good correlation between the implied safety levels in Hiley and Janbu formulae, while those associated with Gates are exceptionally different and grossly conservative.

**Key words:** Stochastic model, dynamic pile capacity, dynamic pile formulae.

**List of symbols**

| | | |
|---|---|---|
| $A$ | = | Pile cross-sectional area |
| $E$ | = | Modulus of Elasticity |
| $e_h$ | = | Hammer efficiency |
| $E_h$ | = | Manufacturers' hammer-energy rating |
| $H$ | = | height of all of ram |
| $K_1$ | = | elastic compression of capblock and pile cap and is a form of PuL/AE |
| $K_2$ | = | elastic compression of pile and is of a form of PuL/AE |
| $K_3$ | = | elastic compression of soil, also termed quake for wave-equation. |
| $L$ | = | Pile length |
| $N$ | = | Co-efficient of restitution |
| $P_u$ | = | Ultimate pile capacity |
| $S$ | = | amount of point penetration per blow. |
| $W_p$ | = | Weight of pile including weight of pile, cap, driving shoe, and capblock (also includes anvil for double-acting steam hammers) |
| $W_r$ | = | Weight of ram (for double-acting hammers include weight of casing). |

## INTRODUCTION

Pile capacity determination is very difficult. A large number of different equations are used, and seldom will any two give the same computed capacity. Organizations which have been using a particular equation tend to stick to it especially when successful data base has been established. It is for this reason that a number of what are believed to be the most widely used (or currently accepted) equations are included in most literature.

Also, the technical literature provide very little information on the structural aspects of pile foundation design, which is a sharp contrast to the mountains of information on the geotechnical aspects. Building codes present design criteria, but they often are inconsistent with criteria for the super structure, and sometimes are incomplete or ambiguous. In many ways this is an orphan topic that neither structural engineers nor geotechnical engineers have claimed as their own (Coduto, 2001).

Dynamic measurements of force and velocity at the upper end of the pile during pile driving, followed by a signal matching procedure, is the most common method for dynamic determination of pile capacity. This method is a convenient tool in the pile driving industry. However, though dynamic methods have been used in practice for years, actual reliability of dynamic methods is vague because their comparison with static loading tests is made incorrectly in most cases.

The well-known dynamic formulae have been criticized in many publications. Unsatisfactory prediction in pile capacity by dynamic formulae is well characterized in the recent published Manual for Design and Construction of Driven Pile Foundations (Hannigan et. al, 1996), in which it was concluded: "Whether simple or more comprehensive dynamic formulas are used, pile capacities determined from dynamic formulae have shown poor correlations and wide scatter when statistically compared with static load test results. Therefore, except where well supported empirical correlations under a given set of physical and geological conditions are available, dynamic formulas should not be used."

There are two attempts to breathe new life into dynamic formulae. First, Paikowsky and Chernauskas (1992) and Paikowsky et al. (1994) have suggested one more simplified energy approach using dynamic measurements for the capacity evaluation of driven piles. Liang and Zhou (1997) have concluded regarding this method: "Although the delivered energy is much more exactly evaluated, this method still suffers similar drawbacks of Engineering News (ENR)". In a second, criticizing the simplified energy approach, Liang and Zhou (1997) have developed a probabilistic energy approach as an alternative to the signal matching technique for effective pile-driving control in the field. Both attempts to improve dynamic formulas, comparison of pile capacity determined by the simplified and probabilistic energy methods with the results of Static Load Tests, are incorrect. Dynamic formulas, including their two new representations, using maximum energy, pile set and maximum displacement from Dynamic Pile Testing do not take into account the time between Static Load Tests and Dynamic Pile Testing (Svinkin,1997).

The purpose of design is the achievement of acceptable probabilities that the structure being designed will not become unfit in any way for the use for which it is intended. Engineering problems of this structure, however, often involve multiple failure modes; that is, there may be several potential modes of failure, in which the occurrence of any one of the potential failure modes will constitute non- performance of the system or component. Recent researches in the area of structural reliability and probabilistic analysis have centered around the development of probabilistic-based design procedures. These include load modeling, ultimate and service load performance and evaluation of current levels of safety/reliability in design (e.g., Farid Uddim, 2000; Afolayan, 1999; Afolayan, 2003; Afolayan and Opeyemi, 2008).

