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MODELLING RIVERS STATE MONTHLY ALLOCATION BY SEASONAL BOX-JENKINS METHODS

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ABSTRACT

Rivers State of Nigeria Monthly Allocation is hereby modelled using seasonal autoregressive integrated moving average (SARIMA) techniques. The period covered is from 2007 to 2012. This realization shall be called RSMA. Its time plot shows a fairly horizontal trend. An outlier is evident in June 2008. Seasonality is not obvious from this time plot. An inspection of the data reveals that yearly minimums tend to occur early in the year and the maximums in the middle of the year. This means that the data are fairly seasonal of period 12 months. A 12-monthly differencing of RSMA yields the series SDRSMA which has a generally horizontal trend too. Augmented Dickey Fuller Test for RSMA is significant whereas that for SDRSMA is highly significant. That means that even though both series could be said to be stationary, SDRSMA is the more stationary. The autocorrelation structure of SDRSMA makes the SARIMA models of orders $(0, 0, 1)x(0, 1, 1)_{12}$ and $(0, 0, 1)x(1, 1, 1)_{12}$ suggestive for RSMA. The estimate of the former is non-invertible whereas that of the latter is not only invertible but possesses uncorrelated residuals that follow the normal distribution; hence, its adequacy.

Keywords: Rivers State Monthly Allocation, SARIMA models, Time Series Analysis.

INTRODUCTION

The Rivers State of Nigeria like every other state receives monthly allocation from the Federation Account. The focus of this write-up is obtaining an adequate model which could be used in forecasting future values of the allocation. In particular, a seasonal autoregressive integrated moving average (SARIMA) approach shall be adopted. Many economic and financial time series, apart from being volatile, are seasonal. Box and Jenkins (1976) proposed SARIMA models for such series. Observation has been made herein that the particular realization analysed shows some seasonal tendencies of annual periodicity. There has been a growing interest in the adoption of SARIMA techniques in modelling such time series. A few of the researchers who have published works on SARIMA modelling include Appiah and Adetunde (2011), Etuk and Igbudu (2013), Osarumwense(2013), Bako et al.(2013), Singh (2013), Ali(2013), Abdul-Aziz et al.(2013), Etuk et al. (2012), Lee et al.(2012).

MATERIALS AND METHODS

The data for this work are seventy two values of Rivers State Monthly Allocation (RSMA) from 2007 to 2012 obtainable from the Rivers State Ministry of Finance, Port Harcourt. The actual values used in the analysis are corrected to the nearest tenth million of naira.

Sarima Models

A time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q* designated ARMA (p,q) if it satisfies the following difference equation

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \qquad (1)$$

Where α's and the β's are constants such that model (1) is both stationary and invertible. Model (1) may be written more specifically as

$$A(L)X_t = B(L)\varepsilon_t \qquad (2)$$

Where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$ and L is the backshift operator defined by $L^k X_t = X_{t-k}$. Stationarity and invertibility conditions are such that the zeros of $A(L) = 0$ and $B(L) = 0$ lie outside the unit circle respectively.

Many real-life time series are not stationary. For such a time series, Box and Jenkins(1976) proposed that differencing of sufficient order d could render it stationary. That means, the series $\{\nabla^d X_t\}$ is stationary. Here the symbol ∇ is the difference operator defined by $\nabla = 1 - L$. If the d^{th} difference $\{\nabla^d X_t\}$ follows an ARMA(p,q) model then $\{X_t\}$ is said to follow an *autoregressive integrated moving average model of orders p, d and q* designated ARIMA(p, d, q).

If a time series $\{X_t\}$ is seasonal in nature, Box and Jenkins (1976) also proposed that it could be modelled more specifically by

$$A(L)\Phi(L^s)\nabla^d \nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \qquad (3)$$

Where $\Phi(L)$ and $\Theta(L)$ are the seasonal autoregressive and moving average operators respectively and s is the period of seasonality. Here ∇_s^D is the seasonal difference operator and is defined by $\nabla_s^D = 1 - L^s$. Suppose they are respectively polynomials of orders P and Q, then model (3) is said to be a *multiplicative seasonal autoregressive integrated moving average model of orders p, d, q, P, D, Q and s designated SARIMA(p, d, q)x(P, D, Q)$_s$*.

