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MHD BOUNDARY LAYER FLOW PAST A STRETCHING PLATE AND HEAT TRANSFER AND ITS NUMERICAL STUDY

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ABSTRACT

In the present paper, the boundary layer flow of viscous incompressible and electrically conducting fluid past a stretching plate has been considered. The boundary value problem governing the flow has been solved as initial value problem for velocity function using shooting method first, then Runge-Kutta method of order four. The values of velocity function so obtained have been used in the solution of heat transfer problem. The results have been discussed graphically.
Keywords: *Boundary layer equation, Stretching plate, heat transfer.*

INTRODUCTION

The flow past a stretching plate is of great importance in many industrial applications such as polymer industry to draw plastic films and artificial fibers. In the process of drawing artificial fibers, the polymer solution emerges from an orifice with a speed which increases from almost zero at the orifice up to a plateau value at which it remains constant. The moving fiber produces a boundary layer in the medium surrounding the fiber, which is of great technical importance in that, it governs the rate at which the fiber is cooled and this in turn affects the final properties of the yarn. Crane [1] investigated boundary layer flow past a stretching sheet whose velocity is proportional to the distance from the slit. Carragher [2] reconsider the problem of Crane [1] to study heat transfer and calculated Nusselt number for the entire range of Prandtl number Pr. Naseem Ahmad et al. [3] extended the work of Carragher to viscoelastic fluid (Walter liquid B) with heat transfer and discussed the related results. Naseem Ahmad [4] studied hydro- magnetic boundary layer flow past a stretching porous plate and heat transfer. The aim of the present paper is to develop a method to convert the boundary layer flow past a stretching plate which is defined in an infinite domain of definition to a finite domain. The boundary value problem defined in a finite domain has been converted to initial value problem by shooting method and hence solved by Runge-Kutta method of order four for $m = \frac{1}{y_\infty} = M$, where m is the stretching factor, y_∞ means $y \to \infty$ and M is magnetic parameter. Later, applying the finite difference method, the problem of heat transfer has been solved. The results so obtained have been discussed graphically.

Formation of the problem

Two dimension flow of a viscous incompressible and electrically conducting fluid past a linear stretching plate under the transversely applied magnetic field has been considered. It is assumed that induced magnetic field is negligible in comparison to applied magnetic field.

For geometrical configuration, x-axis be along the moving plate and y-axis to be normal to the direction of motion of the plate. If u and v are the velocity components along and normal directions, respectively, then under the usual boundary layer approximations, MHD steady flow is governed by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \qquad (2.2)$$

where ν is the kinematic viscosity.
The relevant boundary conditions are:

$$y = 0,\ u = mx,\ v = 0 \qquad m > 0 \qquad (2.3)$$
$$y \to \infty,\ u = 0,$$

To solve this problem, we define the following dimensionless variables:

$$\bar{y}=\frac{y}{h},\ \bar{u}=\frac{uh}{\nu},\ \bar{x}=\frac{x}{h},\ \bar{v}=\frac{\nu h}{\nu}$$

where h is reference length.
Substituting all these dimensionless variables in equations (2.1) and (2.2), we have the following equations in dimensionless form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.4)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-Mu \qquad (2.5)$$

where $M=\frac{\sigma B_0^2 h^2}{\mu}$, the magnetic parameter.

The boundary conditions are as follows:

$$y = 0,\ u = mx,\ v = 0 \qquad m > 0 \qquad (2.6)$$
$$y \to \infty,\ u = 0,$$

where dash has been dropped for convenience.
Setting the similarity solution of the form

$$u = mxf'(\eta) \qquad (2.7)$$

where $\eta = \frac{y}{y_\infty}$, and substituting u in the equation (2.4) and using the boundary conditions (2.6), we have

$$v = -m\, y_\infty \{f(0) - f(\eta)\}$$
$$= -m\, y_\infty f(\eta) \qquad (2.8)$$

where f(0) = 0 without loss of generality.
Using u and v in the equation (2.5), we have

$$m\left(f'^2(\eta)-f(\eta)f''(\eta)\right)=\frac{1}{y_\infty}f'''(\eta)-Mf'(\eta) \qquad (2.9)$$

with boundary conditions:

$$y = 0,\ f = 0,\ f' = 1 \qquad (2.10)$$
$$\eta = 1,\ f' \to 0$$

Here $y_\infty >> 1$ so by applying magnitude analysis, $\frac{1}{y_\infty} << 1$. Therefore, the term involving $\frac{1}{y_\infty}$ may be neglected. Thus, we have the following boundary value problem:

$$m\left(f'^2(\eta)-f(\eta)f''(\eta)\right)+Mf'(\eta)=0$$
$$\eta = 0,\ f = 0,\ f' = 1 \qquad (2.11)$$
$$\eta \to 1,\ f' \to 0$$

