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# COMPARATIVE ANALYSIS OF RICE AND TWO-WAVE WITH DIFFUSE POWER (TWDP), FADING MODELS IN TELECOMMUNICATION

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## ABSTRACT

This paper present the comparative analysis of Rice and Two wave with Diffuse power (TWDP) fading models in terms of their probability density function (pdf) that describe small scale, local area fading experienced by narrow band wireless receivers in telecommunication. The paper also reviewed the basic method of generating the probability density function (pdf) for Rice and Two wave with Diffuse power (TWDP) model. Results of analysis from the plot of the probability density function (pdf ) against received envelope for worse distortion detection is also presented. Deduction from the graphical analysis on the nature of distortion observed at the presence of Strong Line of Sight (LOS) components in fading channels was also presented.

**Keywords:** *Fading channel, Small scale effect, Random vector, probability density function, Multipath, Line of sight.*

## INTRODUCTION

Fading in telecommunication is the phenomenon of loss of signal (Rajesh, 2009). Fading channel has been successfully classified into three. They include.

(i) Large scale, fading
(ii) Medium scale fading
(iii) Small scale fading

A lot of effort has been made to resolve the issue of fading with regard to the modulated carrier wave (CW) in telecommunication environments. Modeling is the most convenient approach for evaluating fading channel since it deals with the conceptual and mathematical representation of object of study. So many fading models have been proposed which include Rice and the recent Two – wave with Diffuse Power (TWDP) models. These two models are for small scale fading. Small scale fading experienced by a wireless receiver causes a lot of dramatic fluctuation in received signal strength as a receiver moves over a relatively small local area (Bertoni, 2000). The most common method of characterizing a fading channel is the use of probability density function (pdf) of the received signal strength. The shape of the pdf determines the performance of a wireless recover in the present of noise and interference (Ossana, 1964). The effect is due to multipath delay to the receiving antenna. The carrier wave (CW) complex baseband voltage V, as seen by the receiving antenna is of the following form:

$$V = \sum_{i=1}^{N} V_i \exp\left( j\phi_i \right) \quad \text{.......(1)}$$

Where $V_i$ is the amplitude of multipath waves and the

$\Phi_i$ (s) are their corresponding phases. Phase variable, $\emptyset_i$ are treated as statically independent random phase variables, uniformly distributed over the interval (O, $2\pi$ ) for local area propagation (Rice, 1945). This paper presents the comparative analysis of Rician and TWDP pdf(s) for imperative analysis of worse case of fading for Rayleigh distributed envelope of received signal.

## METHODS

### Theory

In comparative analysis of Rice and TWDP models, the probability density function (pdf) of the models are sampled and investigated for the model similarity and deviation from each other. The graphical analysis of the two pdf(s) is carried out to investigate for their compliance with Rayleigh distribution and possible worse distortion observed in Rayleigh distributed signal in the presence of Gussian noise.

### Derivation of PDF for Rice and TWDP fading models

Probability density functions (pdf) of these models are recovered from the transformation of characteristic function of equation (1). The characteristic function of equation (1) is discussed in terms of in-phase and quadrature components which is given as

$$V = \sum_{i=1}^{l} v_i \exp\left(j\phi_i\right) = X + jY \qquad \text{.................................................(2)}$$

Where the real part is in-phase, imaginary part is quadrature while X and Y are independent zero-mean Guassian random variables, each identically distributed with variance $\sigma^2$, the envelope characteristic function is derived below in which the multipath waves are assumed to be random vectors.

Thus, let us consider *n* random vectors with length $A_{i'}$(s) and angle $\phi_i S$, for $i = 1, 2 \ldots n$; $\phi_i$ are independent random variable with uniform pdf(s) on (0, $2\pi$ ). $A_l{}'s$ are independent of $\phi_l{}'S$. Summation of these n random vectors results in defining equation (2) as:

$$X = \sum_{i=1}^{n} A_i \cos\phi_i \qquad \text{.....................................................................3}$$

$$Y = \sum_{i=1}^{n} A_i \sin\phi_i \qquad \text{.........................................................................4}$$

The joint characteristic function of X and Y are defined as

$$\Psi_{XY}\left(\eta,\zeta\right) = E_{XY}\left[\exp\left(j\eta x + j\zeta Y\right)\right] \qquad \text{....................................................5}$$