In this paper, a first-order reliability assessment of dynamic pile capacity using Hiley, Janbu and Gates formulae is reported.

**Predicted Dynamic Pile Capacities**

Estimating the ultimate capacity of a pile while it is being driven in the ground at the site has resulted in numerous equations being presented to the engineering profession. Unfortunately, none of the equations is consistently reliable or reliable over an extended range of pile capacity. Because of this, the best means for predicting pile capacity by dynamic means consists in driving a pile, recording the driving history, and load testing the pile. It would be reasonable to assume that other piles with a similar driving history at that site would develop approximately the same load capacity.

Dynamic formulae have been widely used to predict pile capacity. Some means is needed in the field to determine when a pile has reached a satisfactory bearing value other than by simply driving it to some predetermined depth. Driving the pile to a predetermined depth may or may not obtain the required bearing value because of normal soil variations both laterally and vertically.

The basic dynamic pile-capacity formula termed the *rational pile formula* depends upon impulse – momentum principles (Bowles, 1988). The available dynamic pile capacity predictions include:

(a) Canadian National Building Code (use a safety factor, SF = 3)

$$P_u = \frac{e_h E_h C_1}{s + C_2 C_3} \quad (1)$$

in which

$$C_1 = \frac{W_r + n^2(0.5W_p)}{W_r + W_p},$$

$$C_2 = \frac{3P_u}{2A},$$

and

$$C_3 = \frac{L}{E} + 0.0001$$

(b) Danish formula (use SF = 6 )

$$P_u = \frac{e_h E_h}{s + C_1} \quad (2)$$

where

$$C_1 = \sqrt{\frac{e_h E_h L}{2AE}}$$

(c) Gates formula (use SF = 3)

$$P_u = a\sqrt{e_h E_h}(b - \log s) \quad (3)$$

(d) Janbu (use SF = 3 to 6)

$$P_u = \frac{e_h E_h}{k_u s} \quad (4)$$

in which

$$k_u = C_d (1 + \sqrt{1 + \frac{\lambda}{C_d}})$$

where

$$C_d = 0.75 + 0.15\frac{W_p}{W_r}$$

and

$$\lambda = \frac{e_h E_h L}{AEs^2}$$

(e) Modified ENR formula (use SF = 6)

$$P_u = \frac{1.25 e_h E_h}{s + 0.1}\frac{W_r + n^2 W_p}{W_r + W_p} \quad (5)$$

(f) AASHTO (use SF = 6); primarily for timber piles.

$$P_u = \frac{2h(W_r + A_r p)}{s + 0.1} \quad (6)$$

(g) Navy-McKay (use SF = 6)

$$P_u = \frac{e_h E_h}{s(1 + 0.3C_1)} \quad (7)$$

where

$$C_1 = \frac{W_p}{W_r}$$

(h) Pacific Coast Uniform Building Code (PCUBC) (use SF = 4)

$$P_u = \frac{e_h E_h C_1}{s + C_2} \quad (8)$$

where

$$C_1 = \frac{W_r + kW_P}{W_r + W_p}$$

in which

k = 0.25 for steel piles
k = 0.10 for all others
and

$$C_2 = \frac{P_u L}{AE}$$

(i) Hiley

$$P_u = \frac{e_h E_h}{s + \frac{1}{2}(k_1 + k_2 + k_3)} \frac{W + n^2 W_p}{W + W_p} \qquad (9)$$

**First Order Reliability Method (FORM)**

The general problem to which FORM provides an approximate solution is as follows. The state of a system is a function of many variables some of which are uncertain. These uncertain variables are random with joint distribution function $F_x(x) = P(\bigcap_{i=1}^{n}\{X_i \le x_i\})$ defining the stochastic model. For FORM, it is required that $F_X$ (***x***), is at least locally continuously differentiable, i. e., that probability densities exist. The random variables ***X***$= (X_1, ...X_n)^T$ are called basic variables. The locally sufficiently smooth (at least once differentiable) state function is denoted by g(***X***). It is defined such that g(***X***)>0 corresponds to favourable (safe, intact, acceptable) state. g(***X***)=0 denotes the so-called limit state or the failure boundary. Therefore, g(***X***)<0 (sometimes also g(***X***)$\le$0) defines the failure (unacceptable, adverse) domain, F. The function g(***X***) can be defined as an analytic function or an algorithm (e.g., a finite element code). In the context of FORM it is convenient but necessary only locally that g(***X***) is a monotonic function in each component of ***X***. Among other useful information FORM produces an approximation to

$$P_f = P(X \in F) = P(g(X) \le 0) = \int_{g(x) \le 0} dF_X(x) = \phi(-\beta_R) \qquad (10)$$

in which $\beta_R$ = the reliability or safety index, (Melchers, 2002).

