

STEADY STATE ACCELERATION MODEL OF COSMIC RAY IN THE ATMOSPHERE

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ABSTRACT

Hydrodynamic equations were simplified to a layman understanding together with the diffusion-convection transport equation for the cosmic-ray distribution function  $f(t, r, p)$ , which depends on time  $t$ , radial distance from the point of supernova explosion  $r$  and the particle momentum  $p$ . From the results, gas density  $\rho$  is inversely proportional to the gas velocity  $u$ ; flux tubes have a tendency to rise at an appreciable speed; pressure of the atmospheric gas  $P_g$  depends on the cosmic-ray pressure  $P_c$  and the internal energy of the electron  $U$  is directly proportional to the gas density  $\rho$ . Based on what Vladimir *et al.* (2010) said: "More work is needed to understand how robust our results are", this has helped in making their bulky equations clearer and showing how robust their work are. In the cause of breaking down or simplifying the high energy equations, it was found that cosmic rays move with Alfvén speed.

INTRODUCTION

Cosmic rays are high energy sub-atomic particles from outside of the solar system which contains mostly protons and alpha particles (Usoskin *et al.*, 2004). Cosmic rays are associated with electromagnetic radiations and can travel at nearly the speed of light with enormous energy in the range 0.1–15 GeV (Devendraa and Singh, 2010). Most cosmic rays are thought to originate outside the solar system, with many coming from within our Milky Way galaxy, and a few arriving from other galaxies. Cosmic ray causes ionization in the atmosphere. When cosmic rays enter the earth's atmosphere, they collide with ambient atmospheric gas molecules thereby ionizing them. In this process they may produce secondary particles which can be sufficiently energetic to contribute themselves to further ionization of the neutral gases. This leads to the development of an ionization cascade (or shower). The intensity and penetration depth of the cascade depends on the energy of the primary cosmic particles. Cascade of particles with several hundred MeV of kinetic energy may reach the ground. However, due to their charges, cosmic ray particles are additionally

deflected by the geomagnetic field. Almost all particles can penetrate into the polar region, where the magnetic field lines are perpendicular to the ground, whereas only the rare most highly energetic particles with energy above 15 GeV are able to penetrate the lower atmosphere near the equator.

It is interesting to note that solar energy flux reaching the earth's orbit  $F_s = 1.36 \times 10^3 \text{ Wm}^{-2}$  whereas the cosmic energy flux (particles with energy  $> 0.1 \text{ GeV}$ )  $F_{CR} = 10^{-5} \text{ Wm}^{-2}$  (Frohlich and Learn, 1997). Thus, energy input by cosmic rays in the Earth's atmosphere is about  $10^{-9}$  times that of solar energy and hence it is unlikely that cosmic rays could influence the atmosphere processes. However, cosmic rays are the only source of ion production in the lower atmosphere (Devendraa *et al.*, 2007), which is confirmed from the measurements of Ermakov and Komotkov (Ermakov *et al.*, 1992). Therefore, the processes, depending on the electrical properties of the atmosphere such as atmospheric electric current, lightning production, cloud and thunder cloud formation etc, can be affected by cosmic rays. For studying cosmic ray ground based observatories covering large areas are needed because the flux of cosmic rays is too low at these energies for direct measurement by balloons or satellite based experiment (Apel and Arteaga, 2011).

Cosmic rays are the atomic nuclei (mostly protons) and electrons that are observed to strike the Earth's atmosphere with exceedingly high energies. They are moving at nearly the speed of light. Since the discovery of the cosmic rays, scientists have measured a great number of cosmic rays with different energies with detectors all over the world (Smith *et al.*, 2005). The standard belief is that some cosmic rays are accelerated in the Galaxy and some are accelerated outside the Galaxy. Supernova remnants in the Milky Way galaxy are most promising accelerators of cosmic rays showering on the Earth (Dorman *et al.*, 2011).

In a paper presented by Yamazaki (2009), he reviewed current unresolved problems and recent progress of both observational and theoretical works on the cosmic-ray acceleration at supernova remnants. He focuses on the fact that recent X-ray and very-high-energy (TeV) gamma-ray observations tell us important information on this issue, especially on acceleration efficiency, evidence for proton acceleration, and so on. Here we show how robust the systems of equations governing the acceleration of cosmic rays in the atmosphere are. Also simplify elementarily, to a layman understanding the bulky relations accompanying cosmic ray acceleration. To show how the physical parameters vary or relate to one another. This work is limited to the acceleration of cosmic rays within the atmosphere, using hydrodynamic equation.