Sarima Model Fitting

The fitting of a model of the form (3) begins with the determination of the orders p, d, q, P, D, Q and s. Knowledge of the theoretical properties of the ARMA model family is necessary for this purpose. The seasonality period s might be directly suggestive from the seasonal nature of the time series. The autocorrelation function (ACF) may be useful in the determination of s: the ACF of an s-periodic seasonal series shows a sinusoidal pattern of the same periods.

The differencing orders d and D are chosen such that $d + D < 3$. Often for stationarity that is enough. At each stage stationarity might be tested using a technique like the Augmented Dickey-Fuller (ADF) Test. The autoregressive orders p and P are usually estimated by the non-seasonal and seasonal cut-off lags of the partial autocorrelation function (PACF) respectively. Similarly the moving average orders q and Q are determined by the non-seasonal and the seasonal cut-off lags of the ACF. After order determination the model parameters might be estimated by non-linear optimization techniques like the least error sum of squares technique. In this write-up the statistical and econometric software Eviews 7 which is based on the least squares procedure shall be used. To choose between models the Akaike Information Criterion (AIC) may be used. Etuk (2009) empirically demonstrated that AIC is one of the order determination criteria for full-order auroregressive modelling.

RESULTS AND DISCUSSIONS

The time plot of RSMA in Figure 1 reveals a generally horizontal trend. Seasonality is not so obvious. There is an outlier at June 2008. It is observed not on the time-plot but by direct inspection that the yearly minimums occur in the first four months of the year. For 2007 it is in January; 2008 February; 2009 April; 2010 April; 2011 February; 2012 January. Similarly the maximums occur between March and July; 2007 May; 2008 June; 2009 March; 2010 May; 2011 July; 2012 March. These tendencies are evidence of seasonality. It was therefore necessary to difference the time series seasonally. This yielded the series SDRSMA. The time plot of SDRSMA in Figure 2 shows a generally horizontal trend. The ADF Test statistic for RSMA and SDRSMA are equal to -3.19 and -4.34 respectively. With the 1%, 5% and 10% critical values equal to -3.53, -2.90 and -2.59 respectively, the non-stationarity hypothesis test is highly significant with SDRSMA and just significant and not highly so, for RSMA. This means that at I% level of significance RSMA might not be considered stationary whereas SDRSMA would be. The correlogram of SDRSMA in Figure 3 indicates seasonality of period 12 months and the presence of a seasonal moving average component of order one. Moreover, the autocorrelations at lags 11 and 13 are comparable, suggesting a SARIMA(0, 0, 1)x$(0, 1, 1)_{12}$ model for RSMA. The significant spike at lag 12 in the PACF suggests the involvement of a seasonal autoregressive component of order one. In conjunction with the ACF structure this indicates a SARIMA (0, 0, 1)x(1, 1, 1) model for RSMA. This model as estimated in Table 1 is given by

$$SDRSMA_t = \varepsilon_t + .2127\varepsilon_{t-1} - .9742\varepsilon_{t-12} - .2387\varepsilon_{t-13}$$

Which is non-invertible and therefore unacceptable. The other model, the SARIMA(0, 0,1)x$(1, 1, 1)_{12}$, as estimated in Table 2, is given by

$$SDRSMA_t + .3110SDRSMA_{t-12} = \varepsilon_t + .3998\varepsilon_{t-1} - .3343\varepsilon_{t-12} - .8888\varepsilon_{t-13} \quad (4)$$

Model (4) has been found to be adequate on the following grounds:

1) The residuals are mostly uncorrelated. See Figure 4.
2) The residuals are normally distributed. See the Jarque-Bera test of Figure 5.

