MODELLING THE DYNAMIC MECHANISM OF A METAL BULB TEMPERATURE TRANSMITTER

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ABSTRACT: *A mathematical model for the dynamic mechanism of a metal bulb temperature transmitter is developed. The model was formulated based on the mechanism of heat transfer, transmission of pressure along the capillary tube and the bellows movement. The combination of these three mechanisms produced the desired result of developing a transfer function that relates changes in bellows movement to changes in the temperature of the liquid in which the gas filled metal bulb is immersed. To obtain this transfer function requires that the inflow rate into the bellows be expressed as the ratio of difference in pressure to resistance at the entrance to the bellows. The transfer function obtained was translated to real time expression using the inverse Laplace transform technique.*

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INTRODUCTION

This paper is based on the dynamic response of a simple temperature transmitter consisting of gas filled metal bulb connected bellow via a metal capillary tube. The aim is to obtain an expression for the response of bellows movement to changes in temperature of the fluid surrounding the metal bulb. The metal bulb is filled a gas and immersed in a liquid whose temperature is to be controlled. Any change in the temperature of the liquid surrounding the bulb causes a change in the pressure of the gas inside the metal bulb and this change in pressure is transmitted in turn via a metal capillary tube to the bellows. This change in pressure upon arriving at the bellows will cause the bellows to move. To achieve this, the principles of heat transfer are used to set up an unsteady state energy balance across the metal bulb. Since the metal is filled with a gas and immersed in a liquid, the gas side and the liquid heat transfer coefficient together with the thermal conductivity of the metal tube are necessary to setup the overall heat transfer coefficient. The change in temperature of the gas inside the metal bulb (whose PVT behaviour is known) can now be determined and the consequence of this change in temperature of the gas on the pressure of the gas inside the bulb can then be determined. The formulation of mechanism by which these changes in gas pressure is transmitted via the metal capillary tube to the bellows enables us then to derive an expression that relate the response of bellows movement to changes in temperature of the liquid surrounding the metal bulb. In the literature the heat transfer coefficient from the liquid in the vessel to the gas in the bulb are usually lumped together and expressed in terms of an overall transfer coefficient, without showing these individual resistances combine to yield the overall heat transfer coefficient [1]. Expressions for the liquid side transfer coefficient, the thermal conductivity of metal bulb and the gas side transfer coefficient were obtained and use to estimate the overall heat transfer coefficient [2], [3]. The gas side transfer coefficient is obtained by assuming that, gas flow is laminar up the wall of the metal bulb and liquid side transfer coefficient is as a result of forced convection against the wall of the metal bulb due to impeller agitation of the liquid in which the metal bulb is immersed [4]. In most consideration the significance of frictional effects on the transmission pressure via the metal capillary tube is taken into consideration [5]. However some experimental works on short capillary tube have shown that in such cases friction could be neglected with minimal error [6]. So this study neglects the effects of friction in the transmission of pressure along the capillary tube. Most authors assume that the transmission of pressure along the capillary tube takes place isothermally because it avoids the added complication that the introduction of the energy equation would course [7], [8]. So, isothermal transmission of pressure along the capillary tube will be assumed and the energy equation will be neglected. This leaves only the equation of continuity and motion as pertinent equations for the transmission of pressure along the capillary tube [9]. Bellows are extensively used in the mechanism of pneumatic controllers. For bellows movement, a mass balance expression for inflow into the bellow is considered and combined with a force balance expression involving a bellows spring constant to obtain a transfer function that relates changes in bellows pressure to changes in pressure of the capillary tube [9]. To obtain this transfer function requires that the inflow rate into the bellows be expressed as the ratio of difference in pressure to resistance at the entrance to the bellows. Since, no author has considered combining the mechanism of heat transfer, transmission of pressure along the capillary tube and the bellows movement. In this paper, we shall show how combining all these three mechanisms achieve the desired result of producing the transfer function that relates changes in bellows movement to changes in the temperature of the liquid in which the gas filled metal bulb is immersed.

