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RESIDUE CALCULUS AND ITS SOLUTIONS

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ABSTRACT

In this paper Solution of definite integrals of real variable functions and its solution and solve complicated complex integrals and its solution. The solution is used to solve some problem in science and technology. Based on the findings the integral of $-\infty \; to \; \infty$ is good on solving of problem science and technology than $0 \; to \; 2\pi$. In the solution of calculus residue theory on electrical circuit if t > 0 it complete the contour by a large semicircle in the upper half plane therefore we conclude that there is a current flowing in the circuit.

INTRODUCTION

The residue of a function f (z) at the pole z = a is defined to be the coefficient of $(z-a)^{-1}$ in the Laurent's expansion of the function $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{m} b_n (z-a)^{-n}$ Where

$$b_n = \frac{1}{2\pi} \int_{\gamma} (z-a)^{n-1} f(z) dz \text{ in particularly } b_1 = \frac{1}{2\pi} \int_{\gamma} f(z) dz$$

The coefficient b_1 is called the residue of f(z) at pole z = a. The circle γ is arbitrary and may therefore be replaced by any closed contour C (small or large). (Spiegel)

One of the most remarkable application of integration in the complex plane in general, and Cauchy's theorem in particular, is that it gives a method for calculating real integral that up until now, would have been difficult or even impossible. The residue theorem allows us to evaluate integral without actually physically integrating it allow us to evaluate an integral just by knowing the residue theorem to evaluate certain real integral which are not possible using real integrations techniques from single variable calculus and how to find the values of certain infinite sums. Residues are the building blocks of a general method for computing contour integrals of analytic functions. (Awasthi).

AIM AND OBJECTIVES

The aim and objectives of these papers is to solve calculus of residue and its solution to:

(i) Solve definite integrals of real variable functions and its solution

(ii) Solve complicated complex integrals and its solution.

METHODOLGY

Integral of trigonometric functions from 0 to 2π :

$$\int_0^{2\pi} (\text{trigonomet ric function })d\theta$$

By "trigonometric function" we mean a function of $\cos\theta$ and $\sin\theta$. The obvious way to turn this into a contour integral is to choose the unit circle as the contour, in other words to write $z = \ell^{\theta}$, and integrate with respect to θ. On the unit circle, both $\cos\theta$ and $\sin\theta$ can be written as simple algebraic functions of Z.

$$\cos\theta = \tfrac{1}{2}\left(z + \tfrac{1}{z}\right) \quad \sin\theta = \tfrac{1}{2i}\left(z - \tfrac{1}{z}\right)$$

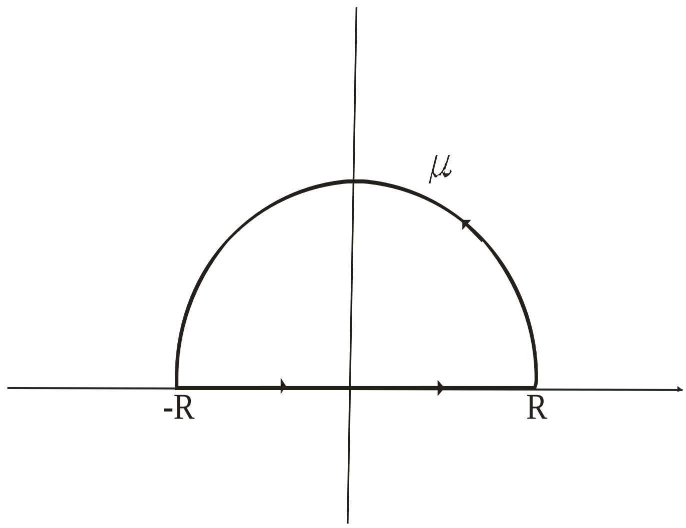
And making this replacement turns the trigonometric function into algebraic function of Z whose poles can be easily found

Integrals of the type $I = \int_{-\infty}^{+\infty} f(x)dx$

Or, equivalently, in the case where f(x) is an even function of x

$$I = \int_0^{+\infty} f(x)dx$$

can be found quite easily by inventing a closed contour in the complex plane which includes the required integral. The simplest choice is to close the contour by a very large semi-circle in the upper half-plane. Suppose we use the symbol "R" for the radius. The entire contour integral comprises the integral along the real axis from –R to +R together with the integral along the semi-circle arc. In the limit as $R \to \infty$ the contribution from the straight line part approaches the required integral I, while the curved section may in some cases vanish in the limit. (Fisher, 2002)



RESULT AND DISCUSSION

Example Used method of contour integration to prove that

$$\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2}, \qquad 0 < a < 1$$

Solution

$$z = e^{i\theta}$$

$$dz = ie^{i\theta} d\theta$$

$$dz = id\theta \qquad \cos\theta = \frac{1}{2}\left(z+\tfrac{1}{z}\right)$$

$$d\theta = \frac{dz}{iz}$$

$$\oint_c \frac{\frac{dz}{iz}}{1+a^2-\frac{2a}{z}\left(z+\frac{1}{z}\right)} = -\tfrac{1}{iz}\oint_c \frac{zdz}{z+za^2-az^2-a}$$

$$-\tfrac{1}{i}\oint_c \frac{dz}{az^2-z\left(1+a^2\right)+a} = -\tfrac{1}{i}\int_c f(z)dz$$

