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MODELLING OF MONTHLY NIGERIAN EXPORT COMMODITY PRICE INDICES BY SEASONAL BOX-JENKINS METHODS

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ABSTRACT

The time plot of a realisation ECPI of the series in Figure 1 reveals a slightly upward secular trend with no clear seasonal component. Seasonal (i.e. 12-monthly) differencing yields the series SDECPI which has a fairly horizontal trend and still no clear seasonality (see Figure 2). Augmented Dickey Fuller (ADF) unit root test adjudges both series ECPI and SDECPI as non-stationary. Non-seasonal differencing of SDECPI yields the series DSDECPI. Its time plot of Figure 3 reveals an overall horizontal trend and no clear regular seasonality. The ADF test shows that DSDECPI is seasonal. Its autocorrelation function in Figure 4 exhibits a significant negative spike at lag 12, an indication of 12-monthly seasonality and the presence of a seasonal moving average component of order one. Applying Surhatono's (2011) modelling steps, the initial $(0, 1, 1)x(0, 1, 1)_{12}$ SARIMA fit is found to be adequate.

Keywords: Export Commodity Price Indices, SARIMA models, Nigeria

INTRODUCTION

Most economic and financial time series are known to exhibit some seasonality as well as volatility. Price indices are inclusive. For instance, Etuk (2012a) observed Monthly Nigerian Composite Consumer Price Indices to be seasonal of period 12 months. Seasonal series may be modelled by seasonal Box-Jenkins or seasonal autoregressive integrated moving average (SARIMA) methods (Box and Jenkins, 1976).

Of recent a lot of researchers have shown renewed interest in the application of SARIMA models to model seasonal time series. A few of these are Ismail and Mahpol (2005), Linlin and Xiaorong (2012), San-Juan *et al.,* (2012), Saz(2011), *Luo et al.,* (2013), Arumugam and Anithakumari (2013), Surhatono (2011), Surhatono and Lee(2011), Etuk(2012b, 2013a, 2013b), Osabuohien-Irabor (2013), Etuk and Amadi (2013) and Etuk *et al.,* (2012).

In this work Nigerian Export Commodity Price Indices are to be modelled using SARIMA methods. Etuk (2014) has shown that for intrinsically seasonal models SARIMA techniques do better than just autoregressive integrated moving average (ARIMA) ones.

MATERIALS AND METHODS

The data for this work are 156 monthly Nigerian Export Commodity Price Indices from 2000 to 2012 obtainable from the website of the Central Bank of Nigeria, www.cenbank.org. It is published in the 2012 Statistical Bulletin – Section D (Read Only) as All SITC Product Export Price Index of Table D. 4.1. The Base Period is January 2007.

Sarima Model

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of order p and q*, denoted by ARMA(p, q), if it satisfies the following difference equation.

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \qquad (1)$$

Or

$$A(L)X_t = B(L)_{\varepsilon t} \qquad (2)$$

Where; $\{\varepsilon_t\}$ is a white noise process and the α's and the β's are constants such that the model is both stationary and invertible. $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$ where L is the backward shift operator defined by $L^k X_t = X_{t-k}$.

Many real-life time series are not stationary. For such a series $\{X_t\}$, Box and Jenkins(1976) proposed that differencing to a sufficient degree could make it stationary. Let d be such a degree. That is, the d^{th} difference of X_t, namely $\nabla^d X_t$, is stationary. If the series $\{\nabla^d X_t\}$ follows an ARMA (p, q) model, then $\{X_t\}$ is said to follow an *autoregressive integrated moving average of orders p, d and q,* designated ARIMA(p, d, q).