The Transmission Mechanism

The metal bulb contains air whose PVT behaviour is considered to be ideal. Changes in temperature of the liquid that surrounds the metal bulb will cause changes in the temperature of the air inside the metal bulb which in turn causes the pressure of the air inside the metal bulb to change. Hence, we shall obtain a relationship that connects the changes in air pressure inside the metal bulb to changes in temperature of the liquid surrounding the metal bulb. The gas filled metal bulb is immersed in a liquid whose temperature is T_T. When heat moves from the liquid across the metal bulb to the gas inside the metal bulb, it encounters three resistances in series.

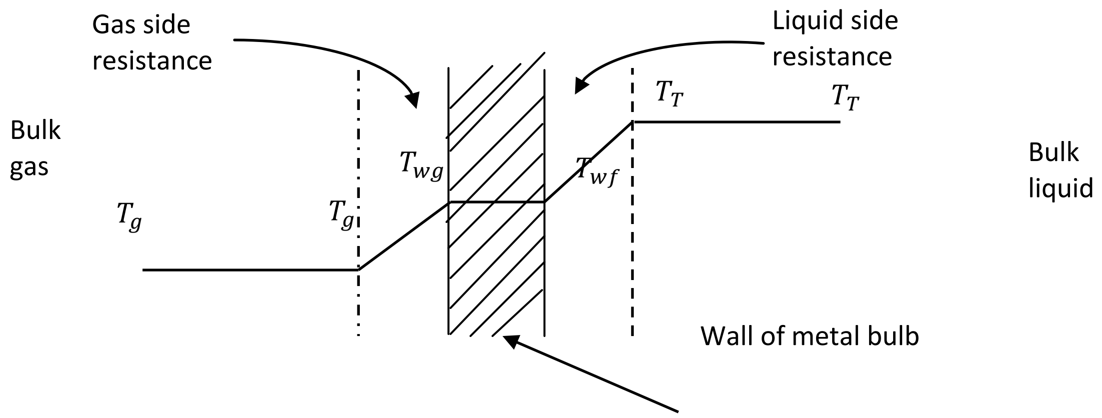

Fig. 1: Mechanism of Heat Transfer across the Metal Bulb

Where
T_g = temperature of the bulb gas at any time
T_{wg} = metal temperature of the gas side
T_{wf} = metal temperature of the liquid side
T_T = temperature of the bulk liquid at any time

The model illustrated in Fig. 2, assumes that an interface exists between the liquid and the metal wall, and an interface exists between the gas and the metal wall. It is also assumed that, the temperature any such interface is the same as that in the bulk of either the liquid or gas. The temperature difference $(T_T - T_g)$ between the liquid side interface and that of the metal wall drives heat across the liquid side resistance. The difference $(T_{wf} - T_{wg})$ drives heat across the metal wall and $(T_{wf} - T_g)$ drives heat across the gas side resistance. In order to obtain the overall resistance to heat transfer across the metal bulb (U), the gas side heat transfer coefficient (h_g), the liquid side heat transfer coefficient (h_f), the thermal conductivity of the metal wall (K_w), are necessary for estimating U, where the area (A) available for heat transfer is chosen as the outside area of the metal bulb [1]:

$$\frac{1}{UA} = \frac{1}{h_f A} + \frac{\Delta X}{K_w A} + \frac{1}{h_g A} \quad .....(1)$$

Since the metal bulb is immersed in a liquid that is continuously stirred, the liquid side heat transfer coefficient is given by [2]:

$$\frac{h_f D_j}{K} = 0.87 \left(\frac{L_p^2 N_t \rho}{\mu_w}\right)^{0.62} \left(\frac{C_p \mu}{K}\right)^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \quad ....(2)$$