The nonlinear differential equation in boundary value problem (2.11) has singularity at η = 0. Therefore, it requires special attention. To overcome this difficulty, we solve the boundary value problem given by the equations (2.9) through (2.10) by considering m = $\frac{1}{y_\infty}$ = M. Now, the boundary value problem (2.9) through (2.10) reduces to

$$f'''(\eta)=f'^2(\eta)-f(\eta)f''(\eta)+f'(\eta)$$
$$\eta = 0,\ f = 0,\ f' = 1 \qquad (2.12)$$
$$\eta \to 1,\ f' \to 0$$

For the sake of numerical solution, we convert this nonlinear boundary value problem into its equivalent initial value problem by applying shooting method. The guess for f″(0) by shooting method has been obtained by the formula

$$M_i = M_{i-2}+\frac{M_{i-1}-M_{i-2}}{f'(M_{i-1};\ 1)-f'(M_{i-2};\ 1)}\left(f'(1)-f'(M_{i-2};\ 1)\right)$$

where M_i are approximations for M = $5f''^{(2}(0)+164f''(0)+250$.
We get M = 0, that is, $5f''^2(0)+164f''(0)+250$ = 0 which has the roots – 1.6027 and – 13.1973. Taking f″(0) = – 1.6027, we solve the following initial value problem equivalent to the boundary value problem given by (2.9) through (2.10):

$$f'''(\eta)=f'^2(\eta)-f(\eta)f''(\eta)+f'(\eta)$$
$$\eta = 0,\ f = 0,\ f' = 1,\ f'' = -1.6027 \qquad (2.13)$$

This initial value problem is solved for f(η), f ′(η) by Rubge-Kutta method of order four employing C+ computer programming . Trends of variation of f(η) and f ′(η) has been shown in Figure -1 and -2, respectively.

Heat Transfer Problem

Under the usual boundary approximations, the heat transfer between stretching plate and the surrounding fluid is governed by the following equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \qquad (3.1)$$

with boundary conditions

$$y = 0,\ T = T_p$$
$$y \to \infty,\ T = T_\infty \qquad (3.2)$$

Defining dimensionless temperature field

$$\theta = \frac{T - T_\infty}{T_p - T_\infty}$$

We have the equation (3.1) and boundary conditions (3.2) in dimensionless form as follows:

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{p_r}\frac{\partial^2 \theta}{\partial y^2}$$
$$y = 0,\ \theta = 1$$
$$y \to \infty,\ \theta = 0 \qquad (3.3)$$

where P_r is Prandtl number.

Using the transformation $\eta = \frac{y}{y_\infty}$ where y_∞ means $y \to \infty$, we have,

$$\frac{d^2\theta}{d\eta^2} + y_\infty P_r f(\eta)\frac{d\theta}{d\eta} = 0$$
$$\eta = 0,\ \theta = 1 \qquad (3.4)$$
$$\eta \to \infty,\ \theta \to 0$$

In the process of solving boundary value problem (3.4) numerically, we get the following tri - diagonal system of linear equations

$$\begin{bmatrix} -2 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & -2 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & -2 & \alpha_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_4 & -2 & \alpha_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_5 & -2 & \alpha_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_6 & -2 & \alpha_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_7 & -2 & \alpha_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_8 & -2 & \alpha_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_9 & -2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9 \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\alpha_i = 1+0.05y_\infty P_r f_i$ and $\beta_i = 1 - 0.05y_\infty P_r f_i$, I = 1, 2, 3, ···, 9.

DISCUSSION AND CONCLUSIONS

In the course of study of MHD boundary layer flow past a stretching plate with heat transfer for m = $\frac{1}{y_\infty}$ = M, we draw the following conclusions:

1. Using the transformation $\eta = \frac{1}{y_\infty}$, where y_∞ means $y \to \infty$, we have converted the boundary value problem defined in an infinite domain y = 0 to $y \to \infty$ to the boundary value problem defined in a finite domain $\eta = 0$ to $\eta = 1$. By changing the problem in a finite domain, we are able to apply numerical approach. Therefore, employing shooting method first, then Runge-Kutta method of order four, the problem has been solved numerically.

2. According to the numerical scheme which we employed in the computation of $f'(\eta)$, the error comes out to be 4% which may be admissible.

3. From Figures 1 and 2, we observed that velocity function $f(\eta)$ and $f'(\eta)$ satisfy the boundary conditions approximately. So, our numerical approach may be quite valid for solving MHD boundary layer flow past a stretching plate.