CONCLUSION

It may then be concluded that Rivers State Monthly allocation follows a SARIMA(0, 0, 1)x$(1, 1, 1)_{12}$ model. This might be used as basis for its forecasting.

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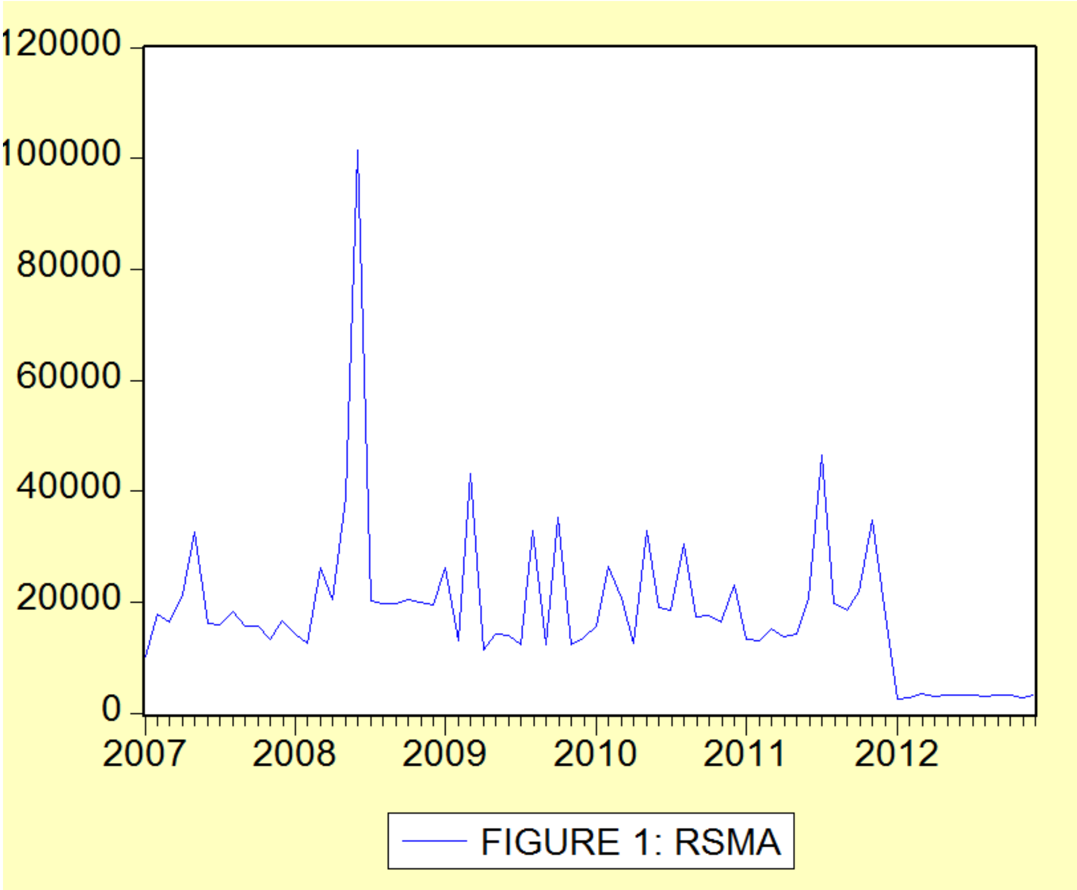
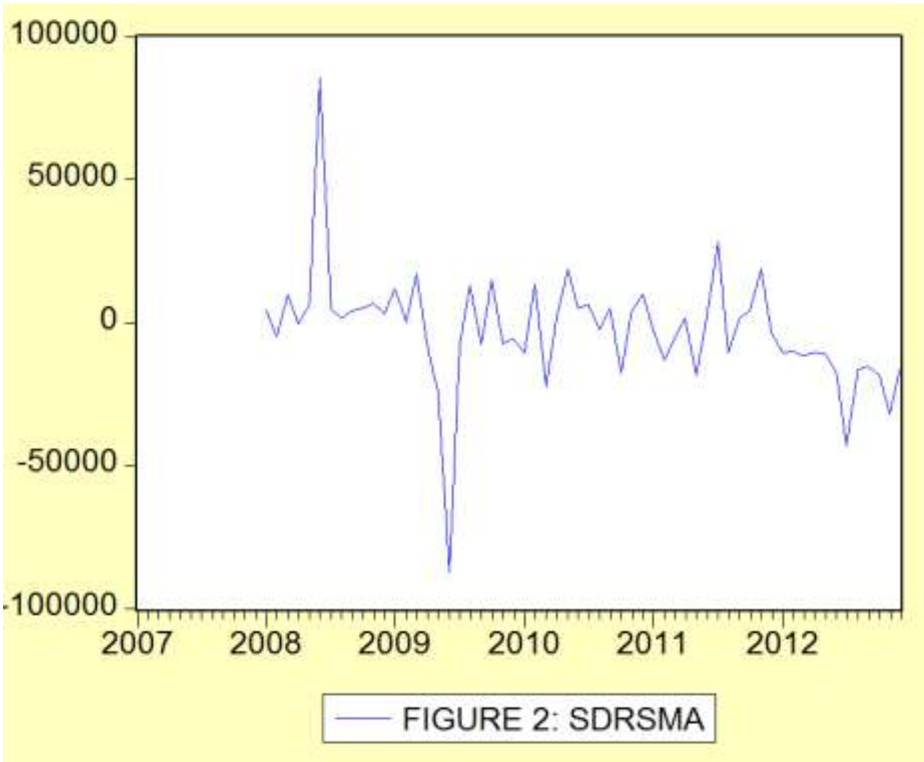
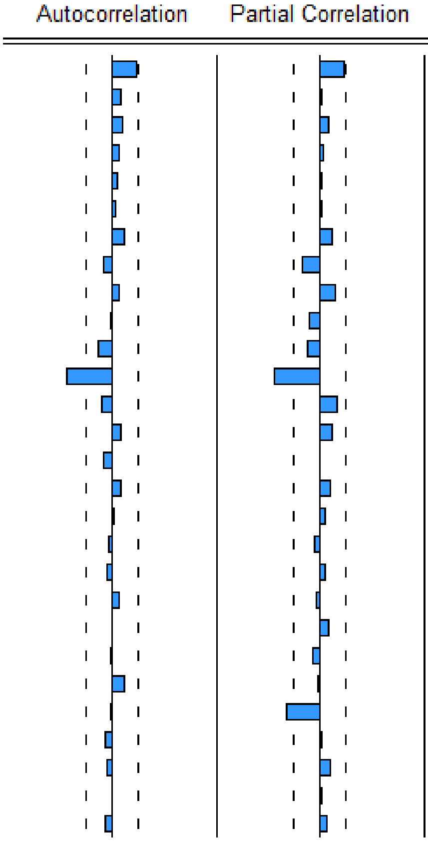

