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APPLICATION OF GAUSS-LAGUERRE QUADRATURE FORMULA IN DETERMINATION OF DIMENSIONLESS PRESSURE DISTRIBUTION IN A VERTICAL WELL

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ABSTRACT
In this paper, application of Gauss-Laguerre quadrature formula was used to determine dimensionless pressure distribution in a vertical well based on the following assumptions: *Infinite acting reservoir that is, the reservoir is infinite size. *The well is producing at constant flow rate. *The reservoir is at a uniform pressure, when production begins. *The well, with a wellbore radius of radius of r_w is cantered in a cylindrical reservoir of radius r_e *No flow across the outer boundary. In work, the result shows that as r_D increases, P_D decreases. This indicates that when the radius of wellbore is increasing, productivity decreases for a vertical well and also at early t_D the productivity is very insignificant. When P_D and t_D was plotted on log-log at late time the slope was 1.1513.This indicates that fully penetrating vertical well in an infinite reservoir and boundaries have been felt. The values of E_i at various r_D, shows that as r_D increases, E_i decreases. The results also shows that dimensionless pressure P_D increases with dimensionless time t_D and log approximation was applied when $\alpha \geq 0.25$ and the smaller alpha, the higher the productivity. This method has ability to handle all homogeneous porous media, ability to handle all shapes of boundaries, regular or irregular and ability to eliminate trial-and error procedure.
Key words: *Dimensionless, reservoir, wellbore, pressure, radius*

INTRODUCTION
A lot of work has been carried out on determination of dimensionless pressure distribution in both vertical and horizontal well using different methods as shown in Refs(1-3).In this work, conditions proposed by Mathews and Russel solution to the diffusivity equation called the exponential integral E_i was used: Infinite acting reservoir that is, the reservoir is infinite size, the well is producing at constant flow rate, the reservoir is at a uniform pressure , when production begins, the well, with a wellbore radius of radius of r_w is cantered in a cylindrical reservoir of radius r_e and no flow across the outer boundary .
Employing the above conditions, the authors presented their solution in the following form:

$P = P_i + 70.6\frac{q\beta\mu E_i}{kh}\left(\frac{-948\theta\mu c_t r^2}{kt}\right)$-------------------------1.o

And

$E_i(-x) = -\int_x^\infty \frac{e^{-u}}{u}du$ -----------------2.0

A further simplification of the solution to the flow equation is possible for <0.02, $E_i(-x)$ can be approximated with an error less than 0.6% by[4]

$E_i(-x) = In(1.78x)$-------------------------------3.0

Theis(1935) has given the governing equation for the line solution:

$P_D = -\frac{1E_i}{2}\left(-\frac{r_D{}^2}{4t_D}\right)$---------------------------4.0

Equation 4.0 can be written as

$$p_D = \frac{1}{2}e^{-\alpha}\left(\frac{w_1}{z_1+\alpha}+\frac{w_2}{z_2+\alpha}.---\frac{W_n}{Z_n+\alpha}\right)$$ ---------------------------- **(5.0)**

Equation 5.0 is called Gauss-Laguerre quadrature formula[6]

Where

W_1= upper weight factor (Three-Point Formula)

W_2= Lower weight factor (Three-Point Formula)

Z_1 = upper root

Z_2=Lower root

$\alpha = \left(\frac{r_D{}^2}{4t_D}\right)$=apha

W_1,W_2, Z_1 and Z_2 can be obtained from table.

