Journal of Science and Multidisciplinary Research
Volume 5, Number 1, 2013.
ISSN: 2277-0135

Determination of the Most Appropriate Least Squares Method for Position Determination in a Triangulation Network

Omogunloye O.G.[1], Qaadri J. A.[2], Oladiboye O.E[3] and Oshode J. O[4]
[1]Dept. of Surveying and Geoinformatics, University of Lagos, Lagos
[2]Dept. of Surveying and Geoinformatics, Federal School of Surveying, Oyo, Oyo State
[3]Dept. of Surveying and Geoinformatics, Yaba Tech, Lagos
[4]Dept. of Surveying and Geoinformatics, University of Lagos, Lagos
E-mail: gabolushohan@yahoo.com

ABSTRACT

The Fundamental Least Squares Observation Equation Method of Adjustment is reviewed using a case study of a Triangulation network. Four Alternative techniques were applied. They are: Simultaneous, Sequential, Phase and Combined (Phase & Sequential). Based on computational results obtained from these techniques suggestions were made as to the merit of one method over the other.

Keywords: Least Square Adjustment, Triangulation Network, Observation Equation Method

INTRODUCTION

In recent years, considerable attention has been given to Least Squares Adjustment Computations in Geodesy, Surveying, Photogrammetry, Remote Sensing and all other related disciplines (Ayeni 1980, 1985). This has given rise to a number of Least Squares Method. The researcher or the practitioner is often therefore faced with the difficulty of choosing the most appropriate method, out of several alternatives for solving a particular geodetic positioning problem. It is the objective of this paper to discuss briefly the four techniques of Least Square Adjustment using the Observation Equation Method, and to demonstrate their computational suitability for certain control survey problem such as a Triangulation network. The authors are aware of the number of different nomenclatures, terminologies and notations used in Least Squares Method which vary from one author to another. All the symbols and notations used in this paper are therefore defined in the text.

MATHEMATICAL METHODS

Simultaneous Method

Observation Equation Method which is used for Simultaneous approach is defined by equation (1).

$$L^a = F(X^a) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1a)$$

linearised form is given by equation (2)

$$V = AX + L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2,2a)$$

Where;

$L^a = L + V$, $L = Vector\ of\ observation$
$X^a = Adjusted\ parameters$

V is the vector of residuals, X is the solution vectors, A is the design matrix dF/dX^a , X^0 is the approximate of the parameters, $L = F(X^0) - L$

The resulting normal equation is shown in equation

$$X = -(ATPA)^{-1}ATPL \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3a)$$

P = Weight matrix of observations

Iterative solution is necessary to compensate for linearization in equation (2) giving rise to equation (4)

$$X^a i + 1 = X^0 i + Xi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4a)$$

Where; i represent ith iteration which converges as Xi = 0

Fig.1.1 shows the triangulation figure and Table 1.1 the observed angles used for simultaneous adjustment for the whole network.

Tables 1. A & B shows the result of simultaneous adjustment.

Sequential Method

Observation Equation method which is used for Sequential approach is defined by equation (5) and (6).

$$L^a1 = F1(X^a1) \ldots\ldots (2.5a)$$
$$L^a2 = F2(X^a1, X^a2) \ldots\ldots (2.6a)$$

STEP 1

The models above can be considered for phase adjustment, in which case equation (5) is adjusted first to obtain

$$X^*1 = -(A1TPA1)^{-1}A1TPL1 \ldots\ldots (2.7a)$$

STEP 2

$$L^aX1 = X^a1 \ldots\ldots (2.8a)$$
$$L^a2 = F2(X^a1 + X^a2) \ldots\ldots (2.9a)$$

Xª1 & Xª2 are adjusted parameters, Lª1 & Lª2 are adjusted observations LX1 & Xª2 are additional observation s and parameters respectively

$L1 = F1(X^01) - L1, L2 = F2(X^01, X^02) - L2$

Linearising equation (8) & (9)

$$VX1 = X2 + LX1 \ldots\ldots (2.10a)$$
$$V2 = A2X1 + A3X2 + L2 \ldots\ldots (2.11a)$$

Where; LªX1 = LX1 + VX1, Lª2 = L2 + V2

Vx1 & V2 are the vectors of residuals, X1 & X2 are the solution vectors, A1, A2 & A3 are the respective design matrices. X1º & X2º are the approximate values of the parameters.