If $\{X_t\}$ is seasonal of period s, let $\nabla_s X_t = X_t - X_{t-s}$ be the seasonal difference of X_t once. Then $\nabla_s = 1 - L^s$. Suppose that the minimum order to which the series {Xt} would be differenced seasonally for stationary is D. Box and Jenkins (1976) proposed that $\{X_t\}$ may be modelled by

$$A(L)\Phi(L^s)\nabla^d \nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \qquad (3)$$

Where; $\Phi(L)$ and $\Theta(L)$ are polynomials. Their coefficients are such that the model is both stationary and invertible. Suppose that they are of orders P and Q respectively. The model (3) is called a *multiplicative seasonal autoregressive integrated moving average model of orders p, d, q, P, D, Q and s,* designated $(p, d, q)x(P, D, Q)_s$ SARIMA model.

Surhatono (2011), using moving average symbolism distinguishes between three types of SARIMA models. He says for a seasonal period of s, a subset SARIMA is of the form

$$\nabla^d \nabla_s^D X_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_s \varepsilon_{t-s} + \beta_{s+1} \varepsilon_{t-s-1} \qquad (4)$$

If $\beta_{s+1} \neq \beta_1 \beta_s$. Otherwise it is multiplicative. If $\beta_{s+1} = 0$ then the model is said to be additive. He then proposed the following modelling steps:

Fit a subset SARIMA model. If $\beta_{s+1} = 0$, then fit the additive model. Otherwise, check for multiplicativity.

Sarima Model Fitting

Order determination is invariably the first step of model building. Often it is done by graphical approach. The time plot could show up the period s of seasonality if the seasonal pattern, if existent, is of sufficient regularity. Often this is not the case. The correlogram could better reveal a seasonal tendency by a significant spike at the corresponding lag. If the spike is negative, the presence of a seasonal moving average (MA) component is suggestive; if positive, a seasonal autoregressive (AR) component is suggestive.

The orders of differencing, both seasonal D and non-seasonal d, are usually such that they sum up to at most 2. Traditionally, putting $D = d = 1$ is enough to yield a stationary series. At each stage, before and after differencing, the Augmented Dickey Fuller (ADF) unit root test shall be used to test for stationarity. The autocorrelation function ACF could give indication of an estimate of q as the cut-off point. Similarly the partial autocorrelation function, PACF, cuts off, if at all, at a lag estimating p. The numbers P and Q are similarly estimated by the seasonal cut-off points on the PACF and the ACF respectively.

With the orders determined the parameters could be estimated. Invariably the involvement of items of a white noise process in the model necessitates the use of non-linear optimization techniques in its estimation. An initial estimate is usually made and by an iterative process the estimate is sequentially improved upon until optimality is attained. The optimization criterion could be the least error sum of squares, the maximum likelihood or the maximum entropy procedure, etc. There is a linear optimization technique based on the Yule-Walker equations for the fitting of purely AR models. The duality relationship of AR and MA models is exploited to fit purely MA models by the same principles (See, e.g. Oyetunji, 1985).

After model fitting the fitted model is subjected to goodness-of-fit tests to ascertain its adequacy. A good fit is indicative if the residuals are uncorrelated with zero mean and are normally distributed. In this work use is made of the software Eviews for all the data analysis. For model estimation this package uses the least error sum of squares technique.

RESULTS

The time plot of ECPI in Figure 1 shows a slightly positive secular trend with no clear seasonality. Seasonal (i.e. 12-monthly) differencing yields the series SDECPI which has an overall horizontal trend with no clear seasonality (see Figure 2). A non-seasonal difference of SDECPI produces the series DSDECPI with an overall horizontal trend and no clear seasonal component (See Figure 3). With statistic values of -2.3 for ECPI, -2.8 for SDECPI and -6.43 for DSDECPI and the 1%, 5% and 10% critical values of -3.5, -2.9 and -2.6 respectively, the ADF unit root test confirms both ECPI and SDECPI as non-stationary and DSDECPI as stationary.