COMPUTATION OF P_D

From Gauss-Laguerre quadrature table[4],

W_1=0.85355,W_2=0.14644,Z_1=0.58578, and Z_2=3.414221

Substitute $\alpha = \frac{r_D^2}{4t_D}$ into equation 5.0

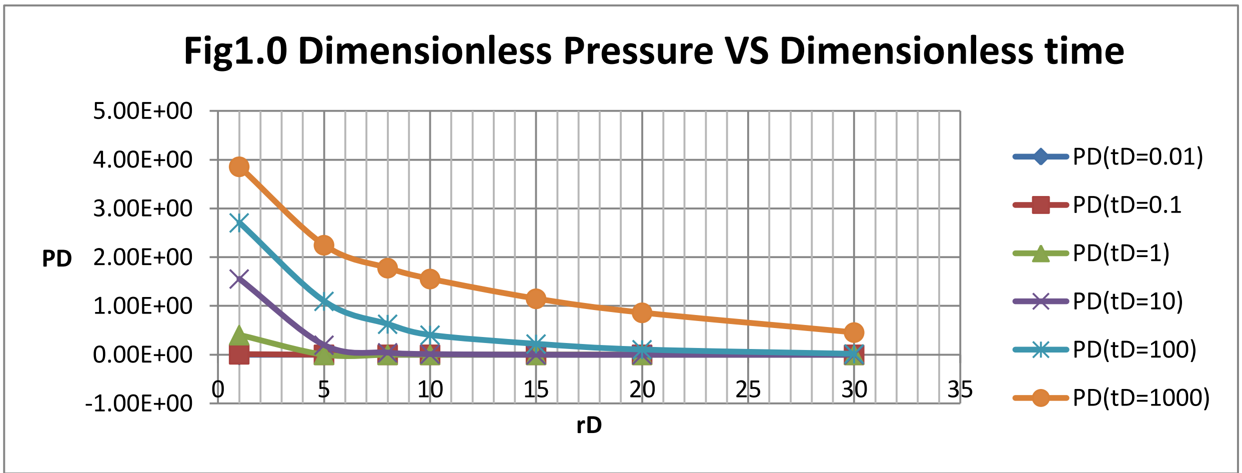
For r_D =1, at t_D = 10^{-3} or 0.001, then $\alpha = \frac{r_D^2}{4t_D}$ = $\frac{1^2}{4x0.001} = 250$. This is value of alpha is substituted into (5.0) above, to give P_D. This calculation is repeated for values of t_D and r_D as shown below.

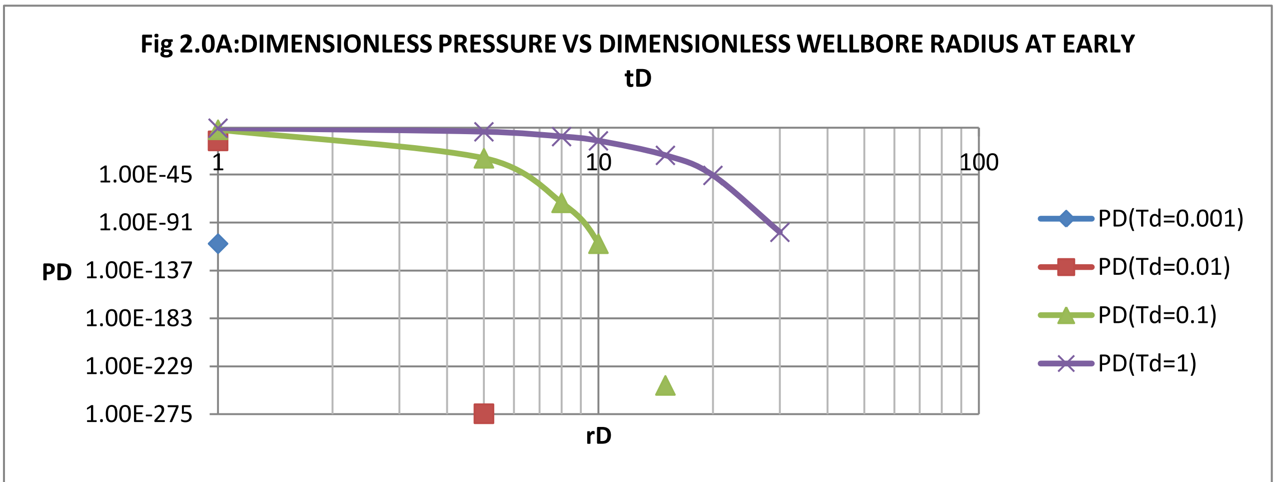
RESULTS AND DISCUSSION

Results of dimensionless pressure behaviour of vertical well P_D at a particular t_D for both early and late flow are illustrated as shown in Table1.0 . As r_D increases P_D decreases as shown in Table1.0 .and Figure 1.0,2.0A and 2.0B. . This indicate that when the radius of wellbore is increase, productivity decreases for a vertical well and also at early t_D the productivity is very insignificant as shown in Fig4.0A and 4.0B. When P_D and t_D was plotted on log-log at late time the slope was 1.1513 as illustrated in Fig3.0 below. This indicates that boundaries have been felt. Table2.0 shows the values of E_i at various r_D.Fig5.0A and 5.0B illustrate the effect of r_D on E_i.As r_D increases, E_i decreases. The transient condition is only applicable for a relatively short period after some pressure disturbance has been created in the reservoir. If pressure is reduced at the wellbore, reservoir fluids will begin to flow near the vicinity of the well. The pressure disturbance and fluid movement will continue to propagate radially away from the wellbore. In the time for which the transient condition is applicable, it is assumed that the pressure response in the reservoir is not affected by the presence of the outer boundary, thus the reservoir appears infinite in extent