**Reliability Estimates**

**Dynamic pile capacity using Hiley formula**

The functional relationship between allowable design load and the allowable dynamic pile capacity using Hiley formula can be expressed in terms of the safety margin given as:
G(x) = Allowable Design Load – Allowable Pile Capacity
which implies,

$$G(x) = 0.35 f_y A_p - \left[ \frac{e_h E_h}{s + 0.5(k_1 + \frac{P_u L}{AE} + k_3)} \frac{W + n^2 W_p}{W + W_p} \right] f^* , \qquad (11)$$

$$f^* = \frac{1}{SF}$$

Table 1 shows the assumed statistical values and their corresponding probability distributions.

**Dynamic pile capacity using Janbu Formula**

On the basis of the dynamic pile capacity predicted by Janbu, the level of safety margin may be given as:

$$G(x) = 0.35 f_y A_p - \left[ \frac{e_h E_h}{(0.75 + 0.15\frac{W_p}{W_r})} \left( 1 + \sqrt{1 + \frac{\frac{e_h E_h L}{AEs^2}}{0.75 + 0.15\frac{W_p}{W_r}}} \right) s \right] f* \quad (12)$$

From Eqn. (12), the statistical and probabilistic descriptions of the variables in the functional relations are presented in Table 2.

**Table 1- Stochastic model for dynamic pile capacity using Hiley formula**

| Variables | Probability density function | Mean values | Coefficients of variations |
|---|---|---|---|
| $F_y$ | Lognormal | 460 x $10^3$kN/m$^2$ | 0.15 |
| $A_p$ | Normal | 1.60 x $10^{-2}$ m$^2$ | 0.06 |
| $e_h$ | Normal | 0.84 | 0.06 |
| $E_h$ | Lognormal | 33.12 kN/m | 0.15 |
| S | Lognormal | 1.79 x $10^{-2}$ m | 0.15 |
| $K_1$ | Lognormal | 4.06 x $10^{-3}$ m | 0.15 |
| $P_u$ | Lognormal | 950kN | 0.15 |
| L | Normal | 12.18m | 0.06 |
| E | Lognormal | 209 x $10^6$ kN/m$^2$ | 0.15 |
| $K_3$ | Lognormal | 2.54 x $10^{-3}$ m | 0.15 |
| W | Gumbel | 80 kN | 0.30 |
| N | Lognormal | 0.5 | 0.15 |
| $W_p$ | Lognormal | 18.5 kN | 0.15 |
| SF | Lognormal | 4.0 | 0.15 |

**Table 2- Stochastic model for dynamic using Janbu formula**

| Variables | Probabiity density function | Mean values | Coefficient of variations |
|---|---|---|---|
| $F_y$ | Lognormal | 460 x $10^3$kN/m$^2$ | 0.15 |
| $A_p$ | Normal | 1.60 x $10^{-2}$ m$^2$ | 0.06 |
| $e_h$ | Normal | 0.84 | 0.06 |
| $E_h$ | Lognormal | 33.12 kN/m | 0.15 |
| $W_p$ | Lognormal | 18.5 kN | 0.15 |
| $W_r$ | Gumbel | 35.58 kN | 0.30 |
| L | Normal | 12.18m | 0.15 |
| E | Lognormal | 209 x $10^6$ kN/m$^2$ | 0.06 |
| S | Lognormal | 1.79 x $10^{-2}$ m | 0.15 |
| SF | Lognormal | 6.0 | 0.15 |

**Dynamic pile capacity using Gates Formula**

Similar to Hiley and Janbu, the functional relationship between the allowable design load and the allowable dynamic pile capacity using Gates formula can be expressed as:

$$G(x) = 0.35 f_y A_p - \left[ a\sqrt{e_h} E_h (b - \log s) \right] f^* \quad (13)$$

The statistical and probabilistic descriptions of the variables in the functional relations are presented in Table 3.