Where E is the expected value

Equation (5) can be expressed in terms of $A_i{}'s$ and $\phi_l{}'S$, as

$$\Psi_{XY}\left(\eta,\zeta\right) = E_{A_1 \ldots A_n \phi_1 \ldots \phi_n}\left[\exp\left(jnx + j\zeta Y\right)\right]$$

$= E_{A_1 \ldots A_n}\left[E_{\phi_1 \ldots \phi_n}\left[\exp\left(jnx + j\zeta Y\right) | A_1 \ldots A_n\right]\right]$ ..........................................6

Since $A_l$'s are independent of $\phi_l$'s, the condition can be omitted in (6) thus:

$\psi_{XY}(\eta,\zeta) = E_{A_1 \ldots A_n}\left[E_{\phi_1 \ldots \phi_n}\left[\exp\left(j\eta X + j\zeta Y\right)\right]\right]$ .............................................7

Substituting for X and Y in (3, 4) using (6) gives

$\psi_{XY}(\eta,\zeta) = E_{A_1 \cdots A_n}\left[E_{\phi_1 \ldots \phi_n}\left[\prod_{i=1}^{n} \exp\left(j\eta A_i \cos\phi_i + j\zeta A_i \sin\phi_i\right)\right]\right]$

Due to independent of $\phi_l$'s, it is reduced to

$\psi_{XY}(\eta,\zeta) = E_{A_1 \ldots A_n}\left[\prod_{i=1}^{n} E_{\phi_i}\left[\exp\left(j\eta A_i \cos\phi_i + j\zeta A_i \sin\phi_i\right)\right]\right]$ .............................8

Introducing new variables $\rho$ and $\theta$ in terms of $\eta$ and $\zeta$ as

$\eta = \rho \cos\theta$, $\zeta = \rho \sin\theta$ and using the trigonometric identity, (8) can be written as:

$\psi_{XY}(\eta,\zeta) = E_{A_1 \ldots A_n}\left[\prod_{i=1}^{n} \int_0^{2\pi} \exp\left(jA_i\rho\cos\left(\phi_i - \theta\right)\right) f_{\phi_i}(\phi_i)\, d\phi_i\right]$ ...........................9

Where $f_{\phi_i}(\phi) = pdf$ of $\phi_i$ and it is given by $f_{\phi_i}(\phi) = \frac{1}{2}$, i.e. uniform pdf of each $\phi_i$ on $(0, 2\pi)$. Using this fact, the portion of equation (9) can be expressed in term of integral form of zero order Bessel function of first kind i.e. $J_0(Z) = 1/2\pi \int_0^{2\pi} \exp\left(jz\cos\xi\right) d\xi$

This implies that equation (9) reduces to the following forms:

$\Psi_{XY}(\eta,\zeta) = E_{Ai \cdots A_n}\left[\Pi_{i=1}^{n} J_0\left(A_i\rho\right)\right]$..........................................10

Equation (10) is the random vector envelope characteristic function which can be expressed in term of baseband complex voltage of received signal in a fading channel as

$\phi_{XY}(v) = v\left[\Pi_{i=1}^{n}\left(v_i v\right)\right]$...................................................11

Equation (11) is a convenient joint characteristic function for in-phase and quadrature component of the received wave signal. However, according to (Durgin et al, 2002), the received envelope probability density function (pdf) and its characteristic function $\phi_{XY}(v)$ are defined as Hankel transform pairs given as

$f_R(r) = r\int_0^{\infty}\left[\Phi_{XY}(v) J_0(v,r) v du\right]$...............................................12

And the characteristic function can also be retrieved from the probability density function (pdf) through Hankel transformation of the pdf given as

$$\Phi_{XY}(v)=\int_0^\infty F_R(r)rJ_0(v,r)dr \qquad 13$$

This transformation is convenient as the integration is from zero to infinity obeying the theorem of zero-mean Gussian random variables and is also true because Hankel transform of a function f(r) under the limit $0\leq r<\infty$ is given as

$$\bar{f}(\rho)=\int_0^\infty(r).rJ_0(\rho,r)dr \qquad 14$$

Here $rJ_0(\rho,r)$ is regarded as Kernel of the transformation (Gupta, 2004), and J = zero-order Bessel function of the first kind.

