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r- PERMUTATION AND r-COMBINATION TECHNIQUES FOR k-INCLUSION CONDITION

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ABSTRACT

In this work, we studied the r-permutation and r-combination of a set of n distinct elements such that a fixed group k of elements ($k \leq n$) must be in each permutation or combination. For the purpose of simplicity in solving problems of this nature, we derived a generalized formula for obtaining the solution to such problems.

KEY WORDS AND PHRASES. r- permutation, r-combination, k- inclusion.

INTRODUCTION AND PRELIMINARIES

Let $X = \{x_i : i = 1,2,\ldots, n\}$ be a collection. We consider the various combinations, permutations and rearrangements (see e.g. [1], [2], [3], [4]) of the objects in X, such that, a fixed group k of elements are all included. If n is small (say 2, 3 or 4) it is easy to exhaustively list and count all the possible outcomes in this arrangements for the inclusion case (see [3],[4]). In this paper, we provide formulae, which are rather easy to apply and are applicable in general situations for the case of combinations, permutations and rearrangements of n objects with k of them being included all at a time. Our formulae are novel, interesting and of general application. In our subsequence paper, we shall look at further techniques in solving permutation and combination problems.

Definition 1.1

A selection of r elements from n ($r \leq n$) order being significant in such a ways that the k-elements ($k \leq r \leq n$) are always included in the r permutation is called k inclusion permutation. The number of this permutation is noted by

$$P_{i(n,r,k)}$$

Definition 1.2

A selection of r elements from n ($r \leq n$) order not being significant in such a way that k elements ($k \leq r \leq n$) are always included in the r combination is called k inclusion combination is denoted

$$C_{i(n,r,k)}$$

MAIN RESULTS

a) r-permutation

THEOREM 2.1

Let $X = \{x_i : i = 1,2,..., n\}$, the r-permutation of n distinct elements ($r \leq n$) of X with the inclusion of a fixed k number of elements all at a time ($k \leq r \leq n$) is

$$P_{i(n,r,k)} = \frac{r!(n-k)!}{(r-k)!(n-r)!}$$

Proof

If k elements are picked from n elements, then we look at the possible ways of arranging the (n – k) element to combine with the k elements for the r-permutation. However, this could be done as follows;

Clearly, we shall arrange the (n – k) elements such that the last element will have (n-r+1) ways of arrangements.

Applying First Counting Principle (FCP) [1,2,3], we have

$$\frac{(n-k)!}{(n-r)!} = P_{(n-r,r-k)} \qquad (2.1)$$

we shall arrange the k elements such that the last element will have 1 ways of arrangement.

By FCP, we have

$$k! = P_{(k,k)} \qquad (2.2)$$

Next, each of the $P_{(n-k,r-k)}$ elements combines with the k elements. This could be done such that the last element will have (r-k+1) ways of arrangement.

By FCP we have

$$\frac{r!}{(r-k)!} = P_{(r,k)}$$

By taking care of the repetition in the above permutation, we have

$$\frac{P_{(r,k)}}{k!} \qquad (2.3)$$

Thus, by FCP we have from (2.1), (2.2) and (2.3)

$$k!P_{(\text{n-k,r-k})} \cdot \frac{P_{(r,k)}}{k!}$$

$$P_{i(n,r,k)} = \frac{r!(n-k)!}{(r-k)!(n-r)!} \qquad (2.4)$$

b) r-combination

THEOREM 2.2

Let $X = \{x_i : i = 1,2,\ldots, n\}$, then the r-combination of n distinct elements of X with the inclusion of a fixed k number of elements all at a time ($k \le r \le n$) is

$$C_{(n-k,r-k)} = \frac{(n-k)!}{(r-k)!(n-r)!}$$

Proof

Applying Def. 1.1, theorem 2.2 follows from theorem 2.1

$$C_{i(n,r,k)} = \frac{P_{i(n,r,k)}}{r!}$$

$$= \frac{(n-k)!}{(r-k)!(n-r)!}$$

$$= C_{(n-k,r-k)} \qquad (2.5)$$

COROLLARY 2.3

Let n, r and k be nonnegative integers with $k \le r \le n$, if k=0, theorem 3.1 reduces to the well known r-permutation formula. That is

$$P_{i(n,r,0)} = P_{(n,r)}$$

Proof

Let k = 0

$\Rightarrow$

$$P_{i(n,r,0)} = \frac{r!(n-0)!}{(r-0)!(n-r)!}$$

$$= \frac{r!n!}{r!(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= P_{(n,r)} \qquad (2.6)$$