Table 3- **Stochastic model for dynamic using Gates formula**

| Variables | Probability density functions | Mean values | Coefficients of variables |
|---|---|---|---|
| $F_y$ | LN | 460 x $10^3$kN/m$^2$ | 0.15 |
| $A_p$ | N | 1.60 x $10^{-2}$ m$^2$ | 0.06 |
| a | N | 1.05 x $10^{-1}$ | 0.06 |
| $e_h$ | N | 0.85 | 0.06 |
| $E_h$ | LN | 33.12 kNm | 0.15 |
| b | N | 2.4 x $10^{-3}$ m | 0.06 |
| S | LN | 1.79 x $10^{-2}$ m | 0.15 |
| SF | LN | 3.0 | 0.15 |

Using eqns. (11), (12) and (13) together with the assumed stochastic parameters in Tables 1 to 3, the reliability levels associated with the predictions of Hiley, Janbu and Gates for dynamic pile capacity are estimated.

The implied safety level associated with piling capacity using Gates' formula is grossly conservative, even much more than Hiley and Janbu formulae. The safety level does not change with the area of pile, hammer efficiency, hammer-energy rating and point penetration per blow (Figs. 1 to 4). As is common in practice, the areas of piles, hammer efficiency, hammer energy rating and point penetration per blow are subjected to variations and the results of the assessment are as displayed in Figures 1to 5.

Hiley formula generally and grossly provides a very conservative pile capacity as seen in Figs. 1 to 5. Nevertheless, the safety level does not change with area of pile (Fig. 1) and the point penetration per blow (Fig. 4). As hammer efficiency and hammer energy rating increase, the safety level reduces significantly as in Fig. 2 and Fig.3 respectively. On the other hand, safety level grows with increasing factor of safety as normally expected (Fig. 4).

Just like the Hiley formula, Janbu formula leads to a grossly conservative pile capacity. However, Janbu's prediction is not as conservative as Hiley's with respect to hammer efficiency and hammer-energy rating. Generally Gates' formula yields the most grossly conservative prediction compared to Hiley and Janbu. It is noted that safety level is not dependent on the area of pile, hammer efficiency, hammer-energy rating and point penetration per blow (Figs.1 to 5).