The liquid moves as a result of forced convection due to impeller motion and equation 2 takes impeller characteristics such as impeller length (L_p) and impeller speed (N_l) into consideration when evaluating the heat transfer coefficient. Also, viscosity variations as a result of temperature difference between the bulk liquid and liquid near the wall of the metal bulb (μ_w, μ_b) are accounted for. The gas side heat transfer coefficient is given as [4]:

$$\frac{h_g l}{K_{av}} = 0.59 \left[\frac{L^3 \rho_{av} g \beta_{av} (-\Delta T_b)}{\mu_{av}^2} \left(\frac{C_p \mu}{K_{av}}\right)_{av}\right]^{\frac{1}{4}} \quad .....(3)$$

Where $\Delta T_b = \frac{T_{wg} - T_g}{2}$ and all quantities carrying subscript av are evaluated at $T_{av} = \frac{T_{wg} + T_g}{2}$

Equation 3 is justifiable because the gas inside the metal bulb moves by natural convention in the laminar range. Development of Unsteady State Energy Balance Equation. The unsteady state energy balance equation across the metal bulb gives us:

$$UA(T_T - T_g) = MC_p \frac{dT_g}{dt} \quad ......(4)$$

By introducing the deviation variables $T_T = \bar{T}_T + \Delta T_T$ and $T_g = \bar{T}_g + \Delta T_g$ into the unsteady state energy balance equation and simplifying it gives us:

$$UA(\Delta T_T - \Delta T_g) = MC_p \frac{d\Delta T_g}{dt} \quad ....(5)$$

Taking Laplace transform of equation 5 yields:

$$\frac{\Delta T_{g(s)}}{\Delta T_{T(s)}} = \frac{1}{\tau s + 1} \quad ....(6)$$

Where τ, the time constant is given by $\frac{MC_p}{UA}$ and s is the Laplace variable

Equation 6 relates changes in gas temperature inside the metal bulb to changes in temperature of the liquid in which the metal bulb is immersed. Thus, considering the gas which fills the metal bulb has an ideal gas PVT behaviour for a fixed volume.

$$\Delta T_g = \frac{\bar{T}_g}{\bar{P}_g}\Delta P_g \qquad \text{...............(7)}$$

By substituting equation 7 into 6, the result is a transfer function that relates changes in pressure inside the metal bulb to changes in temperature of the liquid in which the metal bulb is immersed.

$$\frac{\Delta P_{g(s)}}{\Delta T_{T(s)}} = \left[\frac{1}{\tau s+1}\right]\frac{\bar{T}_g}{\bar{P}_g} \qquad \text{....(8)}$$

In order to achieve steady state, the metal bulb should be immersed sufficiently long enough in the vessel, so that the temperature at the interfaces and the temperature at the inside and outside surfaces of the metal bulb arrive at their steady state values $\bar{T}_T$, $\bar{T}_g$, $\bar{T}_{wf}$, and $\bar{T}_{wg}$. The attainment of steady state assures that the rate at which heat is transferred across the liquid side resistance is the same rate at which heat is transferred across the metal wall and is the same rate at which heat is transferred across the gas side resistance.

So,

$$h_f A(\bar{T}_T - \bar{T}_{wf}) = h_g A(\bar{T}_{wg} - \bar{T}_g) \qquad \text{......(9)}$$

Transmission of Pressure along the Capillary Tube

The gas filled metal bulb is connected to bellows via a metal capillary tube (a pneumatic controller). The equations of motion, continuity and energy are pertinent to the transmission of pressure along the capillary tube. But, our interest here is the pressure movement along the capillary tube and based on the assumption that the movement of pressure along the capillary tube is isothermal, the energy equation is of minimal consequence. Thus, only the equations of continuity and motion shall be considered. So, the equation of continuity is written as:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v)}{\partial x} = 0 \qquad \text{.....(10)}$$

Neglecting the gradient of velocity along the radial and angular directions, equation 9 becomes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \qquad \text{..............(11)}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\rho \partial v}{\partial x} - \frac{v \partial \rho}{\partial x} \qquad \text{..............(12)}$$