COROLLARY 2.4

Let n, r and k be nonnegative integers with $k \le r \le n$, if k = 0, theorem 3.2 reduces to the well known r-combination formula. That is

$$C_{i(n,r,0)} = C_{(n,r)}$$

Proof

k = 0

$\Rightarrow$ $$C_{i(n,r,0)} = \frac{(n-0)!}{(r-0)!(n-r)!}$$

$$= \frac{n!}{r!(n-r)!}$$

$$=C_{(n,r)} \tag{2.7}$$

■

COROLLARY 2.5
Let n, r and k be nonnegative integers with $k \le r \le n$.
Then

$$C_{(n-k,r-k)} = C_{(n-k,n-r)}$$

Proof

$$C_{(n-k,r-k)} = \frac{(n-k)!}{(r-k)!\left[n-k-(r-k)\right]!} = \frac{(n-k)!}{\left[n-k-(r-k)\right]!(r-k)!}$$

$$\frac{(n-k)!}{(n-r)!(r-k)!} = \frac{(n-k)!}{(n-r)!\left[n-k-(n-r)\right]!} = C_{(n-k,n-r)}$$

■

Now, we are ready to consider some examples to illustrate the use of our results.

Example 3.1
How many four-letter words can be formed from {A, B, C, D} for the case when order is (a) significant and (b) insignificant if

i. A, B and C must be considered (included)
ii. C and D must be considered (included)

Solution

i. With the aid of tree diagram, it was observed that the number of arrangement containing A, B and C are 24. Thus, there are 24 four letter words formed containing A, B and C.
If order is not important, then all the 24 four letters words is not important, Hence we have one (1) four-letter word for combination use.

ii. Similarly, the diagram shows that there are 24 four letter word containing C and D. If order is not important, then all the 24 four letter words are identical. Hence we have one (1) four letter word for the case of combination.
Using the **Generalized Techniques** approach,

I n = 4, r = 4, k = 3

$$P_{i(n,r,k)} = P_{i(4,4,3)} = \frac{4!(4-3)!}{(4-3)!(4-4)!} = 24$$

(a)
(b) By definition

$$C_{i(n,r,k)} = \frac{P_{i(n,r,k)}}{r!} = \frac{4!}{4!} = 1$$

ii. n = 4, r = 4, k = 2

(a) $$P_{i(n,r,k)} = P_{i(4,4,2)} = \frac{4!(4-2)!}{(4-2)!(4-4)!} = 24$$

(b) By definition
$$C_{i(4,4,2)} = 1$$

Example 3.2
How many five-letter words can be formed from {A, B, C, D,...,H}, for the case when order is (a) significant and (b) insignificant. If
i. A, B and C must be included.
ii. C and D must be included

Solution
i With the aid of tree diagram we observed that there are 6 three-letter words containing A, B and C if order is significant, and one letter word when order is insignificant.
ii. Observe similarly, there are 12 three letter words containing C and D if order is significant, and there are 2 three letter words when order is insignificant.

Using the **Generalized Techniques** approach,
i. n = 8, r = 5, k = 3

(a) $$P_{i(n,r,k)} = P_{i(8,3,3)} = \frac{3!(8-3)!}{(3-3)!(8-3)!} = 1,200 \ ways$$

(b) By definition
$$C_{i(8,5,3)} = \frac{5!}{2!3!} = 10\,ways$$

ii. n = 8, r = 5, k = 2

(a) $$P_{i(8,5,2)} = \frac{5!(8-2)!}{(5-2)!(8-5)!} = 2,400\, ways$$

(b)

By definition

$$C_{i(8,5,2)} = 20 \ ways$$

CONCLUSION

We can clearly see from the example 3.2 that if n become bigger, the task of exhaustively listing and counting all the possible outcomes of the arrangements becomes very difficult. We cann't effectively carry out this without the use of computer. To determine the number of possible outcomes we need an elegant mathematical formulae like the results we have obtain in this paper.

REFERENCES

[1]. Gaveth, Williams, Practical Finite Mathematics. Toronto. Allyn and Bacon Inc, pg. 208-210, (1976).

[2]..Kenneth H. Rosen, Discr. Math. & its Applications, Singapore. McGraw –Hill, 232-296, (1991).

[3] C. Moore M. Laisin and O.C. Okoli, Generalized r-permutation and r-combination techniques for k-separable inclusion to appear in International Journal of Applied Mathematics and Statistics (IJAMAS), 2010.

[4] O.C. Okoli , M. Laisin, and W. Uwandu, A generalized K-Permutation and k-Combination techniques for k-non-inclusion condition, Natural and Applied Science journal (Vol. 10 No. 1) pg. 36-44 (2009)

[5] O.C. Okoli , M. Laisin, and W. Uwandu, k-partition of k-Permutation and k-Combination for either k-inclusion and k-non-inclusion, Natural and Applied Science journal (Vol. 10 No. 2) pg. 138-145 (2009)

[6]. Richard, J Discr. Math , New York. Macmillian, pg. 43-65, (1984).

[7]. R. Simion and F. Schmidt, *Restricted permutations*, Europ. Journal. Comb. Vol. **6,** pg. 383-406 (1985).