## METHODOLOGY

Hydrodynamic equations laid by Vladimir *et al.* (2010) was used. The equations was accompanied with the diffusion-convection transport equation for the cosmic-ray distribution function  $f(\mathbf{t}, \mathbf{r}, \mathbf{p})$ , which depends on time  $\mathbf{t}$ , radial distance from the point of supernova explosion  $r$  and the particle momentum  $\mathbf{p}$ . The full bulky system of governing equations described by Vladimir *et al.* (2010) is:

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 u \rho \quad (A)$$

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - \frac{1}{\rho} \left( \frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \quad (B)$$

$$\frac{\partial P_g}{\partial t} = -u \frac{\partial P_g}{\partial r} - \frac{\gamma_g P_g}{r^2} \frac{\partial r^2 u}{\partial r} - (\gamma_g - 1)(w - u) \frac{\partial P_c}{\partial r} \quad (C)$$

$$\begin{aligned} \frac{\partial f}{\partial t} = & \frac{1}{r^2} \frac{\partial}{\partial r} r^2 D(p, r, t) \frac{\partial f}{\partial r} - w \frac{\partial f}{\partial r} + \frac{\partial f}{\partial p} \frac{p}{3r^2} \frac{\partial r^2 w}{\partial r} + \frac{\varphi \delta(p - p_{inj})}{4\pi p_{inj}^2 m} \rho(R + 0, t) (\dot{R} - u(R, 0, t)) \\ & \times \delta(r - R(t)) \end{aligned} \quad (D)$$

The equations (A-D) above were simplified elementarily by: assuming a steady state condition in roughly all the equations (A-D), arriving at equation of a bulky fluid behavior of plasma where some interaction terms cancelled each other by Newton's 3<sup>rd</sup> law in equation (B), introducing Ampere's law:  $\left\{ \frac{1}{4\pi} (\nabla \times B) \times B = \nabla P - \rho g \right\}$  in the same equation (B), considering in equation (D) only the momentum distribution function of the particles ( $f(p, t)$ ). The end point of each of the equation was noted, to show how the physical parameters vary or relate with each other; thereby giving insight on the cosmic ray acceleration.

## RESULT

The following workings summarize how the four (4) equations, (equations A to B), accompanying cosmic ray acceleration were broken down to a layman understanding; thus showing how robust the results of the initiators, Vladimir *et al.* (2010) are.

### Governing Equation (A)

From the relation equation governing equation (A), we assume a steady state condition. So that equation (A) becomes:

$$0 = \frac{\partial}{\partial r} r^2 u \rho \quad (1)$$

$$\rho = \frac{\text{constant}}{r^2 u} \quad (2)$$

### Governing Equation (B)

In governing equation (B), we multiply  $\rho$  to both sides of the equation; equation (B) becomes:

$$\rho \frac{\partial u}{\partial t} = -\rho u \frac{\partial u}{\partial r} - \left( \frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \quad (3)$$

Equation (7) gives the energy equation; where  $u$  is the internal heat energy (i.e. energy per unit mass). This equation (7) is similar to the energy equation stated by Einar and Gordon, (1988). Therefore,

$$-u^2 \frac{\partial \rho}{\partial r} = -P \nabla \cdot v + \rho g + \left( \frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \quad (4)$$

$$0 = -P \nabla \cdot v + \rho g + \left( \frac{\partial P_g}{\partial r} + \frac{\partial P_c}{\partial r} \right) \quad (5)$$

This gives;

$$\frac{\partial(nmv)}{\partial t} - \nabla \cdot (Pv) + nF = 0 \quad (6)$$

With an electric field and magnetic field present in the atmosphere the force in equation (15) becomes:

$$F = e \left( E + \left( \frac{v}{c} \right) \times B_o \right) + mg \quad (7)$$

where  $\phi$ , in the simple case of an isotropic distribution of the random velocities of the particles, makes equation (7) becomes:

$$n_e m_e \frac{\partial v_e}{\partial t} + n_i m_i \frac{\partial v_i}{\partial t} = n_e e \frac{(v_i v_e) \times B_o}{c} - \nabla P + (n_i m_i + n_e m_e) g \quad (8)$$

Going by Newton's third law,

$$\rho \left( \frac{\partial v}{\partial t} \right) = \frac{1}{c} j \times B_o - \nabla P + \rho g \quad (9)$$

If we consider Ampere's law,

$$\frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B} = \nabla \left( \frac{B^2}{8\pi} + P \right) - \rho g, \quad (10)$$

Considering a flux tube of uniform field strength  $B_o \hat{x}$ , placed in a field-free atmosphere, plane stratified in the Z-direction. The left hand side of equation (10) vanishes in this situation.

If the tube is in pressure balance and thermal equilibrium with its surroundings then.

$$2n_I kT = P_I = P_o - \frac{B_o^2}{8\pi} = 2n_o kT - \frac{B_o^2}{8\pi} \quad (11)$$

So that,

$$n_I = n_o - \frac{B_o^2}{16\pi kT}$$

The tube is therefore lighter than its surroundings and experiences an upthrust force of:

$$F = (n_o - n_I) m_H g V = \frac{B_o^2 m_H g V}{16\pi kT} \quad (12)$$

$$U_f = \left( \frac{B_o^2}{4\pi\rho} \right)^{1/2} = V_A \quad (13)$$

that is, the Alfvén speed for the field strength  $B_o$ .