The correlogram of DSDECPI in Figure 4 shows negative significant spikes in the ACF as well as the PACF at lags 1 and 12. There is therefore seasonality of period 12 months as well as a seasonal MA component of order one. By Surhatono's (2011) modelling steps, the initial $(0, 1, 1)\text{x}(0, 1, 1)_{12}$ SARIMA model is estimated as summarized in Table 1 as

$$DSDECPI_t = \varepsilon_t - 0.4227\varepsilon_{t-1} - 0.8575\varepsilon_{t-12} + 0.3248\varepsilon_{t-13} \quad (5)$$
$$(\pm 0.0767) \quad (\pm 0.0656) \quad (\pm 0.3248)$$

which clearly is multiplicative. It is adequate for the following reasons: firstly, the fitted model agrees closely with the data as evident from Figure 5; secondly, the correlogram of the residuals in Figure 6 shows that the residuals are uncorrelated.

CONCLUSION

Nigerian Export Commodity Price Indices have been shown to follow an $(0, 1, 1)x(0, 1, 1)_{12}$ SARIMA model. This has also been demonstrated to be adequate.

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APPENDICES

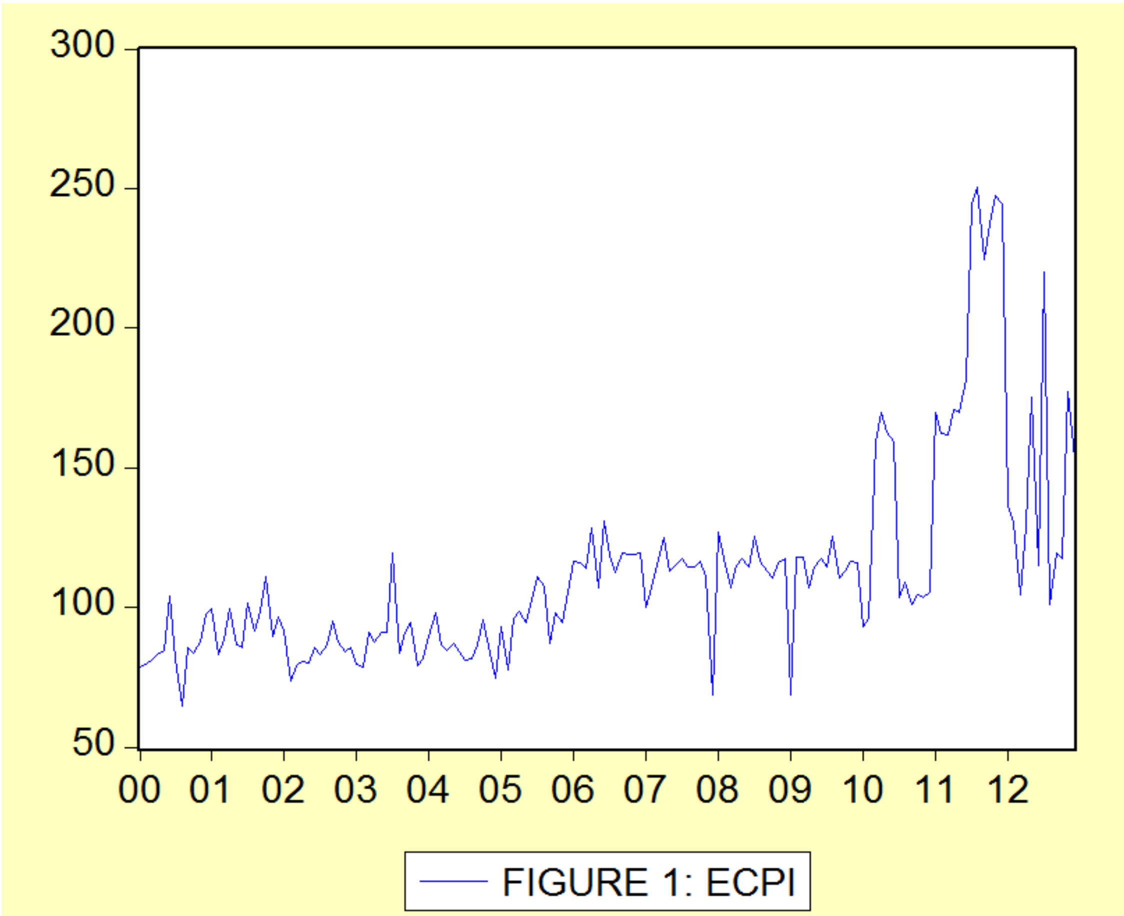
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FIGURE 1: ECPI