As the pressure waves moves from point to point in the capillary tube overall air in the capillary tube is stationary, and consequently the velocity of motion of the gas in the capillary tube is negligible. Thus, equation 11 is simplified as:

$$\frac{\partial \rho}{\partial t} = -\frac{\rho \partial v}{\partial x} \qquad \text{...............(13)}$$

But the volumetric flow rate Q and pressure $P$is given as:

$$Q = Sv \rightarrow v = \frac{Q}{S} \; and \; P = \delta\rho \;\; ..(14)$$

Substituting equation 13 into 12 we obtain:

$$\frac{\partial P}{\partial t} = -\frac{P\partial Q}{S\partial x} \;\; ....(15)$$

Introducing the deviation variables $P = \bar{P} + \Delta P$ and $Q = \bar{Q} + \Delta Q$ into equation 14 gives:

$$\frac{\partial \Delta Q}{\partial x} = -\frac{S}{P}\frac{\partial \Delta P}{\partial t} \;\; ....(16)$$

Note that speed of sound in air C_s is given as $C_s = \sqrt{\frac{P}{\rho}}$ so $P = C_s^2\rho$, and comparing with $P = \delta\rho$ it is seen that the constant of proportionality $\delta = C_s^2$. Therefore, equation 15 becomes:

$$\frac{\partial \Delta Q}{\partial x} = -\frac{S}{\rho C_s^2}\frac{\partial \Delta P}{\partial t} \;\; ....(17)$$

For the equation of motion, we can write:

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = -\frac{\partial P}{\partial x} + \mu\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial x^2}\right] + \rho g_x \;\; ...(18)$$

Neglecting the average velocity terms, equation 17 becomes:

$$\rho\frac{\partial v}{\partial t} = -\frac{\partial P}{\partial x} + \mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \;\; ....(19)$$

Equation 18 consists of three pressure gradient as follows:

i. Due to inertial: $\frac{\partial P_i}{\partial x} = \rho\frac{\partial v}{\partial t}$
ii. Due to friction: $\frac{\partial P_F}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)$
iii. Total pressure: $\frac{\partial P}{\partial x}$

So equation 18 can be written as:

$$\frac{\partial P_i}{\partial x} + \frac{\partial P_F}{\partial x} = -\frac{\partial P}{\partial x} \;\; ....(20)$$

For a frictionless flow, the inertial part alone contributes to the total pressure and substituting for $v$gives:

$$\frac{\partial P}{\partial x} = -\frac{\rho}{S}\frac{\partial Q}{\partial t} \;\; ....(21)$$

The Laplace transform of equations 16 and 20 is:

$$\frac{d\Delta Q}{dx} = -\frac{S_s}{\rho C_s^2}\Delta P \;\; ....(22)$$

$$\frac{d\Delta P}{dx} = -\frac{\rho_s}{S}\Delta Q \;\; ....(23)$$

Differentiating equations 21 and 22

$$\frac{d^2\Delta Q}{dx^2} = -\frac{S_s}{\rho C_s^2}\frac{d\Delta P}{dx} \;\; ....(24)$$

$$\frac{d^2\Delta P}{dx^2} = -\frac{\rho_s}{S}\frac{d\Delta Q}{dx} \;\; ....(25)$$

Substituting equations 21 and 22 into equations 23 and 24 and rearranging gives:

$$\frac{d^2\Delta Q}{dx^2} = -\frac{S^2}{\rho C_s^2}\Delta Q \;\; ....(26)$$

$$\frac{d^2\Delta P}{dx^2} = -\frac{S^2}{\rho C_s^2}\Delta P \;\; ....(27)$$

The Laplace solutions are:

$$\Delta Q(x,s) = C_1\, Cosh\frac{S}{C_s}x + C_2\, Sinh\frac{S}{C_s}x \;\; .....(28)$$

$$\Delta P(x,s) = \frac{-\rho C_s}{S}\left[C_1\, Sinh\frac{S}{C_s}x + C_2\, Cosh\frac{S}{C_s}x\right] \;\; ......(29)$$