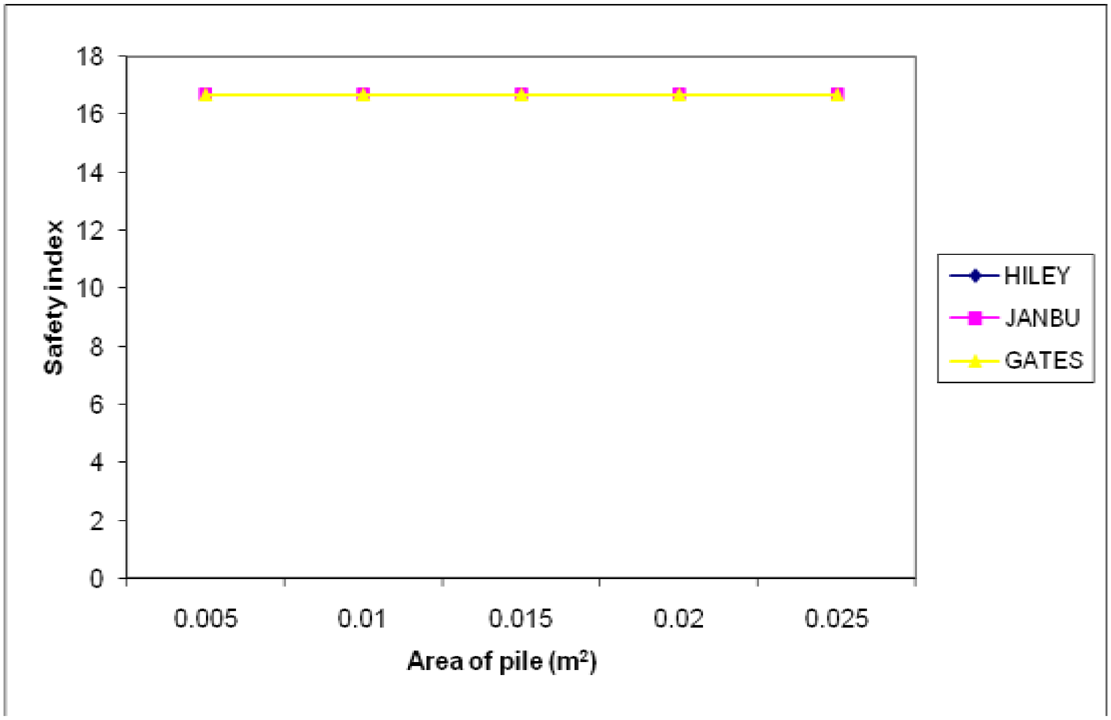


**Fig. 1. - Safety index ( $\beta_R$ ) against Area of pile using Hiley, Janbu and Gates formulae**

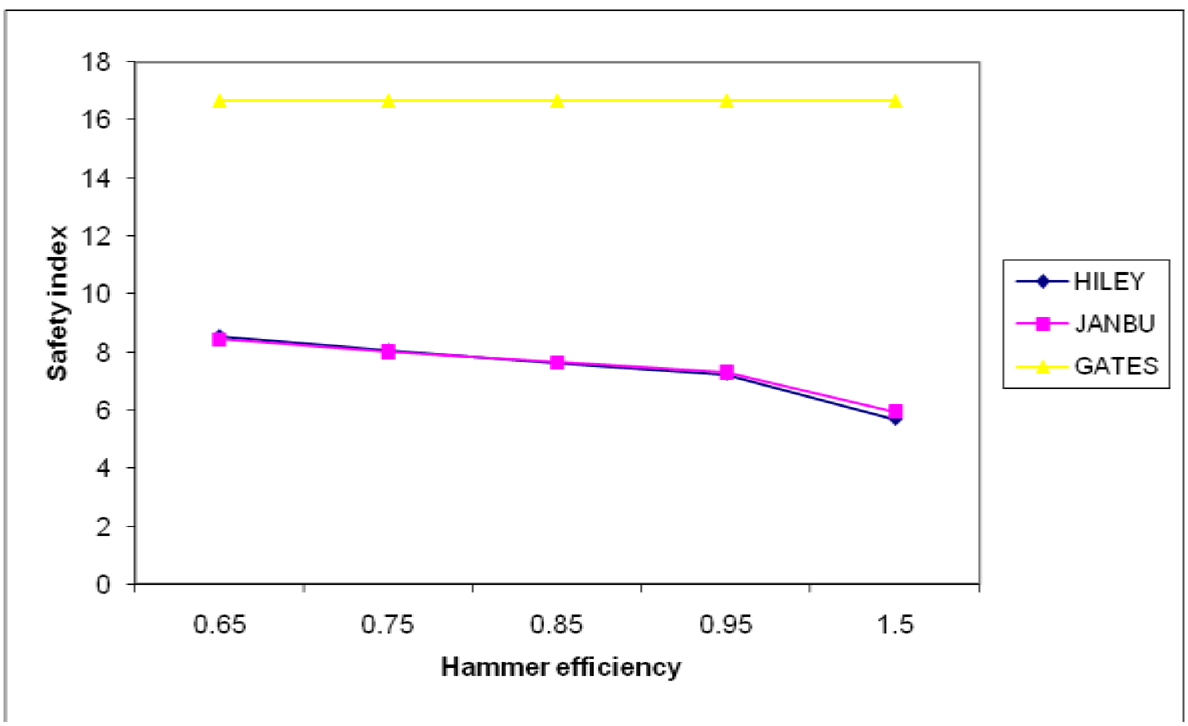


**Fig. 2. - Safety index ( $\beta_R$ ) against Hammer efficiency using Hiley, Janbu and Gates formulae**

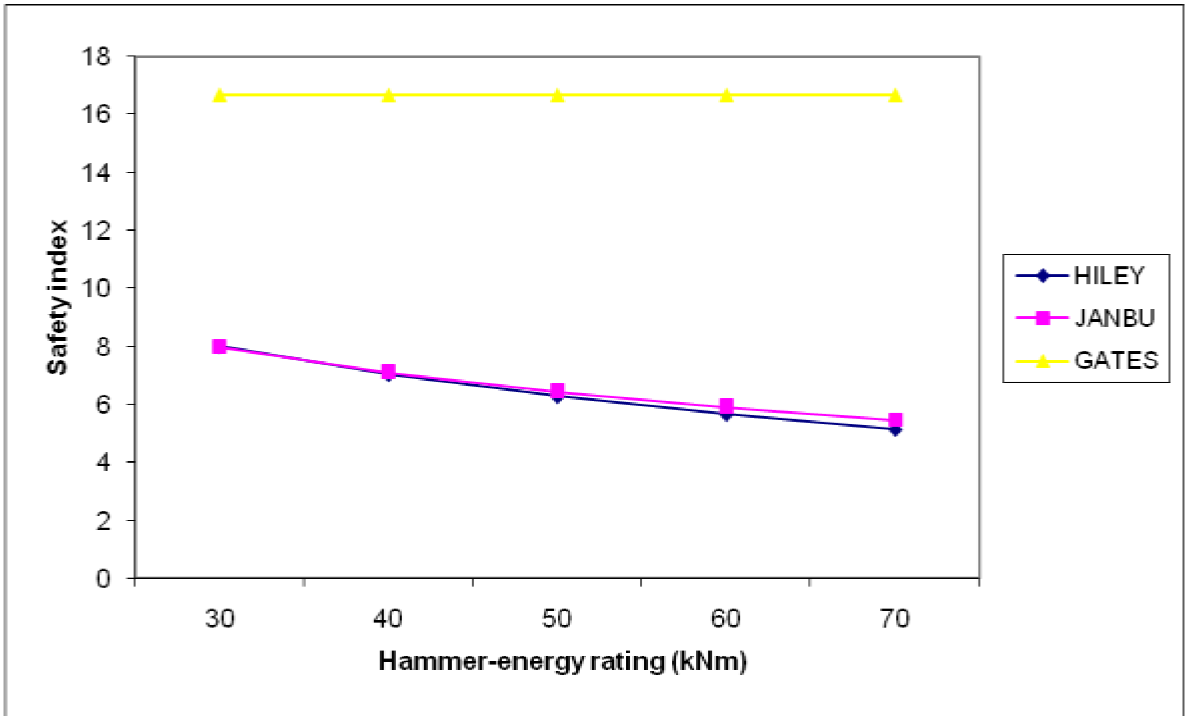


**Fig.3. - Safety index ( $\beta_R$ ) against Hammer- energy rating using Hiley, Janbu and Gates formulae**

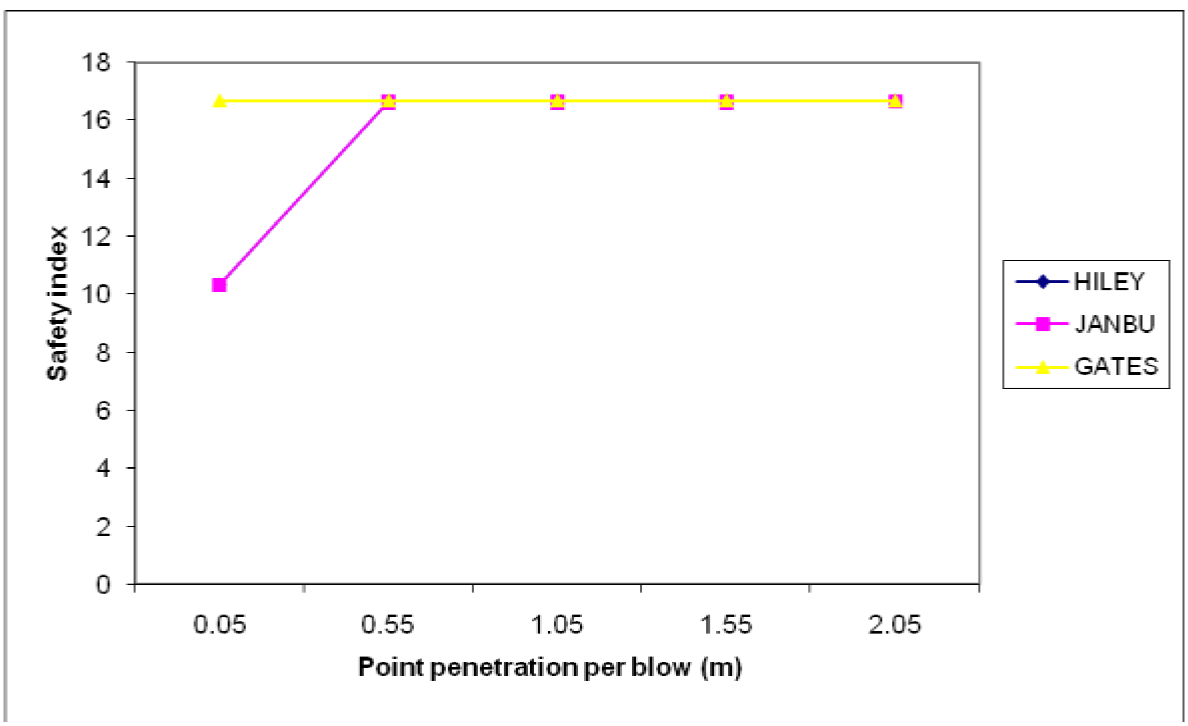