120000
100000
80000
60000
40000
20000
0
2007
2008
2009
2010
2011
2012
FIGURE 1: RSMA


100000
50000
0
-50000
-100000
2007
2008
2009
2010
2011
2012
FIGURE 2: SDRSMA



	AC	PAC	Q-Stat	Prob
1	0.249	0.249	3.9121	0.048
2	0.082	0.021	4.3431	0.114
3	0.104	0.084	5.0545	0.168
4	0.076	0.032	5.4404	0.245
5	0.042	0.010	5.5596	0.351
6	0.039	0.018	5.6668	0.462
7	0.130	0.115	6.8576	0.444
8	-0.097	-0.176	7.5373	0.480
9	0.081	0.150	8.0169	0.532
10	-0.024	-0.109	8.0582	0.623
11	-0.144	-0.121	9.6282	0.564
12	-0.448	-0.453	25.183	0.014
13	-0.110	0.166	26.142	0.016
14	0.085	0.119	26.725	0.021
15	-0.084	0.004	27.315	0.026
16	0.092	0.112	28.027	0.031
17	0.011	0.048	28.037	0.045
18	-0.033	-0.056	28.134	0.060
19	-0.056	0.042	28.416	0.076
20	0.071	-0.040	28.884	0.090
21	0.005	0.085	28.886	0.117
22	-0.023	-0.068	28.936	0.147
23	0.124	-0.021	30.479	0.136
24	-0.020	-0.324	30.520	0.168
25	-0.061	0.008	30.915	0.192
26	-0.044	0.108	31.128	0.224
27	0.001	0.017	31.128	0.266
28	-0.059	0.069	31.537	0.294