4. From Figure-3, we observe that for stretching factor m = 0.66, the contribution of heat to fluid decreases with the increase in Prandtl number P_r. Thus, Prandtl number plays a role of one of the controllers of heat flow from stretching plate to surrounding fluid at a distance y >> 1

5. Figure -4 reveals that for any given Prandtl number P_r, the flow of heat from stretching plate to the surrounding fluid decreases as stretching factor and magnetic parameter both decreases.

Figure- 1 Trend of velocity function

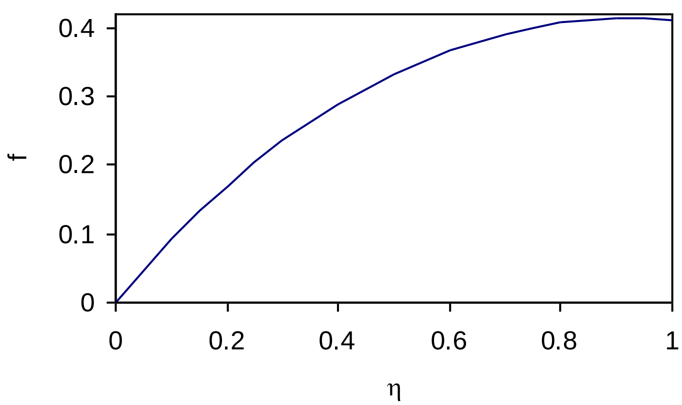


Figure-2 Trend of f'

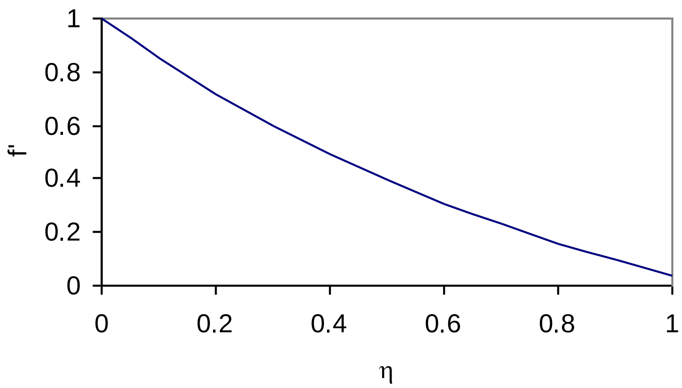


Figure-3 Variation of Temperature field Q for different values of Prandtl number Pr when y=1.5

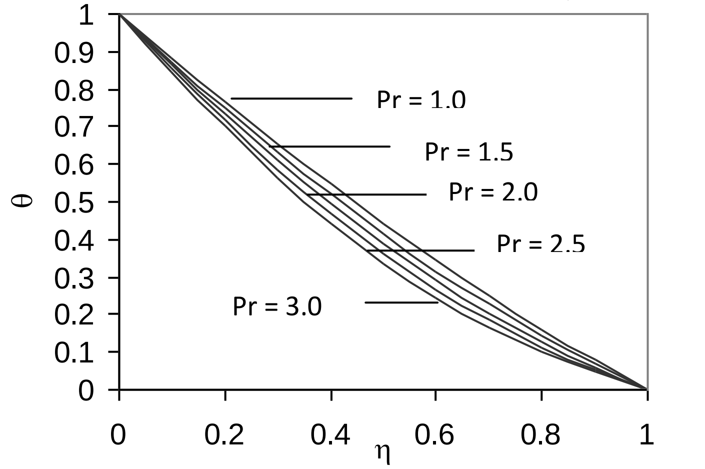
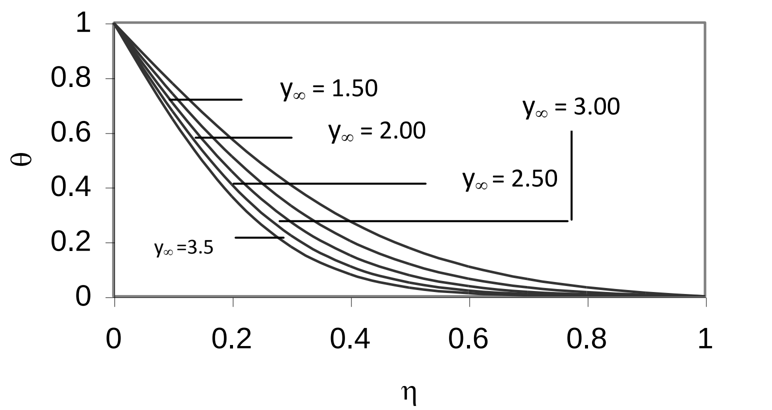


Figure-4 Variation of Temperature field far away from streching plate when Pr = 7



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