The resulting normal equations are shown in equations

$$X1 = X^*1 - P^{-1}X1A2TK2 \ldots\ldots (2.12a)$$
$$X2 = -(A3T(A2P^{-1}X1A2T + P^{-1}2)^{-1}A3T(A2P^{-1}X1A2T + P^{-1}2)^{-1}(A2X^*1 + L2) \ldots\ldots (2.13a)$$

Where; $K2 = (A2P^{-1}X1A2T + P^{-1}2)(A2X2 + A2X^*1 + L2)$

$$PX1 = \delta^2 0\mathcal{E}^{-1}X^a1,\ \ A2 = dF2/dX^a1,\ \ A3 = dF2/dX^a2,\ \ X1 = X^a1$$

$L2 = L^02 - L2,\ L^02 = F2(X^01, X^02), X^a2 = X^02 + X2, P2 = weight\ matrix\ of\ L2$

Iterative solution is necessary to compensate for linearization in equations (2.12a) & (2.13a) giving rise to equation- (2.14a) & (2.15a)

$$X^a1i + 1 = X^01i + X1i \ldots\ldots (2.14a)$$

$$X^a2i + 1 = X^02i + X2i \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . (2.15a)$$

Where; i represent ith iteration which converges as X1i = 0 & X2i = 0

Fig.1.1 shows the triangulation figure and Table 1.1 the observed angles used for Sequential adjustment for the whole network. Tables 2. A & B shows the results of Sequential adjustment.

Phase Method

Observation Equation method which is used for Phase approach is defined by equations (2.12b) and (2.13b)

$$L^a1 = F1(X^a1) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.12b)$$

$$L^a2 = F2(X^a2) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.13b)$$

And their linearised forms are given by equation (2.14b) & (2.15b)

$$V1 = A1X1 + L1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . \quad (2.14b)$$

$$V2 = A2X2 + L2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots . \quad (2.15b)$$

Where $L^a1 = L1 + V1$, $L^a2 = L2 + V2$,

V1 & V2 are the vectors of residuals, X1 & X2 are the solution vectors, A1, & A2 are the respective design matrices.

$X1^o$ & $X2^o$ are the approximate values of the parameters

The resulting normal equations are shown in equations

$$X1 = -(A1TP1A1)^{-1}A1TP1L1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.16b)$$

$$X2 = -(A2TP2A2)^{-1}A2TP2L2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.17b)$$

Iterative solution is necessary to compensate for linearization in equation (2.14b) & (15) giving rise to equation-(2.18b) & (2.19b)

$$X^a1\,I + 1 = X^0I + XI \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.18b)$$

$$X^a2\,I + 1 = X^02 + X2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.19b)$$

Where; I represent ith iteration which converges as XI = 0

Fig.1.1 shows the triangulation figure and Table 1.1 the observed angles used for Phase adjustment for the whole network. Tables 3 A & 3B shows the results of Phase adjustment.

Combined (Phase & Sequential) Method

Observation Equation method which is used for Combined approach is defined by the Sequential method, However

X*1 is use as observations in the adjustment of equation (2.6a) along with weight matrix QX1

$$QX1 = -(A1TPA1^{-1})$$

Fig.1.1 shows the triangulation figure and Table 1.1 the observed angles used for Combined adjustment for the whole network. Table 4A & 4B shows the results of Combined adjustment.

DISCUSSION OF RESULTS

From the results displayed in Table 5A, it should be obvious that the Combined (Phase & Sequential) technique appears to be most suitable for triangulation network from the overall view of the suitability criteria applied. The (VTPV/R), Trace $X^a(\Sigma X^a)$, Trace L^a (ΣL^a) are least for the Combined method. The next suitable is the Simultaneous, followed by the Sequential and lastly the Phase method. It does not however give one of the minimum storage requirement as well as the lowest C.P.U. time.

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that different least squares methods should yield the same results if the same weighting system is used in each method (Ayeni 1980) However this is not so when different techniques are applied to a particular Least square method such as the method of observation Equation. This is obvious from Table 5A. Table 5A is derived from Tables 1A --- Tables 4B.