**Fig. 4. - Safety index ( $\beta_R$ ) against Point penetration per blow using Hiley, Janbu and Gates formulae**

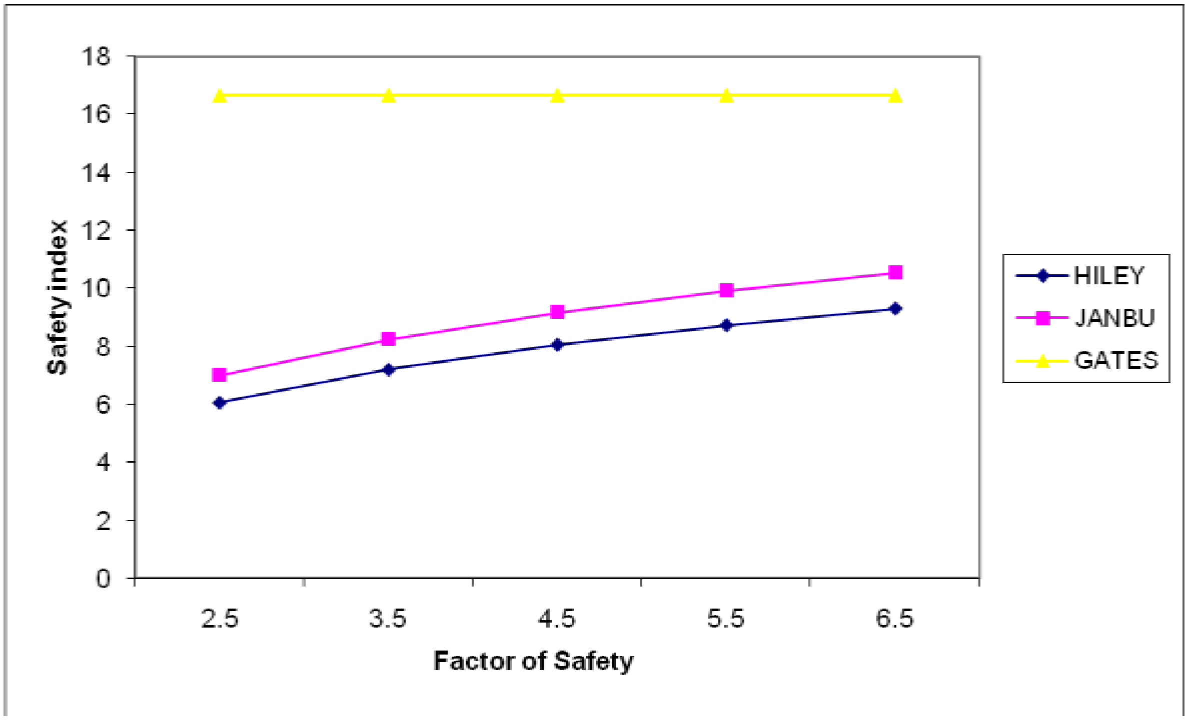


**Fig. 5. - Safety index ( $\beta_R$ ) against Factor of safety using Hiley, Janbu and Gates formulae**

## CONCLUSION

The First-Order Reliability Method has been employed to rate dynamic pile capacity using Hiley, Janbu and Gates formulae. All relevant variables are considered random with assumed probability density distributions. From the results, it can be concluded that there is a correlation between the implied safety levels in Hiley and Janbu formulae. The dynamic predictions of Hiley and Janbu lead to similar safety level while Gates' results in totally different implied safety levels.

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