Table 1.0 Calculated P_D for various values of r_D.

t_D	$P_D(r_{D=1})$	$P_D(r_{D=5})$	$P_D(r_{D=8})$	$P_D(r_{D=10})$	$P_D(r_{D=15})$	$P_D(r_{D=20})$	$P_D(r_{D=30})$
0.001	5.30E-112	0	0	0	0	0	0
0.01	2.67E-13	2.90E-275	0	0	0	0	0
0.1	0.012369	5.66E-30	1.01E-72	5.30E-112	4.50E-248	0	0
1	0.40456	0.000135	3.32E-9	2.67E-13	3.25E-27	1.84E-46	4.30E-101
10	1.555852	0.198371	0.042369	0.012369	0.000277	2.08E-6	3.61E-12
100	2.707145	1.097707	0.627703	0.40456	0.22259	0.105108	0.01722
1000	3.858437	2.24899	1.778996	1.555852	1.150387	0.862705	0.25724





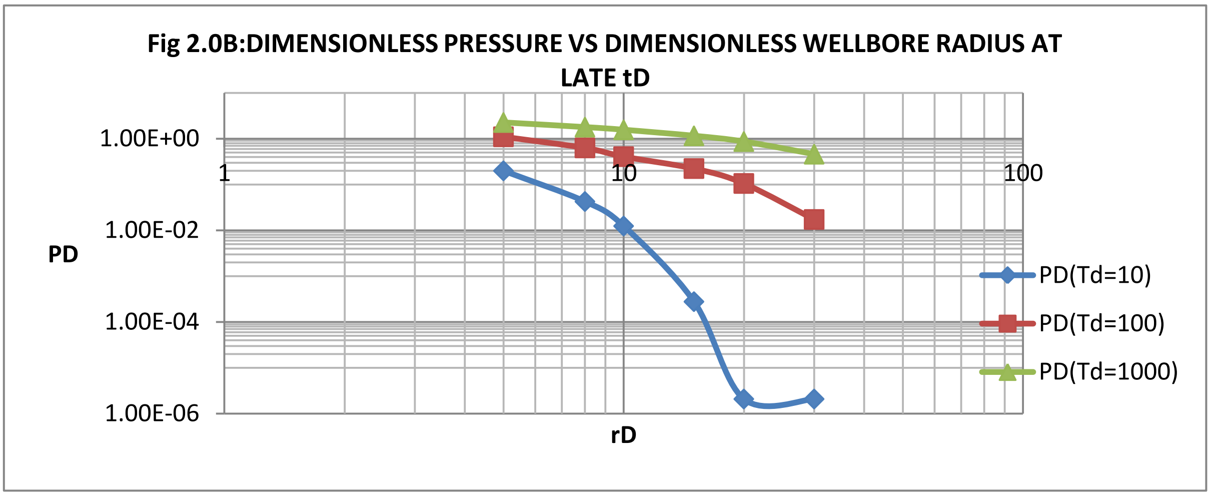


Fig 2.0B:DIMENSIONLESS PRESSURE VS DIMENSIONLESS WELLBORE RADIUS AT LATE tD
1.00E+00
1.00E-02
1.00E-04
1.00E-06
PD
1
10
100
rD
PD(Td=10)
PD(Td=100)
PD(Td=1000)