This implies that probability density function (pdf) of a received enveloped and its characteristic function follows the method of infinite expansion in terms of Fourier Bessel series (Hankel transform) according to (Durgin et al, 2002). Let the characteristic function of a single specula wave with magnitude $V_1$ be

$$J_0(v_{i,}v) \qquad (14a)$$

And the characteristic function of the diffused component with mean square voltage $2\sigma^{2,}$ be given as

$$\exp\left(-v^2\sigma^2/2\right) \qquad 14b$$

Putting equation (14a) and (14b) into (12), then, the general form for the received envelope pdf of equation (1)

i.e. $v=\sum_{i=1}^{L}v_i e\exp(j\phi_i)$ becomes

$$f_R(r)=r\int_0^\infty J_0(v,r)\exp\left(\frac{-v^2\sigma^2}{2}\right)\left[\Pi_{i=1}^{N}J_0(v_{i,}v)\right]vdv \qquad 15$$

The analytical expression for Rician distribution result from integration of (15) under N=0 and non-zero $\sigma$ after applying a definite integral relationship (Gradshteyn and Ryzhic, 1980), the resulting pdf is in the form

$$f_R(r)=\frac{r}{2\sigma^2}\exp\left(\frac{-r^2-v_1^2}{2\sigma^2}\right)J_0\left(\frac{rv_1}{\sigma^2}\right),r\geq 0 \qquad (16)$$

Where $J_0$ is a zeroth order modified Bessel function.

Also the pdf of two wave diffuse power result from (15) when N=2. i.e. expanding the characteristic distribution function of the specular wave in terms of Fourier Bessel series given as;

$$\left[\prod_{i=1}^{2} J_o\left(V_1,\nu\right)\right] = J_o\left(V_1,\nu\right) J_o\left(V_2,\nu\right)$$

Therefore

$$f_R\left(r\right) = r\int_0^{\infty} \exp\left(\frac{-r^2-\nu^2}{2\sigma^2}\right) J_o(V_1,\nu) J_o(V_2,\nu)\nu d\nu \quad ....................................17$$

Which gives the expression below when transformed (Gradshteyn and Ryzhic, 1980)

$$f_R(r) = \frac{r^2}{\sigma}\exp(\frac{-r^2}{2\sigma^2} - K) I(\frac{r}{\sigma}, K, \Delta) \quad \textbf{..............................................................18}$$

This is expression for the probability density function (pdf) of Two-Wave with Diffuse Power (TWDP) in approximate representation.

## RESULTS AND DISCUSSION

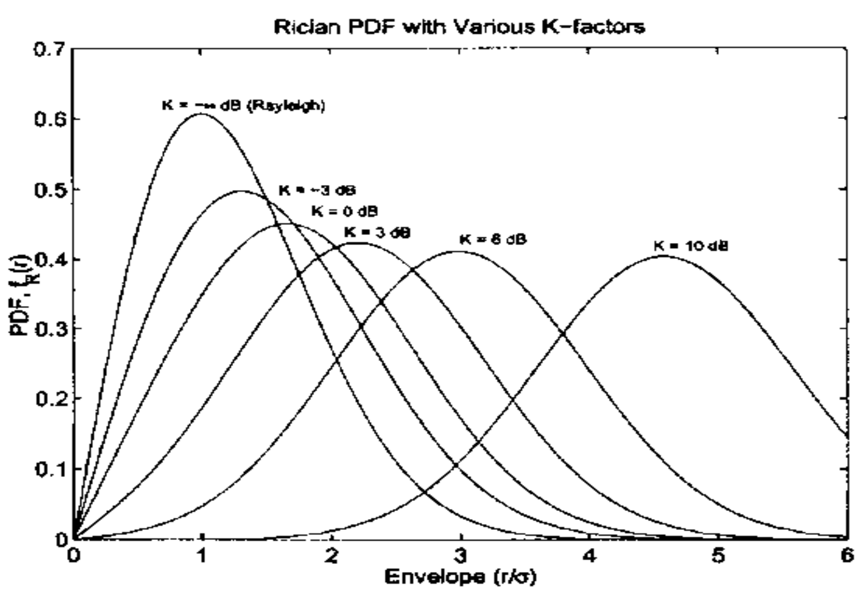
## RESULTS

**Comparison and Difference between the Models:** This is for the reason to investigate the similarities and deviations between the models. The comparison made use of two parameters, K and $\Delta$ which relates the power of specula wave to the diffuse power and relative voltage magnitude to one another respectively. The graph of pdf(s) i.e. ($f_R(r)$) against envelope r/$\sigma$ are ploted to obtain various curves at different K-factor for Rician and various $\Delta$ values for different k-factor for TWDP model as presented in figure 1.