FIGURE 3: CORRELOGRAM OF SDRSMA

TABLE 1: ESTIMATIOM OF SARIMA (0, 0, 1)X(0, 1, 1) MODEL

Dependent Variable: SDRSMA
Method: Least Squares
Date: 08/23/14 Time: 14:02
Sample (adjusted): 2008M01 2012M12
Included observations: 60 after adjustments
Failure to improve SSR after 11 iterations
MA Backcast: OFF (Roots of MA process too large)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	0.212737	0.130642	1.628391	0.1090
MA(12)	-0.974197	0.078849	-12.35519	0.0000
MA(13)	-0.238727	0.146070	-1.634331	0.1077

R-squared	0.432772	Mean dependent var	-2871.328
Adjusted R-squared	0.412869	S.D. dependent var	20529.28
S.E. of regression	15730.45	Akaike info criterion	22.21329
Sum squared resid	1.41E+10	Schwarz criterion	22.31801
Log likelihood	-663.3987	Hannan-Quinn criter.	22.25425
Durbin-Watson stat	1.951955		

Inverted MA Roots	1.00	.87-.50i	.87+.50i	.50+.86i
	.50-.86i	.00+1.00i	.00-1.00i	-.25
	-.50+.86i	-.50-.86i	-.86+.50i	-.86-.50i
	-.99			
	Estimated MA process is noninvertible			

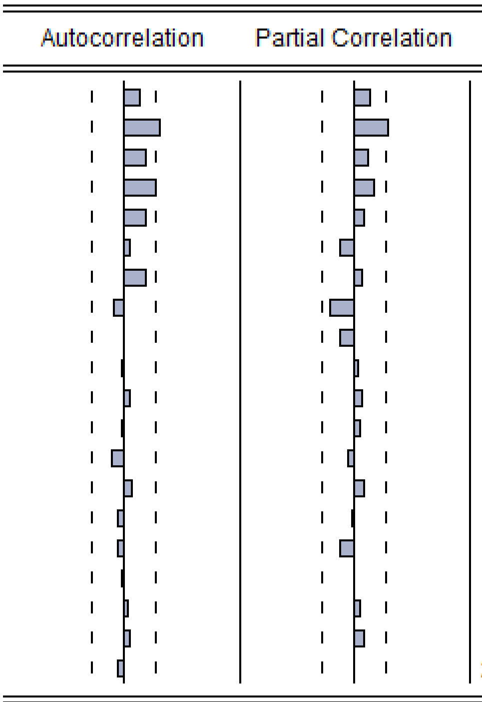
TABLE 2: ESTIMATION OF THE SARIMA (0, 0, 1) X $(1, 1, 1)_{12}$ MODEL

Dependent Variable: SDRSMA
Method: Least Squares
Date: 08/23/14 Time: 14:08
Sample (adjusted): 2009M01 2012M12
Included observations: 48 after adjustments
Convergence achieved after 11 iterations
MA Backcast: 2007M12 2008M12

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(12)	-0.311002	0.102296	-3.040212	0.0040
MA(1)	0.399762	0.081072	4.930971	0.0000
MA(12)	-0.334261	0.077604	-4.307245	0.0001
MA(13)	-0.888798	0.023266	-38.20155	0.0000

R-squared	0.717870	Mean dependent var	-6166.850
Adjusted R-squared	0.698634	S.D. dependent var	18430.30
S.E. of regression	10117.65	Akaike info criterion	21.36161
Sum squared resid	4.50E+09	Schwarz criterion	21.51754
Log likelihood	-508.6785	Hannan-Quinn criter.	21.42053
Durbin-Watson stat	1.664022		

