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APPLYING THE *NAMBU-PSUDOSPIN* AND GREEN' S FUNCTION FORMALISM USING THE WEAK COUPLING RANGE TO THE THEORETICAL INVESTIGATION OF T_C IN DIFFERENT SUPERCONDUCTORS

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ABSTRACT

Mathematical calculation of energy gap equation, transition temperature expression and application to the different superconductors such as Al,Cd,MgB_2, $RbCS_2C_{60}$, K_3C_{60}, and Sr_2RuO_4 are obtained from Nambu psudospin generalization of the Bardeen, Cooper and Schrieffer model (BCS) and results compared with experiments.

Keywords: Applying, *Nambu-Psudospin*, Green' s, Investigation, T_C, Superconductors

INTRODUCTION

The cuprates are the most extensively studied class of compounds in Solid State Physics until date. But the discovery of the new iron based superconductors was a breakthrough of hope to the condense matter physicist [1]. Besides, there are some other superconductors in the HTSC group, such as MgB_2, $RbCS_2C_{60}$, K_3C_{60}, Sr_2RuO_4, and even elements such as (Hg, Al, Cd etc.) that are superconductors. But none of these compounds represented above is a representative of a wide group of compounds like, cuprates and iron pnictides and does not reach as high as these in transition temperature. All these compounds were studied in details with the concept of BCS, its extensions and more [2-4]; while superconductivity in conventional metals is understood by the BCS theory, the mechanism that is responsible for the pairing of carriers in the cuprates and pnictides are far from being clear. In fact research is still going on the pnictides and cuprates superconductors but we shall limit this work to none widely studied compounds to investigate the effect of the weak coupling limit on their T_c.

MEAN FIELD HAMILTONIAN

The Bardeen Cooper and Schrieffer (BCS) model Hamiltonian as generalized by Nambu psudospin theory is as given below[4-6].

$$H = \sum_k \varepsilon_{1k} - u\left(d^+_{1k\uparrow}d_{1k\uparrow} + d^+_{1k\downarrow}d_{1k\downarrow}\right) + \sum_k \varepsilon_{2k} - u\left(d^+_{2k\uparrow}d_{2k\uparrow} + d^+_{2k\downarrow}d_{2k\downarrow}\right) - \sum_{kk'} V_{1kk'}\, d^+_{1k\uparrow}d^+_{1k\downarrow}d_{1k`\downarrow}d_{1k`\uparrow}$$
$$- \sum_{kk'} V_{2kk'}\, d^+_{2k\uparrow}d^+_{2k\downarrow}d_{2k`\downarrow}d_{2k`\uparrow} - \sum_{kk'} V_{12kk'}\left(d^+_{1k\uparrow}d^+_{1k\downarrow}d_{2k`\downarrow}d_{2k`\uparrow} + d^+_{2k\uparrow}d^+_{2k\downarrow}d_{1k`\downarrow}d_{1k`\uparrow}\right)$$
$$- \sum_{kk'} V_{21kk'}\left(d_{2k`\downarrow}d_{2k`\uparrow}d^+_{1k\uparrow}d^+_{1k\downarrow} + d_{1k`\downarrow}d_{1k`\uparrow}d^+_{2k\uparrow}d^+_{2k\downarrow}\right)$$

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Where ε_{ik} (i =1, 2), is the quasi-kinetic energy of the band electrons and u is the chemical potential.$V_{1kk`}$,$V_{2kk`}$,($V_{12kk`}$, $V_{21kk`}$) are wave vector dependent intra-band (inter-band) electron interaction for the 1st (2nd) band electron-pair with crystal vector kk`, measured relatively to the screened Coulomb potential $d^+_{ik}\left(d_{ik}\right)$ represents the creation (annihilation) operators for the electron with spins orientation s $(=\uparrow,\downarrow)$.

$$H = \sum_k \varepsilon_{1k} - u\left(d^+_{1k\uparrow}d_{1k\downarrow} + d_{1k\downarrow}d^+_{1k\uparrow}\right) + \sum_k \varepsilon_{2k} - u\left(d^+_{2k\uparrow}d_{2k\downarrow} + d_{2k\downarrow}d^+_{2k\uparrow}\right) - \sum_K \Delta_{1k}\left(d_{1k\downarrow}d_{1k\uparrow} + d^+_{1k\uparrow}d^+_{1k\downarrow}\right) -$$

$$\sum_k \Delta_{2k}\left(d_{2k\downarrow}d_{2k\uparrow} + d^+_{2k\uparrow}d^+_{2k\downarrow}\right) \qquad 2$$

The singlet pair is preserved by applying Nambu-Gor'kov linearization [6-7] to Eqn(1.0) and we obtain a linearized form of Eqn(1.0) in Eqn(2.0) above.