It has been established that for Triangulation adjustment using the Observation Equation method, the Simultaneous technique is the most suitable for mini desk computer because it requires less C.P.U. time and less storage capacity and it is less rigorous to accomplish compared to the other techniques. From the computational results presented in Table 5A the Combined method yields better values of adjusted observations and high accuracies. It should however be remembered that a good knowledge of the approximate weight is required for the approximate values of the parameters in order to obtain high accuracy (Ayeni 1980, 1985).

It was established that the Trace $X^a(\Sigma X^a)$ in Phase adjustment for each iteration converges at a slow rate compared with other techniques, (Tables 3B).This is so because errors are accumulated from one Phase to another Phase during adjustment due to common parameters between any two phases. However if the dimensional arrays had been properly used for each phase at any time during the adjustment the Phase method might yield the most suitable in terms of storage capacity.

Most of the problem dealt with in these investigations do not required large storage capacity as well as using other methods of adjustment such as Condition Equation and the Generalized mixed model methods. It is therefore recommended for future study that a comparison of these least square methods as well as the four techniques discussed in this paper should be considered for Geodetic control Adjustment involving large nets (Level networks, traverse networks, Trilateration nets and a larger triangulation net). Problems of such magnitude may bring out more clearly the merits and the demerits of these least squares methods.

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Reference to this paper should be made as follows: Omogunloye O.G. *et al.,* (2013), Determination of the Most Appropriate Least Squares Method for Position Determination in a Triangulation Network, *J. of Science and Multidisciplinary Research Vol.5, No.1, Pp. 71-83.*

Biographical Note:

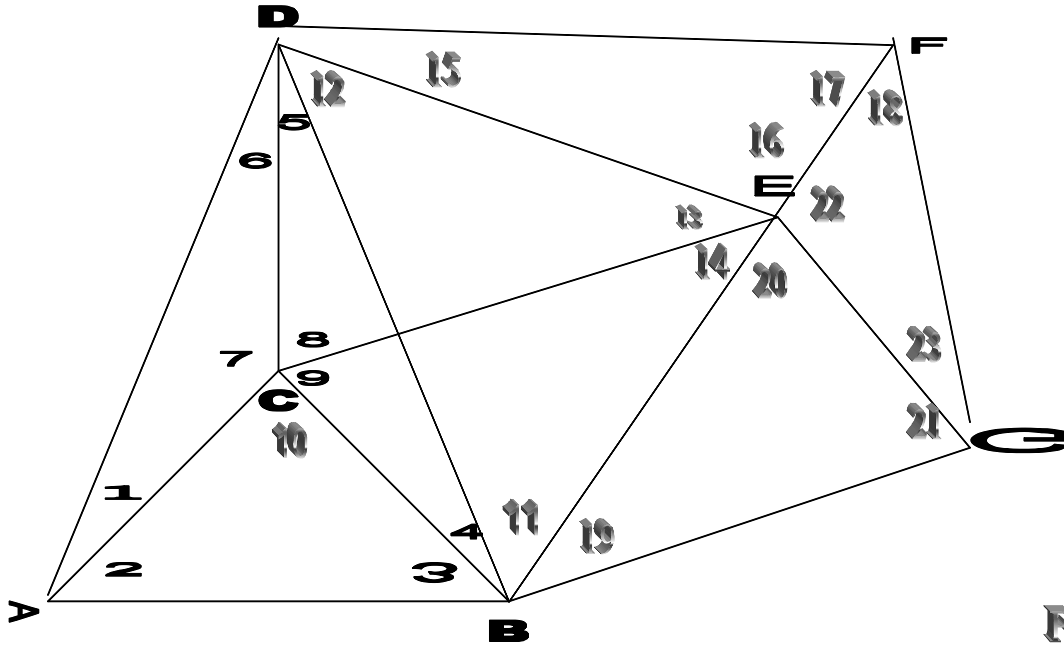

Figure 1.1

Table 1A

SIMULTANEOUS METHOD OF ADJUSTMENT OF TRIANGULATION NET USING OBSERVATION EQUATION METHOD							
ADJUSTED COORDINATES OF STATIONS ITERATED SIX TIMES							
	ITERATION 0	**ITERATION 1**	**ITERATION 2**	**ITERATION 3**	**ITERATION 4**	**ITERATION 5**	**ITERATION 6**
XC	107154.9895885352	107154.9886810447	107154.9880969989	107154.9876338897	107154.9875464861	107154.9875009735	107154.9875009735
YC	275500.001821133	275500.0017749285	275500.0017968321	275500.0018003152	275500.0017758484	275500.0018099622	275500.0018099622
XD	107149.9883385903	107149.9864735747	107149.9855077135	107149.9847310622	107149.9846115818	107149.9845759535	107149.9845759535
YD	276140.0156138032	276140.0155637829	276140.0155138698	276140.0155055454	276140.0154326661	276140.0154717883	108000.00274717883
XE	108000.0045546016	108000.0036613292	108000.0031943865	108000.0028626407	108000.0027571911	108000.0027478947	108000.0027478947
YE	275810.0099033022	275810.0111012802	275810.0117157498	275810.0122250707	275810.0122723453	275810.0123111327	275810.0123111327
XF	108200.0062038602	108200.0047793683	108200.0040290344	108200.0034611224	108200.0033309248	108200.0032985217	108200.0032985217
YF	276200.0166505129	276200.0184305034	276200.0193472177	276200.0201157622	276200.020190955	276200.0202297235	275200.0202297236
XG	108550.0088158806	108550.0090284572	108550.0091026284	108550.0092280779	108550.0091744801	108550.0091799117	108550.0091799117
YG	275400.0136203703	275400.0154155336	275400.0163706857	275400.0171559208	275400.0172399104	275400.0172910491	275400.0172910491

Table 1B

ADJUSTED A- POSTERIORI, TRACE OF COVARIANCE MATRICE OF ADJUSTED PARAMETER & OBSERVABLES ITERATED SIX TIMES							
	ITERATION 0	ITERATION 1	ITERATION 2	ITERATION 3	ITERATION 4	ITERATION 5	ITERATION 6
VTPV / R (δ^2)	0.000261	2.59E-09	3.63E-09	1.07E-08	1.57E-08	1.05E-08	7.94E-10
TRACEX^a ($\sum X^a$)	2730.102	0.027104	0.037984	0.111545	0.163793	0.10994	0.008299
TRACEL^a ($\sum L^a$)	0.075666	7.51E-07	1.05E-06	3.09E-06	4.54E-06	3.05E-06	2.3E-07

Table 2A

SEQUENTIAL METHOD OF ADJUSTMENT OF TRIANGULATION NET USING OBSERVATION EQUATION METHOD							
ADJUSTED COORDINATES OF STATIONS ITERATED SIX TIMES							
	ITERATION 0	ITERATION 1	ITERATION 2	ITERATION 3	ITERATION 4	ITERATION 5	ITERATION 6
XC	107154.9950772433	107154.9963171268	107154.9961647477	107154.9954147002	107154.9944208732	107154.99332471	107154.9922654707
YC	275500.0008378865	275500.003314339	275500.0044559452	275500.0049108751	275500.00501941	275500.0049557727	275500.0048044396
XD	107149.9992710891	107149.9988273606	107149.997569768	107149.995885527	107149.9940807876	107149.9922427622	107149.9905344536
YD	276140.0106483437	276140.0149298419	276140.0172111327	276140.0183735793	276140.019009702	276140.0193026661	276140.0194223952
XE	108000.006892905	108000.0082953586	108000.0086320457	108000.0083291345	108000.0077910545	108000.0070935691	108000.0064102591
YE	275809.9999083501	275810.0050100462	275810.0082813314	275810.0105301977	275810.0122290178	275810.0135924271	275810.0147453148
XF	108200.0103420721	108200.0117842178	108200.0118984796	108200.0112156451	108200.0102873252	108200.009141381	108200.0080280523
YF	276200.0018332942	276200.0087663268	276200.0133207634	276200.0164974005	276200.0189698558	276200.0209700449	276200.022726287
XG	108550.0027757135	108550.0078830225	108550.0107015042	108550.0121855676	108550.0130620662	108550.0135498712	108550.0139131477
YG	275400.0025360493	275400.0062965432	275400.0091239799	275400.011527763	275400.0135171845	275400.015315793	275400.016964749

Table 2B

ADJUSTED A- POSTERIORI, TRACE OF COVARIANCE MATRICE OF ADJUSTED PARAMETER & OBSERVABLES ITERATED SIX TIMES							
	ITERATION 0	**ITERATION 1**	**ITERATION 2**	**ITERATION 3**	**ITERATION 4**	**ITERATION 5**	**ITERATION 6**
VTPV / R (δ^2)	5.23E-05	2.38E-06	7.36E-07	4.47E-07	4.2E-07	4.21E-07	3.67E-07
TRACEX^a ($\sum X^a$)	664.8679	30.28775	9.35037	5.683714	5.334887	5.345153	4.666856
TRACEL^a ($\sum L^a$)	-0.02945	-0.00134	-0.00041	-0.00025	-0.00024	-0.00024	-0.00021