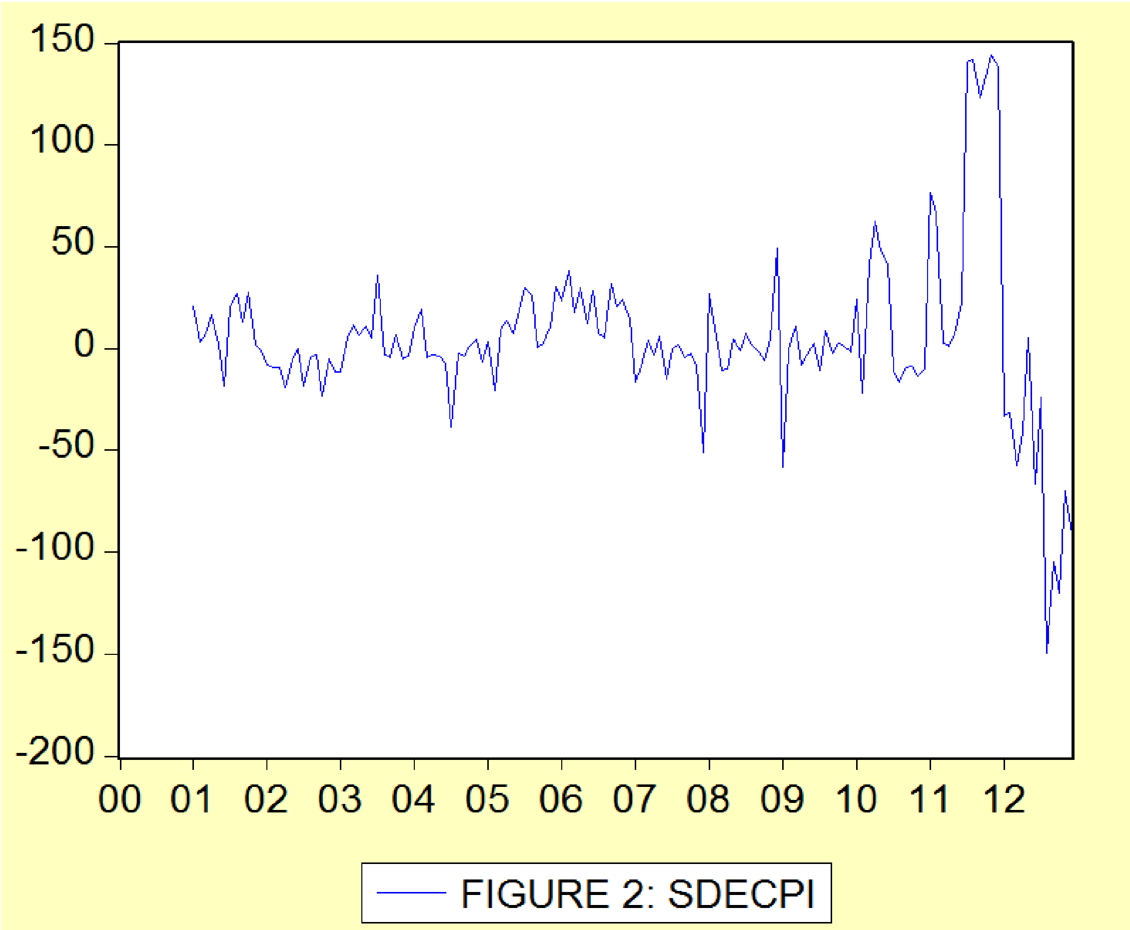
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FIGURE 2: SDECPI