### Governing Equation (C)

From the relation governing equation (C), we have:

$$\frac{\partial P_g}{\partial t} = -u \frac{\partial P_g}{\partial r} - \frac{\gamma_g P_g}{r^2} \frac{\partial r^2 u}{\partial r} - (\gamma_g - 1)(w - u) \frac{\partial P_c}{\partial r} \quad (C)$$

$$\frac{\partial P_g}{\partial t} + \frac{\partial}{\partial r} (u P_g + (\gamma_g - 1)(w - u) P_c) = - \frac{1}{r^2} \frac{\partial r^2}{\partial r} u (P_g \gamma_g) \tag{14}$$

By the steady state assumption,

$$P_g \frac{\partial u}{\partial r} = -P_c \frac{\partial}{\partial r} (\gamma_g - 1)(w - u)$$

$$P_g = P_c, \tag{15}$$

which means that pressure of the gas depends on the pressure of the cosmic ray.

**Governing Equation (D)**

From the relation governing equation (D):

$$\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 D(p, r, t) \frac{\partial f}{\partial r} - w \frac{\partial f}{\partial r} + \frac{\partial f}{\partial p} \frac{p}{3r^2} \frac{\partial r^2 w}{\partial r} + \frac{\varphi \delta(p - p_{inj})}{4\pi p_{inj}^2 m} \rho(R + 0, t) (\dot{R} - u(R, 0, t)) \times \delta(r - R(t)) \tag{D}$$

According to Ramaty (1988), if we consider only the  $f(p, t)$ , momentum distribution function of the particles, we have:

$$\frac{\partial f}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D(p) \frac{\partial f}{\partial p} \right]. \tag{16}$$

equation (33) becomes:

$$\frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D(p) \frac{\partial f}{\partial p} \right] = 0.$$

This means that,

$$\frac{\partial \left( \frac{p^4 U^2}{3lv} \right)}{\partial p} = 0$$

Therefore,

$$U = \frac{2l}{p} \tag{17}$$

**DISCUSSION AND CONCLUSION**

In discussing the acceleration of cosmic ray particles in the atmosphere, hydrodynamic equations were simplified to a layman understanding together with the diffusion-convection transport equation for the cosmic-ray distribution function  $f(t, r, p)$ , which depends on time  $t$ , radial distance from the point of supernova explosion  $r$  and the particle momentum  $p$ .

The governing equations (A–D) above were simplified elementarily by: assuming a steady state condition. From the expression governing equation (A),  $\rho = \frac{\text{Constant}}{r^2 u}$  reveals that gas density  $\rho$  is inversely proportional to the gas velocity  $u$  in equation (3); though it can also be constant only if  $r^2 u$  is constant. According to Osterbrock (1961), the magnetic field at this region may play some guiding and controlling role. Hence, in equation (7) it can be seen that if the component of the equation is parallel to the magnetic field, then there will be a uniform acceleration along the magnetic field-lines.

Further decomposition of the expression governing equation (B) leads to an equation of a bulky particle behavior of the atmosphere, where some Ampere's law was introduced, where the atmosphere was assumed to be in thermal equilibrium (11). From the expression governing equation (C): it entails that pressure of the atmospheric gas  $P_g$  depends on the cosmic-ray pressure  $P_c$  i.e.  $P_g = -P_c$  (15).

Based on what Vladimir et al. (2010) said: "More work is needed to understand how robust our results are", this has helped in making their bulky equations clearer and showing how robust their work are by noting the end point of each of the four equations, how the physical parameters vary or relate with each other. This gives insight on the cosmic ray acceleration and behavior within the atmosphere.

In the result of this work, the acceleration of cosmic ray particles in the atmosphere has been resolved and simplified using high energy equation. These high energy equations are associated with some physical parameter. How these physical parameters vary or relate with each other were clearly shown. This gives insight on the cosmic ray acceleration and behaviours. Atmospheric density is inversely proportional to the particle velocity from the expression governing equation (A). From the expression governing equation (B), cosmic ray flux has a tendency to rise at an appreciable speed. Pressure of the atmospheric gas  $P_g$  depends on the cosmic-ray pressure  $P_c$  as seen from the relation governing equation (C). From the diffusion-convection transport equation for the cosmic-ray distribution function  $f(t, r, p)$ , (equation D), the internal energy of the electron  $U$  is directly proportional to the

gas density  $\rho$ . It also shows that cosmic rays particle undergoes Alfvén speed in the cause of acceleration within the atmosphere.

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