Table 3A

PHASE METHOD OF ADJUSTMENT OF TRIANGULATION NET USING OBSERVATION EQUATION METHOD							
ADJUSTED COORDINATES OF STATIONS ITERATED SIX TIMES							
	ITERATION 0	**ITERATION 1**	**ITERATION 2**	**ITERATION 3**	**ITERATION 4**	**ITERATION 5**	**ITERATION 6**
XC	107154.9915588175	107154.9976125524	107155.0034995119	107155.0093121793	107155.0150088344	107155.0206122725	107155.0261193432
YC	275500.0015563034	275500.0022816808	275500.0030436364	275500.0037824504	275500.004513278	275500.0052354501	275500.005935797
XD	107149.992363802	107150.0022760125	107150.0120601048	107150.0217234593	107150.0311930256	107150.0405015732	107150.0496474489
YD	276140.0119711952	276140.0099444306	276140.0080178549	276140.0061267228	276140.0042993423	276140.0024927455	276140.0007143446
XE	108000.0041571035	108000.0073360629	108000.0105567097	108000.013730633	108000.0168590473	108000.0199133884	108000.0229374164
YE	275810.0063011349	275809.9994970953	275809.9927847934	275809.9861995961	275809.9797036899	275809.9733556709	275809.9671383599
XF	108200.0056011889	108200.0106823856	108200.0158563723	108200.0209135973	108200.0259357482	108200.0308154961	108200.0356459184
YF	276200.0110150758	276200.000696971	276199.9905244136	276199.9805489372	276199.9707312107	276199.9610877649	276199.9516688795
XG	108550.005416283	108550.0025962847	108549.9999130221	108549.9972722091	108549.9947052572	108549.992135558	108549.9897089583
YG	275400.0107778283	275400.0020138631	275399.9932578969	275399.9847463601	275399.9762200001	275399.9680271187	275399.9598948268

Table 3B

ADJUSTED A- POSTERIORI, TRACE OF COVARIANCE MATRICE OF ADJUSTED PARAMETER & OBSERVABLES ITERATED SIX TIMES							
	ITERATION 0	**ITERATION 1**	**ITERATION 2**	**ITERATION 3**	**ITERATION 4**	**ITERATION 5**	**ITERATION 6**
VTPV / R (δ^2)	0.000165	2.07E-05	2.02E-05	1.95E-05	1.9E-05	1.81E-05	1.75E-05
TRACEX^a ($\sum X^a$)	2090.42149314	262.3218	256.8348	247.3916	240.8862	230.2275	222.3556
TRACEL^a ($\sum L^a$)	-1.6E-05	-2.6E-07	-2.5E-07	-2.3E-07	-2.2E-07	-2E-07	-1.9E-07

Table 4A

PHASE & SEQUENTIAL METHOD OF ADJUSTMENT OF TRIANGULATION NET USING OBSERVATION EQUATION METHOD							
ADJUSTED COORDINATES OF STATIONS ITERATED SIX TIMES							
	ITERATION 0	**ITERATION 1**	**ITERATION 2**	**ITERATION 3**	**ITERATION 4**	**ITERATION 5**	**ITERATION 6**
XC	107154.992034765	107154.9984845525	107155.0051268124	107155.0119828432	107155.0190823761	107155.0264694767	107155.0340542612
YC	275500.0013503566	275500.0022593709	275500.0032309048	275500.004265355	275500.0053322009	275500.0064060074	275500.0075519088
XD	107149.9934724101	107150.003535953	107150.0141496229	107150.0250814192	107150.036429993	107150.0482034177	107150.0603351019
YD	276140.0137117007	276140.0129336359	276140.0121846005	276140.0114454703	276140.010703964	276140.009825649	276140.0090291745
XE	108000.0029494454	108000.0069637344	108000.0112429519	108000.0156355639	108000.0202453579	108000.024980364	108000.0298991419
YE	275810.0037736025	275809.9971566127	275809.9902067787	275809.9831062524	275809.9756913002	275809.967951283	275809.9600794411
XF	108199.9808720306	108200.0073197052	108200.0137609565	108200.0205896683	108200.0276429884	108200.0350000896	108200.0425567587
YF	276199.9807393911	276200.0102885221	276200.0004217429	276199.9900039131	276199.9793941422	276199.9682979992	276199.9567389941
XG	108549.9675329482	108550.0059469203	108550.0041289431	108550.002209262	108550.000311872	108549.9983550808	108549.9961899851
YG	275400.0047951938	275400.0093408019	275400.0000327896	275399.9902207362	275399.9801639909	275399.9696078444	