**Fig.1: Rician PDF with various K- factors**

From the graph, it is clearly shown that when k=$\infty dB, v \rightarrow -\infty,$ i.e no line of sight exist and Rician pdf falls back to Raleigh

Recall $k = \frac{v_1^2}{2\sigma^2}$ at v$\rightarrow -\infty, k = -\infty$



And $J_0(r,v/\sigma) = 0$

Therefore $f_R(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2 - v_1^2}{2\sigma^2}\right) J_0\left(r\frac{v}{\sigma}\right)$ will reduced to $f_R(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)$ which is Rayleigh pdf

For this reason, Rician can be used to study other pdfs like Rayleigh, and log-normal which are Gussian distribution functions in nature.

**Two-wave with diffuse power (TWDP) Probability density function (pdf).**

In TWDP, plot of pdf $(f_R(r))$ against envelope r/$\sigma$ was also used but various values of $\Delta$ at different k were considered. i.e at k=odB, 3dB, 6dB and 10dB, various values of $\Delta$ *like* $\Delta$= 0.0,0.5, 0.8 *and* 1.0 were used.

The graph of different k-factor at various $\Delta$= *values*. are presented in figure 2-5

Fig 2 TWDP for K=odB at various $\Delta$

Fig. 3: TWDP pdf for K=3dB various $\Delta$

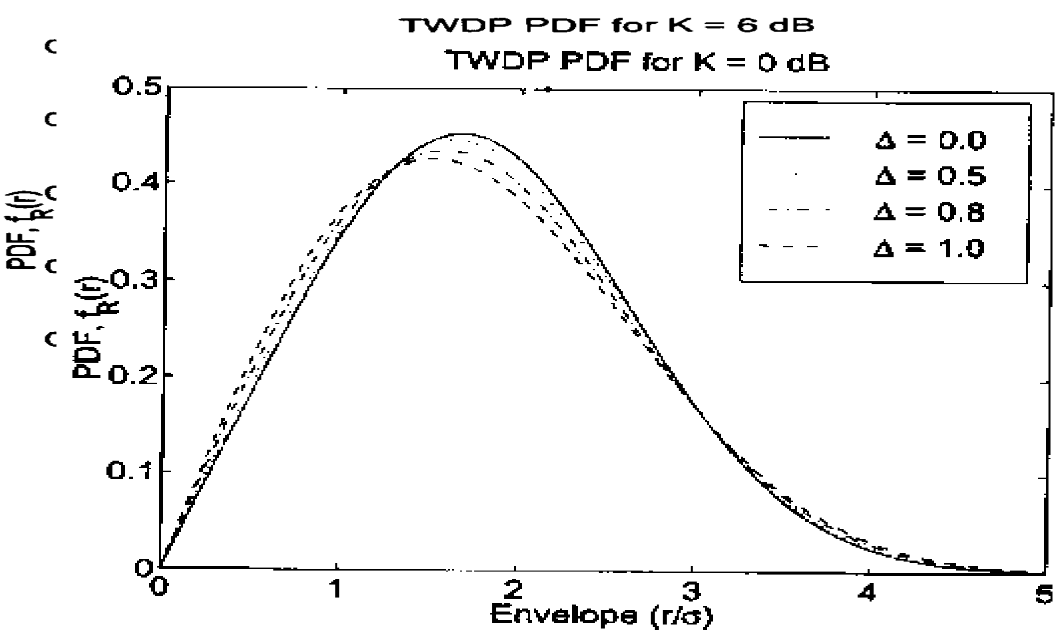


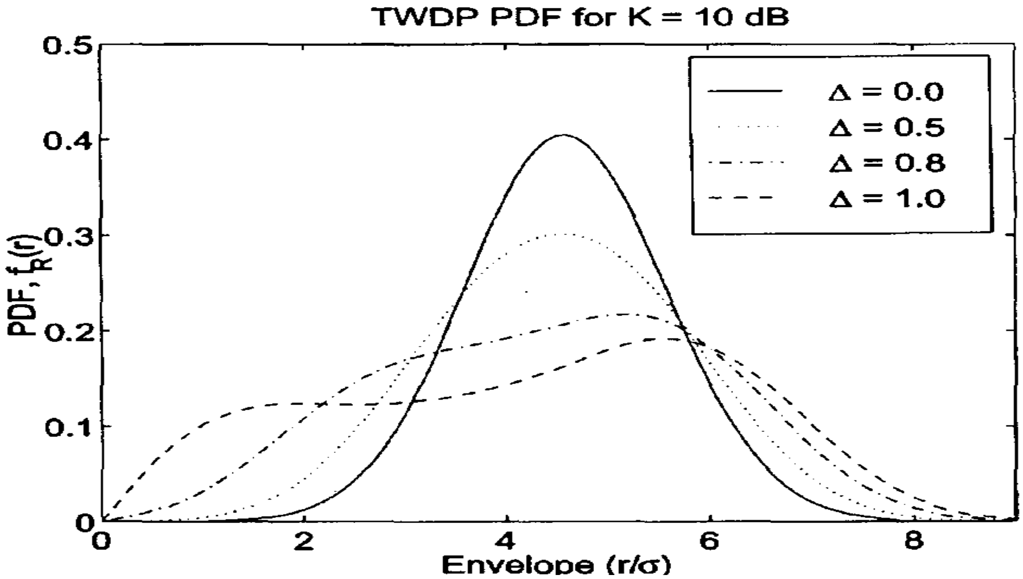

Fig.4: TWDP pdf for k=6dB at various $\Delta$

Fig.5: TWDP pdf for k=10dB at various $\Delta$.

It worth to note from fig. 2-5, that the nature of curve in each k-factor varies at different values of Δ deviation of the TWDP from Rician pdf.

## DISCUSSION

This work has shown a lot of similarities between the two models to the point of establishing




a basic formula from where the pdf of models are derived. But, the fact remains that the condition for which these pdf(s) are to be valid are different. Log-normal made use of standard form of normal curve function while others made use of the Fourier Bessel series (Hankel transformation)(Adbi, 1996). Differences between the models can be also drawn from the graphical analysis of pdfs. It can be seen from plots in fig.2-5 that in fig2, there is a little difference between Rician pdf and TWDP pdf when k is less than 3dB. The difference becomes gradually more pronounced as k increases, particularly when the peculiar power is divided equally between the two discrete components (Δ= 1 ). Fig.5, shows the graph at k=10dB, illustrating a dramatic distortion. In fact