Inverted AR Roots	.88+.23i	.88-.23i	.64+.64i	.64+.64i
	.23-.88i	.23+.88i	-.23-.88i	-.23+.88i
	-.64-.64i	-.64-.64i	-.88+.23i	-.88-.23i
Inverted MA Roots	.99	.87+.48i	.87-.48i	.53+.84i
	.53-.84i	.06+.99i	.06-.99i	-.41+.90i
	-.41-.90i	-.78+.62i	-.78-.62i	-.97+.22i
	-.97-.22i			



	AC	PAC	Q-Stat	Prob
1	0.150	0.150	1.1427	
2	0.325	0.310	6.6623	
3	0.194	0.130	8.6788	
4	0.290	0.187	13.276	
5	0.198	0.087	15.454	0.000
6	0.062	-0.121	15.674	0.000
7	0.193	0.070	17.848	0.000
8	-0.085	-0.209	18.281	0.001
9	0.003	-0.124	18.281	0.003
10	-0.014	0.034	18.295	0.006
11	0.050	0.078	18.454	0.010
12	-0.014	0.057	18.466	0.018
13	-0.107	-0.060	19.253	0.023
14	0.074	0.085	19.640	0.033
15	-0.054	-0.009	19.855	0.047
16	-0.063	-0.137	20.152	0.064
17	-0.019	0.005	20.178	0.091
18	0.028	0.047	20.239	0.123
19	0.050	0.095	20.443	0.156
20	-0.062	0.003	20.770	0.188

FIGURE 4: CORRELOGRAM OF SARIMA $(0,0,1)X(1,1,1)_{12}$ MODEL RESDIDUALS

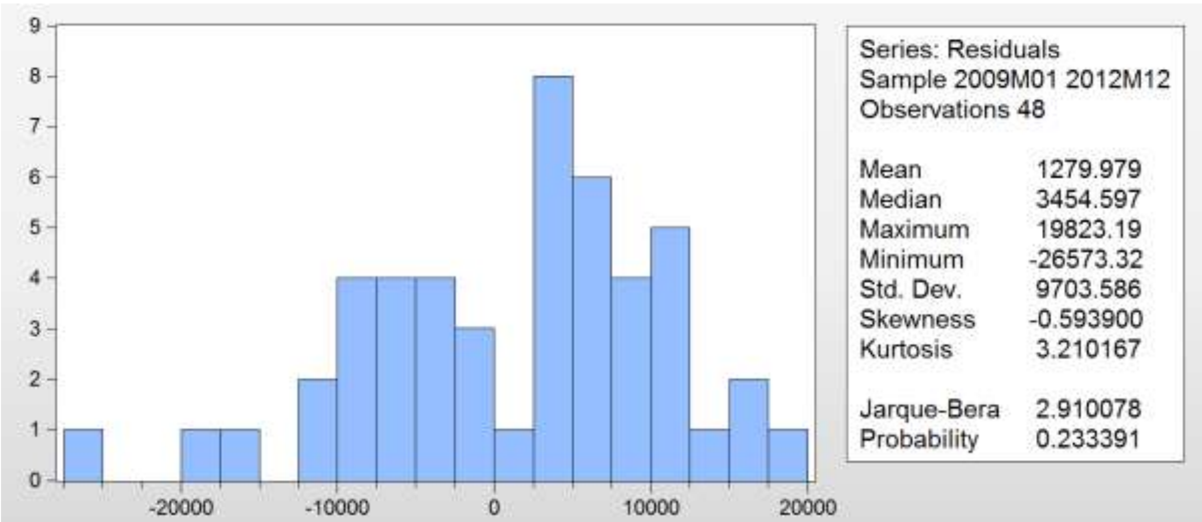


FIGURE 5: HISTOGRAM OF SARIMA $(0, 0, 1)X(1, 1, 1)_{12}$ RESIDUALS

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