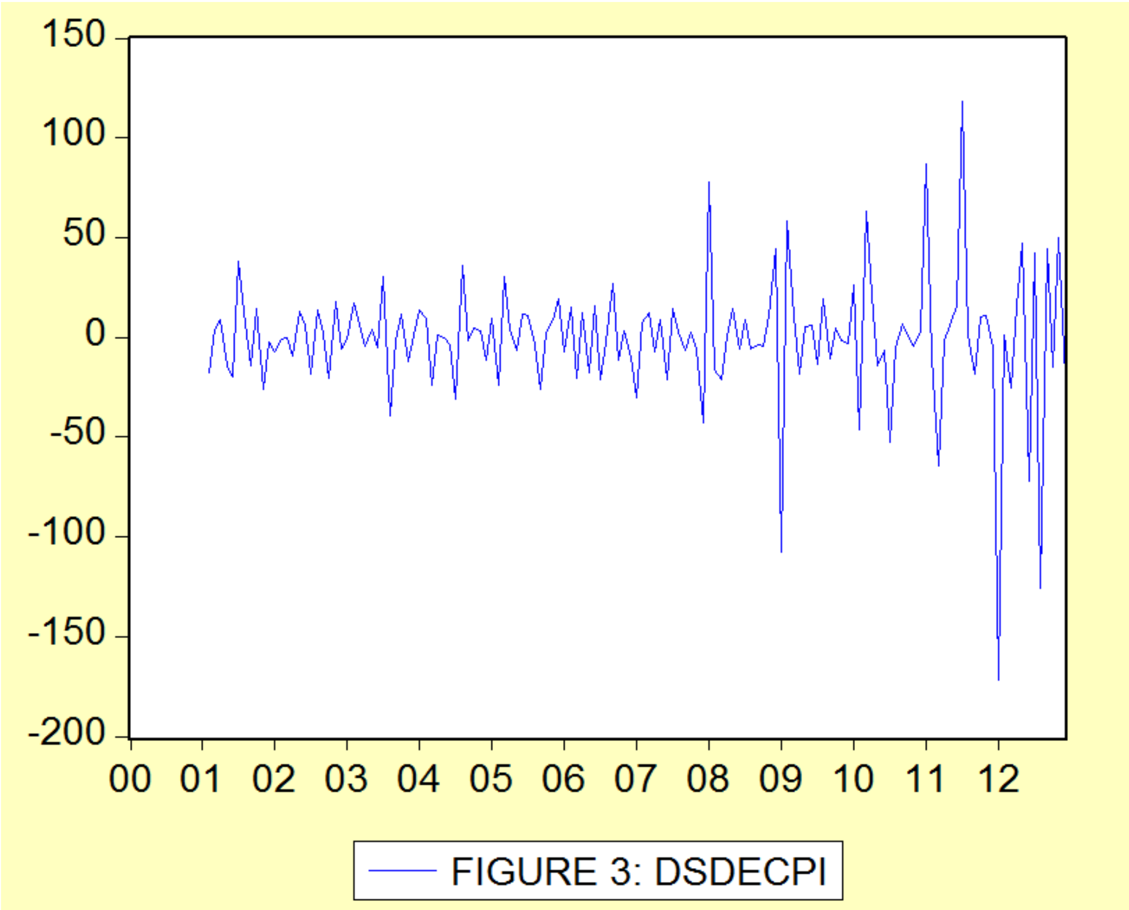
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FIGURE 3: DSDECPI