Where

$$f(z) = az^2 - z(1+a^2) + a$$

$$z = \frac{(1+a^2) \pm \sqrt{(1+a^2) - 4a^2}}{2a} = \frac{(1+a^2) \pm \sqrt{(1-a^2)^2}}{2a}$$

$$z = \frac{(1+a^2) \pm (1-a^2)}{2a}$$

Therefore, $z = \frac{1}{a}, z = a$

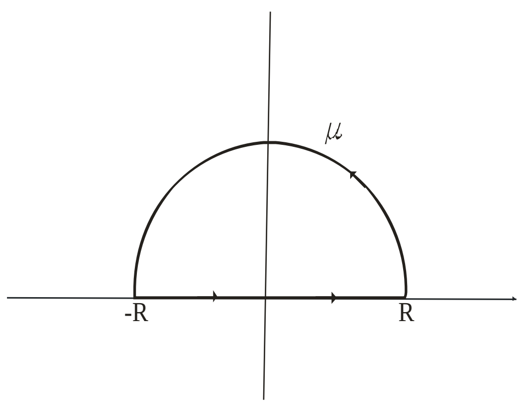
z = a lies inside C residue at the simple pole z = a is given by

$$\lim_{z \to a} \frac{\varphi(z)}{\varphi'(z)} = \lim_{z \to a} \frac{1}{2az - (1+a^2)} = \frac{1}{2a^2 - (1+a^2)} = \frac{1}{2a^2 - 1 - a^2} = \frac{1}{a^2 - 1}$$

$$\frac{-1}{i} \times 2\pi i \times \frac{1}{a^2 - 1} = \frac{2\pi}{1-a^2}$$

Example Show by contour integration that $\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$

Consider the contour C comprising of a semi-circle Centre at origin and radius R from x = -R to x = R,



Where

$$f(z) = \frac{1}{1+z^2}$$

Poles of f(z) are given by $1+z^2=0$ $\Rightarrow z=\pm i$ are simples poles, only z = I lies within the contour, the residue at z = i

$$\lim_{z\to i}\frac{(z-i)}{(z-i)(z+i)}=\lim_{z\to i}\frac{1}{(z+i)}=\frac{1}{2i}$$

Hence by Cauchy residue theorem we have,

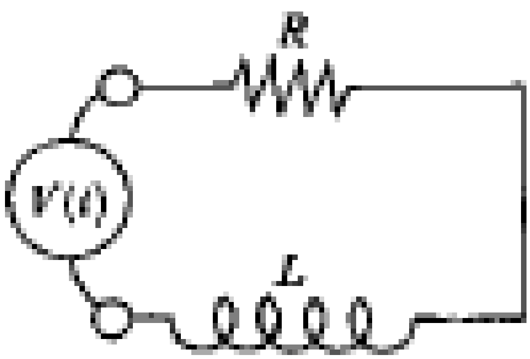
$$\int_c f(z)dz=2\pi i\times sum\, of\ residues\, within\, C=2\pi i\times\frac{1}{2i}=\pi$$

$$\int_{-\infty}^{\infty}\frac{dx}{1+x^2}=\pi$$

$$\int_{0}^{\infty}\frac{dx}{1+x^2}=\frac{\pi}{2}$$

Example Consider a resistance R and inductance L connected in series with a voltage V (t). Suppose V (t) is a voltage impulse, that is, a very high pulse lasting for very short time.

$$V(t)=\frac{A}{2\pi}\int_{-\infty}^{\infty}e^{i\varpi t}d\varpi$$

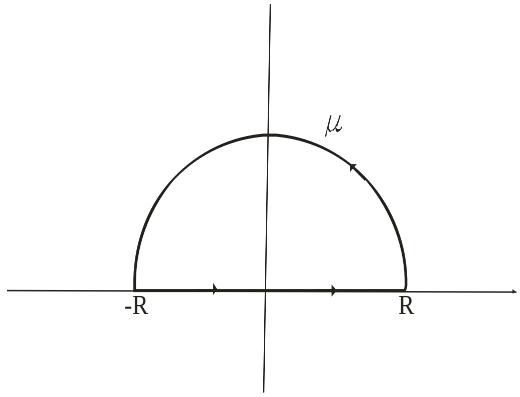


Where A is the area under the curve V (t), the current due to a voltage $e^{i\varpi t}$ is $e^{i\varpi t}/(R+i\varpi L)$. Thus the current due to our voltage pulse is

$$I(t)=\frac{A}{2\pi}\int_{-\infty}^{\infty}\frac{e^{i\varpi t}d\varpi}{R+i\varpi L}$$

$$\frac{e^{i\varpi t}}{R+i\varpi L}=\frac{e^{i\varpi t}}{iL\left(\varpi-\frac{iR}{L}\right)}$$

This has a simple pole at $\varpi=\frac{iR}{L}$



Thus the residue at $\varpi = \frac{iR}{L}$ is given by

$$\lim_{\varpi \to \frac{iR}{L}} \frac{\varphi(\varpi)}{\varphi'(\varpi)} = \lim_{\varpi \to \frac{iR}{L}} \frac{e^{i\varpi t}}{iL} = \frac{e^{-\frac{R}{L}t}}{iL}$$

$$\int_c f(\varpi)d\varpi = 2\pi i \times sum\, of\;\; residue = 2\pi i \times \frac{e^{-\frac{R}{L}t}}{iL} = \frac{2\pi e^{-\frac{R}{L}t}}{L}$$

$$I(A) = \frac{A}{2\pi} \times \frac{2\pi e^{-\frac{R}{L}t}}{L} = \frac{A}{L} e^{\frac{-R}{L}t}$$

SUMMARY/ CONCLUSION

In this work, we have able to solve the definite integrals of real variable functions and to use the solution to some problem in science and technology. Based on the findings the integral of $-\infty\, to\;\; \infty$ is good on solving of problem science and technology than $0\, to\, 2\pi$. In. the application of residue theory on electrical circuit if t > 0 it complete the contour by a large semicircle in the upper half plane therefore we conclude that there is a current flowing in the circuit.

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