Where C_1 and C_2 are constants of integration and are evaluated at boundary condition. The evaluation of the constants of integration provides the transfer function that relates changes in exist pressure from the capillary tube to changes in inlet pressure to the capillary tube. The equation of continuity and motion helps in describing the manner with which pressure is transmitted via the metal capillary tube. Thus, equation 28 is the required equation for pressure transmission along the capillary tube; however equation 27 will be useful in evaluating the constants of integration. So, assuming the change in pressure in the gas filled metal bulb ΔP_gthat enters the capillary tube is $\Delta P_g(s) = \Delta P(0,s)$ and that entering the bellows at the end of the capillary tube $\Delta P_g(s) = \Delta P(L,s)$. Thus substituting the boundary condition for the change in pressure at the entrance to the capillary tube and that at the exit of the capillary tube into equations 27 and 28 gives:

$$\Delta Q(0,s) = C_1 \, Cosh\frac{S}{C_s}(0) + C_2 \, Sinh\frac{S}{C_s}(0) \qquad ....(30)$$

$$\Delta P(0,s) = \frac{-\rho C_s}{S}\left[C_1 \, Sinh\frac{S}{C_s}(0) + C_2 \, Cosh\frac{S}{C_s}(0)\right] \quad ..(31)$$

So,

$$\Delta Q(0,s) \approx 0 \qquad ....(32)$$

$$\Delta P(0,s) = \frac{-\rho C_s}{S} C_2 \;=>\; C_2 = \frac{\Delta P_g(s)S}{\rho C_s} \qquad ....(33)$$

Therefore, the change in pressure entering the bellows is:

$$\boxed{\Delta P(L,s) = \Delta P_g(s) \, Cosh\frac{S}{C_s}L} \quad ...(34)$$

To model the mechanism of bellows movement, the pressure transmitted via the capillary tube moves into the bellows and causes the bellows to move either by extension or compression. Thus, the transfer function that relates bellows movement to changes in pressure at the entrance to the bellows is given by:

$$\boxed{\frac{\Delta X_b(s)}{\Delta P_{(L,s)}} = \frac{A_b}{k_b}\frac{1}{\tau_1 s+1}} \qquad ..(35)$$

Where X_b= distance of the movable ends of the bellows

A_b= cross sectional area normal to direction of bellows movement

k_b = spring constant of the bellows

RESULT AND DISCUSSION

The transfer functions for the various mechanisms of heat transfer, transmission of pressure along the capillary tube and the bellows movement were transformed to real time space equations and the solution evaluated. We shall now show how combining all these three mechanisms achieve the desired result of producing a single transfer function (translated in real time) that relates changes in bellows movement to changes in the temperature of the liquid in which the gas filled metal bulb is immersed. The various pertinent equations 8, 9, 34 and 35 will be combined to obtain the transfer function that relates changes in bellows movement $\Delta X_b$to changes in the temperature of the liquid in which the metal bulb is immersed.

Thus,

$$\Delta X_b(s) = \frac{A_b}{k_b}\cdot\frac{1}{\tau_1 s+1}\cdot\frac{1}{\tau s+1}\cdot\frac{\bar{P}_g}{\bar{T}_g}\, Cosh\frac{S}{C_s}L.\Delta T_{T(s)}$$

For a step change in the temperature of the liquid in the vat i.e. $\Delta T_{T(s)} = \frac{\Delta T_T}{S}$

$$\Delta X_b(s) = \frac{A_b}{k_b}\frac{\bar{P}_g}{\bar{T}_g}\Delta T_T\left[\frac{1}{s(\tau_1 s+1)(\tau s+1)}\, Cosh\left(\frac{S}{C_s}L\right)\right]$$

Noting that $Cosh\left(\frac{S}{C_s}L\right) = \frac{e^{S/C_s L} + e^{-S/C_s L}}{2}$