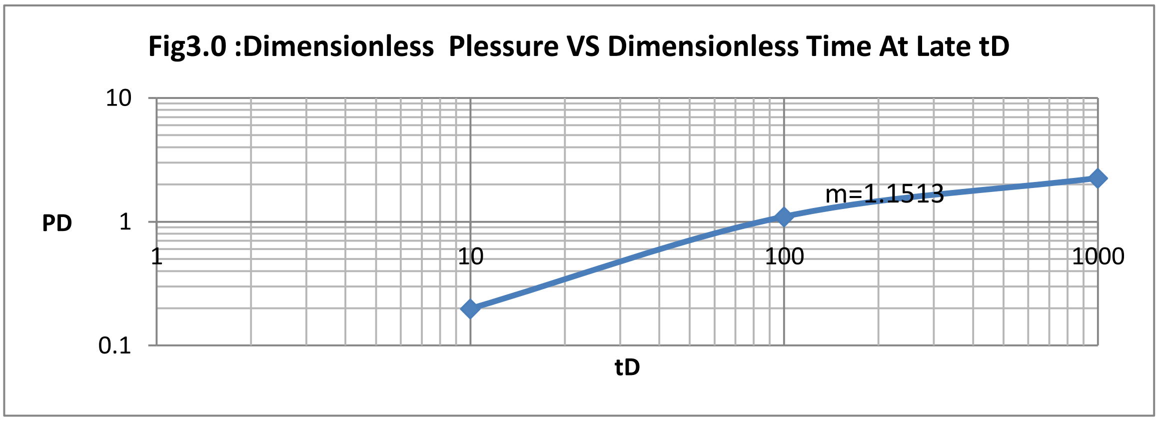


Fig3.0 :Dimensionless Plessure VS Dimensionless Time At Late tD
10
1
0.1
PD
1
10
100
1000
m=1.1513
tD

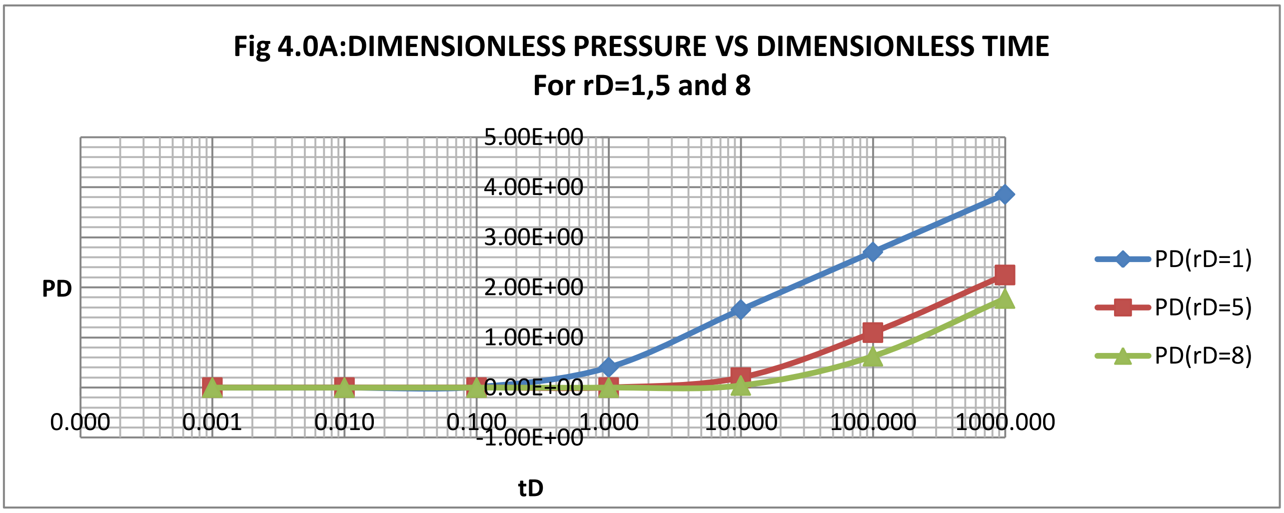


Fig 4.0A:DIMENSIONLESS PRESSURE VS DIMENSIONLESS TIME For rD=1,5 and 8
5.00E+00
4.00E+00
3.00E+00
2.00E+00
1.00E+00
0.00E+00
-1.00E+00
PD
0.000
0.001
0.010
0.100
1.000
10.000
100.000
1000.000
tD
PD(rD=1)
PD(rD=5)
PD(rD=8)