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.292	-0.292	12.463	0.000
		2	0.038	-0.052	12.677	0.002
		3	-0.085	-0.097	13.748	0.003
		4	-0.086	-0.153	14.861	0.005
		5	0.073	-0.003	15.663	0.008
		6	-0.116	-0.122	17.696	0.007
		7	0.126	0.040	20.132	0.005
		8	-0.091	-0.061	21.413	0.006
		9	0.025	-0.033	21.508	0.011
		10	0.156	0.165	25.317	0.005
		11	0.003	0.128	25.318	0.008
		12	-0.319	-0.353	41.394	0.000
		13	0.063	-0.095	42.027	0.000
		14	-0.078	-0.089	43.017	0.000
		15	0.021	-0.125	43.086	0.000
		16	0.135	0.079	46.083	0.000
		17	-0.038	0.011	46.321	0.000
		18	0.081	0.005	47.404	0.000
		19	-0.041	0.108	47.679	0.000
		20	0.012	-0.061	47.702	0.000
		21	-0.026	-0.056	47.819	0.001
		22	-0.078	0.077	48.855	0.001
		23	0.108	0.094	50.880	0.001
		24	-0.094	-0.171	52.409	0.001
		25	0.040	-0.040	52.693	0.001
		26	0.068	-0.012	53.503	0.001
		27	-0.036	-0.066	53.737	0.002
		28	-0.022	-0.001	53.825	0.002
		29	-0.045	-0.047	54.200	0.003
		30	0.012	0.007	54.227	0.004
		31	-0.022	0.080	54.318	0.006
		32	0.075	0.045	55.364	0.006
		33	-0.036	-0.098	55.602	0.008
		34	-0.008	0.008	55.613	0.011
		35	0.071	0.054	56.580	0.012

Dependent Variable: DSDECPI
Method: Least Squares
Date: 10/18/13 Time: 08:33
Sample(adjusted): 2001:02 2012:12
Included observations: 143 after adjusting endpoints
Convergence achieved after 14 iterations
Backcast: OFF (Roots of MA process too large for backcast)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.422748	0.076691	-5.512340	0.0000
MA(12)	-0.857537	0.065634	-13.06544	0.0000
MA(13)	0.324845	0.094857	3.424568	0.0008

R-squared	0.422328	Mean dependent var	-0.780839
Adjusted R-squared	0.414075	S.D. dependent var	31.86430
S.E. of regression	24.39076	Akaike info criterion	9.247041
Sum squared resid	83287.27	Schwarz criterion	9.309199
Log likelihood	-658.1635	F-statistic	51.17595
Durbin-Watson stat	1.986245	Prob(F-statistic)	0.000000

Inverted MA Roots	.99	.86+.49i	.86 -.49i	.50 -.85i
	.50+.85i	.38	.00+.99i	.00 -.99i
	-.49+.85i	-.49 -.85i	-.85 -.49i	-.85+.49i
	-.98			