So that,

$$\Delta X_b(s) = \frac{A_b}{k_b}\frac{\bar{P}_g}{\bar{T}_g}\Delta T_T\left[\left(\frac{1}{s(\tau_1 s+1)(\tau s+1)}\right)\left(\frac{e^{S/C_s L} + e^{-S/C_s L}}{2}\right)\right]$$

$$\Delta X_b(s) = \frac{A_b}{k_b}\frac{\bar{P}_g}{\bar{T}_g}\Delta T_T\left[\frac{1}{s(\tau_1 s+1)(\tau s+1)}e^{-\left(\frac{-SL}{C_s}\right)} + \frac{1}{s(\tau_1 s+1)(\tau s+1)}e^{\left(\frac{-SL}{C_s}\right)}\right] \quad ..(36)$$

To obtain the inverse Laplace transform of equation 36 (i.e. the real time expression for changes in bellows movement due to sudden changes in the temperature of the liquid in the vat in which the metal bulb is immersed). Let f (t) be the function whose Laplace transform is F(s), from [10]:

$$\mathcal{L}\{f(t-\hat{\alpha})\} = e^{-s\hat{\alpha}}F(s)$$

So that in equation 36, let $F(s) = \frac{1}{s(\tau_1 s+1)(\tau s+1)}$

Then from Laplace transform theory:

$$f(t) = 1 + \frac{1}{\tau-\tau_1}\left(\tau_1 e^{\frac{-t}{\tau_1}} - \tau e^{\frac{-t}{\tau}}\right)$$

Now the convolution

$$\mathcal{L}\{f(t-\hat{\alpha})\} = e^{-s\hat{\alpha}}F(s)$$

So that with $\hat{\alpha} = \frac{L}{C_s}$ then

$$\mathcal{L}\left\{f\left(t-\frac{L}{C_s}\right)\right\} = e^{-S\frac{L}{C_s}}F(s) = e^{-S\frac{L}{C_s}}\frac{1}{s(\tau_1 s+1)(\tau s+1)}$$

$$\mathcal{L}^{-1}\left(e^{-S\frac{L}{C_s}}F(s)\right) = f\left(t-\frac{L}{C_s}\right) = 1 + \frac{1}{\tau-\tau_1}\left(\tau_1 e^{-\frac{\left(t-\frac{L}{C_s}\right)}{\tau_1}} - \tau e^{-\frac{\left(t-\frac{L}{C_s}\right)}{\tau}}\right)$$

Therefore, the time dependent response of bellows movement to a sudden step change in the temperature of the liquid in the vat (ΔT_T) in which the metal bulb is immersed can be expressed as:

$$\Delta X_b(t) = \frac{A_b}{k_b}\frac{\bar{P}_g}{\bar{T}_g}\frac{\Delta T_T}{2}\left[\left(1 + \frac{1}{\tau-\tau_1}\left(\tau_1 e^{-\frac{\left(t+\frac{L}{C_s}\right)}{\tau_1}} - \tau e^{-\frac{\left(t+\frac{L}{C_s}\right)}{\tau}}\right)\right) + \left(1 + \frac{1}{\tau-\tau_1}\left(\tau_1 e^{-\frac{\left(t-\frac{L}{C_s}\right)}{\tau_1}} - \tau e^{-\frac{\left(t-\frac{L}{C_s}\right)}{\tau}}\right)\right)\right] \quad ..(37)$$

CONCLUSIONS

This paper has showed the development of dynamic response of bellows movement to sudden changes in the temperature of the liquid in the vat (ΔT_T) in which the metal bulb is immersed. The transfer functions for the mechanisms of heat transfer, transmission of pressure along the capillary tube and the bellows movement were transformed to real time space equations. Combining all these three mechanisms achieve the desired result of producing a single transfer function (translated in real time) that relates changes in bellows movement to changes in the temperature of the liquid in which the gas filled metal bulb is immersed.

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