as the product of the parameter k and Δ becomes large, the graph of the pdf becomes bimodal and exhibiting two maxima (Durgin et al, 2002). The two models have been tested and seen to have agreed with the Gaussian distribution function condition of fading channel in telecommunications. Rician and Two-Wave with Diffuse Power (TWDP) model in telecommication have been seen from the expression of pdfs to have contained some similarities in the function distribution of their received envelop, R, in the presence of random Gaussian noise. This similarity is practically observed in the graphical analysis of their pdfs as shown in fig.1 and fig.2 respectively for Rice model and TWDP model. In fig. 1 Rician Pdf is plotted against several of the Rician factors (K) for K between -∞dB, -3dB, 0Db, 3db, 6db and 10dB. In fig. 2a-2d, the plot of pdf for TWDP against r/δ for various values of Δ at different were considered for K= 0Db, 3dB, 6dB and 10Db with random values of Δ =0.0, 0.5, 0.8 and 1.0. The graph show that when K is less than 3Db, there is a little difference between the Rician pdf and TWDP pdf. It also observed that the difference becomes gradually pronounced as K increase and become very significant when $\Delta$ =1. The two models conform to the Gussian distribution condition of fading channel in telecommunication. Worthy to note also is that as the product of K and $\Delta$ becomes larger. The pdf becomes bimodal and exhibits two maximal (Durgin et al, 2002).

**CONCLUSIONS**

From the mathematical and graphical analysis in this work, it can be seen that for some cases of limiting parameters, exact TWDP pdf contains Rician pdf and other pdf(s). Rayleigh and Rician pdf(s) are special cases of the TWDP pdf. For this reason, it is useful to know the range of parameters over which TWDP fading may be approximated by these simpler distributions. Careful inspection of the graph of fig. 4-7 reveals the range of K and $\Delta$ over which a Rician pdf approximates to a TWDP pdf which gives speculation of approximating other pdf(s) which are contained in TWDP pdf Rician condition: $K < 2/\Delta$ Rayleigh condition: **k** $< \min\left(\frac{2}{\Delta}, \frac{1}{\sqrt{1-\Delta^2}} - 1\right)$

Table II gives the summary of conditions for Rayleigh and Rician fading models to be approximated to TWDP model.

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