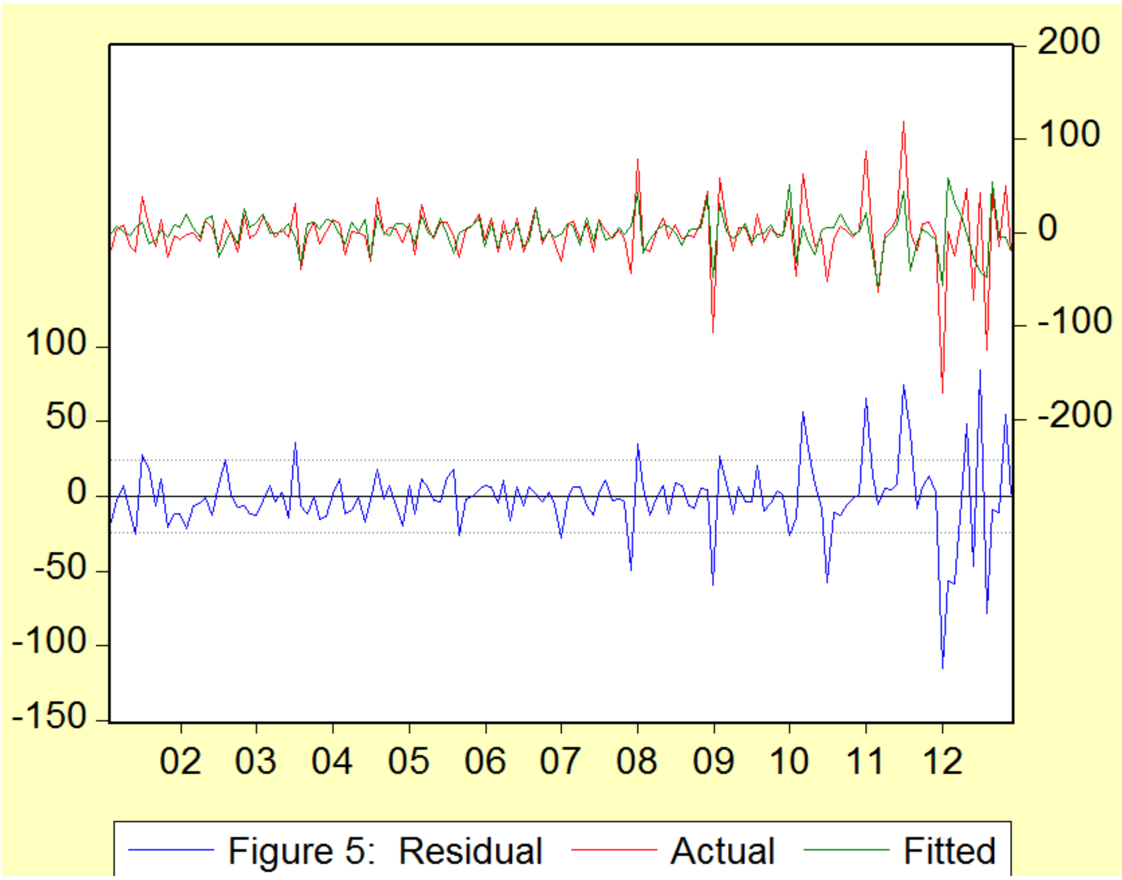
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Figure 5:

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.002	0.002	0.0004	
		2	0.051	0.051	0.3755	
		3	-0.105	-0.105	2.0007	
		4	-0.101	-0.104	3.5327	0.060
		5	-0.051	-0.041	3.9285	0.140
		6	-0.109	-0.112	5.7261	0.126
		7	0.046	0.028	6.0429	0.196
		8	-0.099	-0.113	7.5495	0.183
		9	0.016	-0.023	7.5897	0.270
		10	0.001	-0.008	7.5899	0.370
		11	-0.049	-0.080	7.9619	0.437
		12	-0.116	-0.157	10.105	0.342
		13	-0.071	-0.084	10.903	0.365
		14	-0.122	-0.176	13.290	0.275
		15	0.053	0.001	13.738	0.318
		16	0.132	0.073	16.570	0.220
		17	0.004	-0.090	16.573	0.280
		18	0.146	0.084	20.121	0.167
		19	0.017	0.013	20.167	0.213
		20	0.016	-0.041	20.213	0.263
		21	-0.048	-0.021	20.600	0.300
		22	-0.105	-0.120	22.506	0.260
		23	0.023	0.003	22.601	0.309
		24	-0.034	-0.006	22.802	0.355
		25	0.087	0.004	24.138	0.340
		26	0.049	0.031	24.573	0.373
		27	-0.021	-0.033	24.652	0.425
		28	-0.048	-0.069	25.072	0.458
		29	-0.084	-0.038	26.355	0.444
		30	-0.043	-0.029	26.689	0.481
		31	0.028	0.029	26.839	0.527
		32	0.058	0.070	27.476	0.546
		33	0.009	-0.035	27.492	0.597
		34	-0.073	-0.150	28.503	0.595

Figure 6: Correlogram of the Residuals

Reference to this paper should be made as follows: Ette Harrison Etuk (2013), Modelling of Monthly Nigerian Export Commodity Price Indices by Seasonal Box-Jenkins Methods. *J. of Physical Science and Innovation*, Vol. 5, No. 2, Pp. 91 – 102.