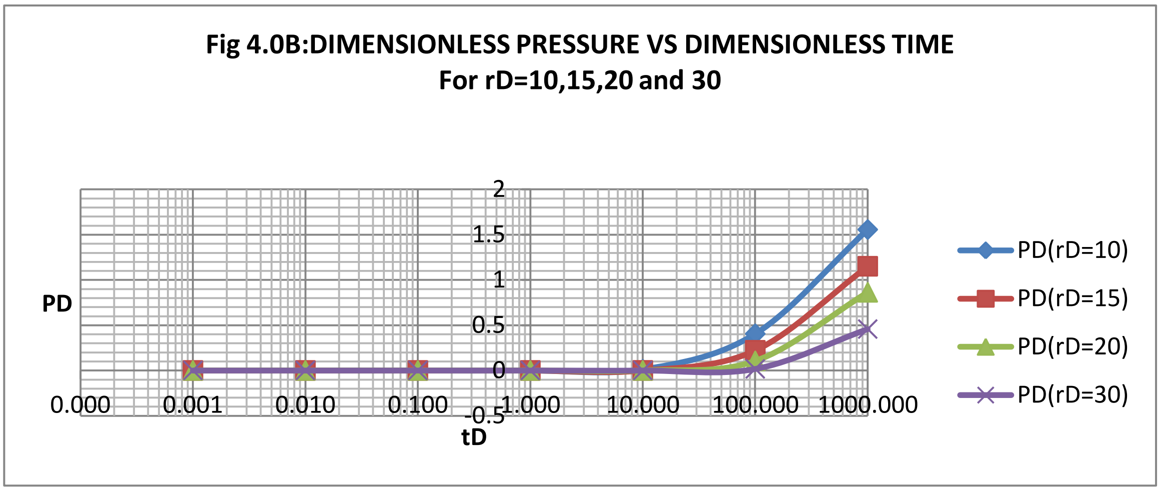


Fig 4.0B:DIMENSIONLESS PRESSURE VS DIMENSIONLESS TIME
For rD=10,15,20 and 30
PD
tD
2
1.5
1
0.5
0
-0.5
0.000
0.001
0.010
0.100
1.000
10.000
100.000
1000.000
PD(rD=10)
PD(rD=15)
PD(rD=20)
PD(rD=30)

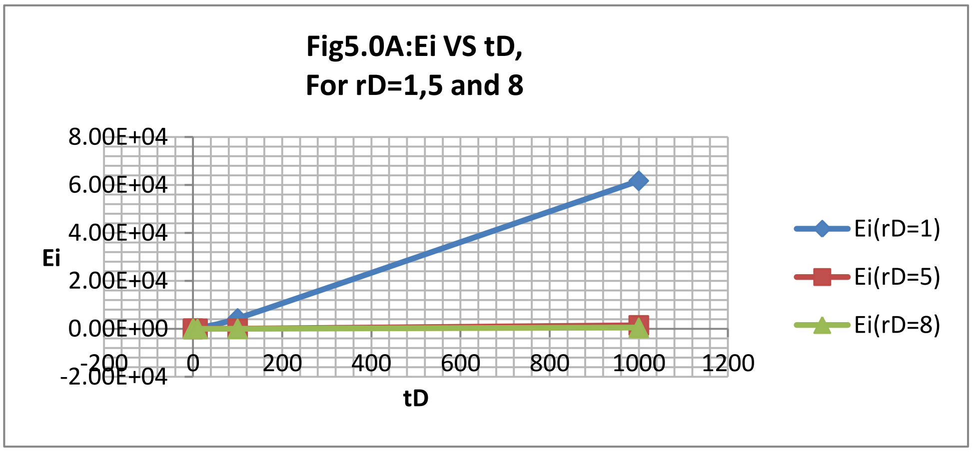


Fig5.0A:Ei VS tD,
For rD=1,5 and 8
Ei
tD
8.00E+04
6.00E+04
4.00E+04
2.00E+04
0.00E+00
-2.00E+04
-200
0
200
400
600
800
1000
1200
Ei(rD=1)
Ei(rD=5)
Ei(rD=8)