Table 4B

ADJUSTED A- POSTERIORI, TRACE OF COVARIANCE MATRICE OF ADJUSTED PARAMETER & OBSERVABLES ITERATED SIX TIMES							
	ITERATION 0	**ITERATION 1**	**ITERATION 2**	**ITERATION 3**	**ITERATION 4**	**ITERATION 5**	**ITERATION 6**
VTPV / R (δ^2)	3.27E-10	1.89E-10	2.2E-10	3.23E-10	5.07E-10	7.8E-10	1.15E-09
TRACEX^a ($\sum X^a$)	0.000479	0.000277	0.000322	0.000473	0.000742	0.001143	0.001683
TRACEL^a ($\sum L^a$)	1.06E-08	6.14E-09	7.14E-09	1.05E-08	1.65E-08	2.53E-08	3.73E-08

Table 5A

SUMMARY & COMPARISON OF THE FOUR METHODS OF TRIGULATION NET ADJUSTMENT USING THE RESULTS OF THE FORTH ITERATION						
SUITABILITY CRITERIA		**SIMULTANEOUS METHOD --- (I)**	**SEQUENTIAL METHOD --- (II)**	**PHASE METHOD ------- (III)**	**PHASE & SEQUENTIAL METHOD ---- (IV)**	**REMARK (ORDER OF SUITABILITY OF CRITERIA)**
MATHEMATICAL MODEL		$L^a = F(X^a)$	$L^a1 = F(X^a1)$ $L^a2 = F(X^a1 \,.\, X^a2)$	$L^a1 = F(X^a1)$ $L^a2 = F(X^a2)$	$L^a1 = F(X^a1)$ $L^a2 = F(X^a1 \,.\, X^a2)$	
STORAGE REQUIRED (KILOBYTES)		23.7	25.9	27.8	28.1	I, II, III, IV
C.P.U. TIME** (SECS)		15	18	20	40	I, II, III, IV
VTPV / R (δ^2)		1.57E-08	4.2E-07	1.9E-05	5.07E-10	IV, I, II, III
TRACEX^a ($\sum X^a$) m		0.163793	5.334887	240.8862	0.000742	IV, I, II, III
TRACEL^a (($\sum L^a$) m		4.54E-06	-0.00024	-2..2E-07	1.65E-08	IV, III, I, II
D.F. = R		N – M N=23 M=10	N1 + N1 +N2 - M1- M1-M2+M1 N1=10,N2=13 M1=4,M2-8	N1+N2-M1-M2 N1=10,N2=13 M1=4,M2-8	N1 + N1 +N2 - M1- M1-M2+M1 N1=10,N2=13 M1=4,M2-8	IV, II, III, I
ADJUSTED COORDINATES OF THE TRIANGULATION STATIONS	XC	107154.9875464861	107154.9944208732	107155.0150088344	107155.0190823761	IV, I, II, III
	YC	275500.0017758484	275500.00501941	275500.004513278	275500.0053322009	
	XD	107149.9846115818	107149.9940807876	107150.0311930256	107150.036429993	
	YD	276140.0154326661	276140.019009702	276140.0042993423	276140.010703964	
	XE	108000.0027571911	108000.0077910545	108000.0168590473	108000.0202453579	
	YE	275810.0122723453	275810.0122290178	275809.9797036899	275809.9756913002	
	XF	108200.0033309248	108200.0102873252	108200.0259357482	108200.0276429884	
	YF	276200.020190955	276200.0189698558	276199.9707312107	276199.9793941422	
	XG	108550.0091744801	108550.0130620662	108549.9947052572	108550.000311872	
	YG	275400.0172399104	275400.0135171845	275399.9762200001	275399.9801639909	