The gaps Δ_{1k}, Δ_{2k} are defined as well as the pair correlations S_{1k}, S_{2k}.

$$\Delta_{1k} = \sum_{K'}\left(V_{1kk'}S_{1K} + V_{12}S_{2K} + V_{21}S_{1K}\right),\ \Delta_{2k} = \sum_{K'}\left(V_{2kk'}S_{2K} + V_{12}S_{1K} + V_{21}S_{2K}\right) \qquad 3$$

Since the bands are not completely separate, we can apply in Eqn(3.0), Nambu two component spinors, and algebra of Pauli matrix [3]defined respectively by ;

$$\psi_{ik}, \psi^+_{ik}, and\ \tau_i\ , i = 1, 2.$$
$$H = \sum_k \psi^+_{1k}\left(\varepsilon_{1k}\tau_3 - \Delta_{1k}\tau_1\right)\psi_{1k} + \sum_k \psi^+_{2k}\left(\varepsilon_{2k}\tau_3 - \Delta_{2k}\tau_1\right)\psi_{2k} \qquad 3.a$$

Applying the method of Green's function [2,8] solution to the pseudospin Hamiltonian.

$$G_s(k, iw_n) = \left[iw_n - H_1\right]^{-1} + \left[iw_n - H_2\right]^{-1} = \left[iw_n - (\varepsilon_{1k}\tau_3 - \Delta_{1k}\tau_1)\right]^{-1} + \left[iw_n - (\varepsilon_{1k}\tau_3 - \Delta_{2k}\tau_1)\right]^{-1} \qquad 3.b$$

Upon simplification we obtain the energy spectra, in addition to the pair correlation function;

$$S_{ik} = \langle d_{ikS}\, d_{ikS} \rangle = \langle d^{+}_{iks} d^{+}_{iks} \rangle \qquad i = 1,2 \ , \langle\langle d^{+}_{iks} d^{+}_{iks} \rangle\rangle = \left(\frac{2\Delta_{ik}}{(iw_n)^2 - (E_k^2)} \right) \tag{3.1}$$

Where this shows the sum total of contributions to the energy gap opened in the 1st band by the two bands in 1 and 2 as a result of pairing and electron pair condensation[9]. We approximate Eqn.(3.1) for the calculation of T_c .
As we solve the k summation, one obtains the limit $\Delta_{ik} \to 0, T \to T_c$

$$\Delta_{ik} = V\beta^{-1} N(\varepsilon) \int_{\hbar w_D}^{\hbar w_D} \frac{\Delta_{ik} d\varepsilon}{(iw_n)^2 - (E_k^2)} \ , where\ Z = iw_n \tag{3.3}$$

$$1 = V\beta^{-1} N(\varepsilon) \int_{\hbar w_D}^{\hbar w_D} \frac{d\varepsilon}{(iw_n)^2 - (E_k^2)} = 2V\beta^{-1} N(\varepsilon) \int_{0}^{\hbar w_D} \frac{d\varepsilon}{z^2 - E^2} \ . \tag{3.4}$$

Within the range of integration, this approximation allows for the analytical evaluation and simplification of the Tc [10-11]. Therefore, we obtain

$$\frac{\Delta(T)}{K_B T_c} = 2 \exp\left(\frac{-1}{VN(0)} \right). \tag{3.5}$$

$$T_c = \theta \exp\left(\frac{-1}{VN(0)} \right). \tag{3.6}$$

Theoretical predictions of transition temperatures

N(0)V	Al	Cd	Ta	MgB_2	$RbCS_2C_{60}$	K_3C_{60}	Sr_2RuO_4
0.10	0.01K	0.009k	0.01k	0.01	0.10	0.10	0.1
0.15	0.54K	0.2k	0.3k	0.52	2.83	2.8	1.4
0.20	2.88K	1.4k	1.6k	2.76	15.0	15	
0.25	7.80K	3.8k	4.3k	7.50	40.0	40	
0.30	15.2k	7.4k	8.5k	14.6	-		
0.35	24.5k	12.0k	13.7k	23.5			
0.40	34.1	17.1k	19.7k	33.6			

SUMMARY AND CONCLUSION

The Nambu- psudospin derivation of Eqn(3.5) and Eqn(3.6) is equivalent to the BCS formulation. It is seen from the equations and above table that the gap and Tc are affected by the nature of the coupling parameter. However, N (o)V affects the Tc values of elements and compounds that exhibits superconductivity. This may well be the case for the group of well known iron pnictides and cuprates compounds. Comparing results with experimental deduced T_c values, it is found in this theory that the Tc of these elements and compounds equal to and approach experiments; The T_c of 2.88 in Al at N(0)V = 0.20K approach experiment, 3.5K ; Tc of 33.6K in MgB_2 approach 39K .The expression of the energy gap and critical transition temperature is seen to be dependent on interaction strength , Debey temperature and density of state. Since NV is much less than unity therefore, the range of 1.0-0.4 applies for NV in this theoretical calculation.

REFERENCE

1. G.F. Chen, Z. Li, G. Li, J. Zhou, D. Wu, J. Dong, W.Z. Hu, P. Zheng, Z.J. Chen, J.L.Luo, N.L. Wang. arXiv: 0803.0128.

2. H. Takahashi, K.Igawa, K. Arii,Y. Kamihara, M. Hirano and H.Hosono, (2008), Nature 453, 376.

3. X. H. Chen, T. Wu, G. Wu, R. H. Liu, H. Chen, and D. F. Fang, (2008) Nature **453**, 761.

4. J. Bardeen, L. N. Cooper, J. R. Schrieffer, (1957), Phys.Rev.108,1175.

5. G. C. Asomba, (1996) Physica C 258 30-40.

6. Y. Nambu, Phys. (1960) Rev. 117,648.
7. L.P. Gor'kov, (1959), Zh. Exsp.teor.Fiz.36, 1918.

8. E.N Economuo, (2006), *Green's Functions in Quantum Physics*, Third Edition, Springerverlag Berlin,.

9. G. C. Asomba, (1995) Physica C 245, 355.

10. G. E. Akpojotor and A.O.E. Animalu, (2009) AJP, 2, 46.

11. A.O. E. Animalu, (1994) Hadronic J. 17, 349.

12. J. B. Ketterson and S.N. Song, 1999, *Superconductivity*,(Cambridge University Press United Kingdom).

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