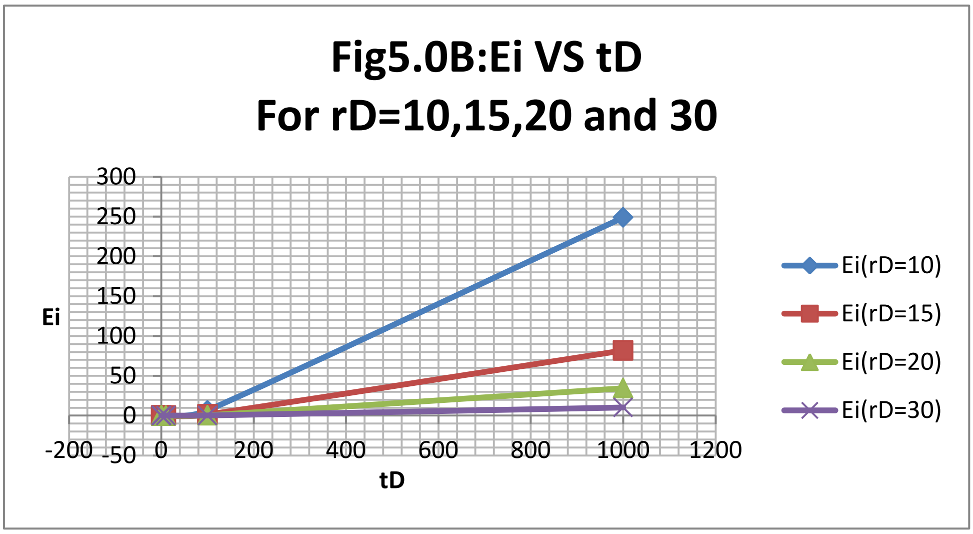


Fig5.0B:Ei VS tD
For rD=10,15,20 and 30
Ei
tD
300
250
200
150
100
50
0
-50
-200
0
200
400
600
800
1000
1200
Ei(rD=10)
Ei(rD=15)
Ei(rD=20)
Ei(rD=30)

Table2.0 :Values of Ei

t_D	Ei(rD=1)	Ei(rD=5)	Ei(rD=8)	Ei(rD=10)	Ei(rD=15)	Ei(rD=20)	Ei(rD=30)
0.001	8.48E-114	0.00E+00	0	0	0	0	0
0.01	4.27E-14	1.86E-277	0	0	0	0	0
0.1	0.01979	3.62E-31	2.53E-74	8.48E-114	3.20E-250	0	0.00E+00
1	6.47296	8.64E-05	8.30E-10	4.27E-14	2.31E-28	7.36E-48	1.72E-103
10	248.9363	1.27E+00	0.105923	0.01979	0.000197	8.32E-07	1.44E-13
100	4331.432	7.03E+01	15.69258	6.47296	1.582862	0.420432	6.89E-02
000	61734.99	1.44E+03	444.749	248.9363	81.8053	34.5082	1.03E+01

Table3.0:Values of alpha at various r_D

S/N	t_D	α (rD=1)	$P_{D(rD=1)}$	α (rD=5)	$P_{D(rD=5)}$	α (rD=8)	$P_{D(rD=8)}$	α (rD=10)	$P_{D(rD=10)}$	α (rD=15)
1	10^{-3}	250	5.3E-112	6250	0.0	16000	0.0	*25000*	*0.0*	56250
2	10^{-2}	25	2.67E-13	625	2.9E-275	1600	0.0	2500	0.0	5625
3	10^{-1}	2.5	0.012369	62.5	5.66E-30	160	1.01E-72	250	5.3E-112	562.5
4	1	0.25	0.40456	6.25	0.000135	16	3.32E-9	25	2.67E-13	56.25
5	10	0.025	1.555852	0.625	0.198371	1.6	0.042369	2.5	0.012369	5.625
6	100	0.0025	2.707145	0.0625	1.097707	0.16	0.627703	0.25	0.40456	0.5625
7	1000	0.00025	3.858437	0.00625	2.248999	0.016	1.778996	0.025	1.555852	0.05625

S/N	t_D	α (rD=20)	$P_{D(rD=20)}$	α (rD=30)	$P_{D(rD=30)}$
1	10^{-3}	100000	0.0	225000	0.0
2	10^{-2}	10000	0.0	22500	0.0
3	10^{-1}	1000	0.0	2250	0.0
4	1	100	1.84E-46	225	4.3E-101
5	10	10	2.08E-6	22.5	3.61E-12
6	100	1	0.105108	2.25	0.01722
7	1000	0.1	0.862705	0.225	0.45724

CONCLUSION

This method has ability to handle all homogeneous porous media, ability to handle all shapes of boundaries, regular or irregular and ability to eliminate trial-and error procedure. The solution gives the pressure at any point in the reservoir where the influence of the finite wellbore radius is insignificant. The solution does not account for skin or wellbore storage making it more appropriate for interference test than a single well test.

Nomenclature

P_D=Dimensionless pressure
r_D=Dimensionless radius
t_D=Dimensionless time

r_w=Wellbore radius
r_{eD}=Dimensionless external radius
r_e=Drainage radius
W_1= upper weight factor (Three-Point Formula)
W_2= Lower weight factor (Three-Point Formula)
Z_1 = upper root
Z_2=Lower root

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