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PHYSICS OF GROUNDWATER FLOW FOR CONFINED AND UNCONFINED AQUIFERS IN RELATION TO MEASUREMENT OF GROUNDWATER LEVEL VARIATION WITH TIME

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ABSTRACT

This research is on physics of groundwater flow which consist of equations of confined and unconfined aquifers these are mathematical equation called steady state or Laplace's equation and Boussinesq of equation respectively .As in many third world countries like Nigeria the need to study groundwater flow is very important , as water is very important for human survival and environment .But , it is also scarce in many developing countries as such to have enough knowledge of the ground water flow and what it takes to have sufficient water supply is very important for living things and the environment in general.

Keyword: Groundwater flow, types of aquifers, sufficient water supply and environment.

INTRODUCTION

Groundwater may be defined as water occurring in the entire void and within a geologic stratum. It can also be defined as water that is found under the soil or rock. To understand the ways in which water occurs; underground, groundwater processes energy in mechanical, thermal and chemical forms and these vary spastically to eliminate the energy differentials, groundwater is forced to move from one region to another, thus the flow of groundwater is controlled by the laws of physics and thermodynamics. The property of a rock of possessing a pores or voids is called porosity. Rocks containing a relatively large proportion of void space are described as " porous" thus soil are also porous , if we dig drill from ground surface down into the saturated zone , water will flow from the rock into the hole until the water reaches a constant level . This will usually be at about the level below which all the pore holes in the rock are filled with water in the wards, the upper unit of the saturated zone, this is called the water table. The distances need to drill or dig to reach the water table varies from one place to another. In general the water table is not flat; it rises and falls with the ground surfaces but in shallower beneath valleys.

All water that occurs naturally below the earth surface is called sub-surface water, whether it occurs in the saturated zone that is to say below the water table, is called "ground water" (Price Micheal 1985).

OCCURRENCE OF GROUND WATER

Ground water generally occurs in an aquifer. An aquifer may be defined as a formation that contains sufficient permeable material to yield significant quality of water to supply wells and springs. It can also be defined as an underground layer of earth or stone that contains water. Aquifers are generally extensive and may be overlain or underlain by a confining bed, which are defined earlier as relatively impermeable. These are aquifers, aqui-fuge and acquired.

FACTORS THAT DETERMINE THE AVAILABILITY OF GROUNDWATER

There are three major factors that determine the availability of groundwater (Price Micheal 1985). These factors are:

1. The extent to which rocks are porous if there are only few small voids, and then the amount of water contained in a given volume of rocks will be limited.
2. The combination of size of the pore and the degree to which the pore are interconnected because the combination will control the case with which water flows through the rocks.
3. The amount of replenishment the degree to which water flows the aquifer is recharge.

THE PHYSICS OF GROUNDWATER FLOW

Groundwater Movements

Groundwater in its natural states is invariably moving; this movement is governed by establishing principle that flows through aquifer, which can be expressed by Darcy's law.

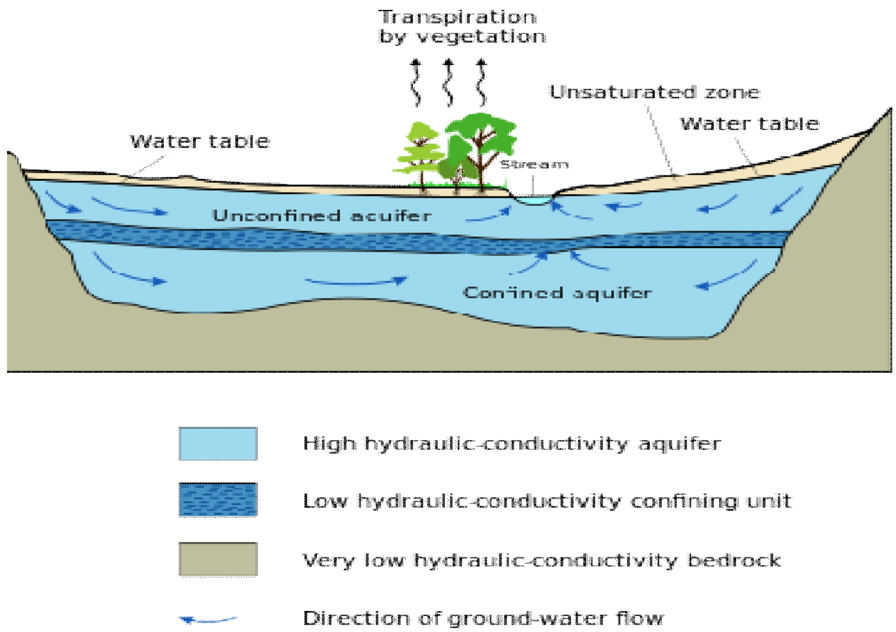
Darcy' Law

This state that flow rate through a porous media is proportional to the head loss and inversely proportional to length of flow. To derive Darcy's law, a cylinder of cross sectional area A, filled with sand which is held between two mesh streams. Water flows through the cylinder at a steady rate Q, measured for example in liters per day, manometers at X and Y a distance L apart along the cylinder measured the head of the water relative to the datum level, the difference of the two head loss h_i .By Darcy's law, Difference in hydraulic head (h) cause water to move from one place to another, water flows from locations of high h. Hydraulic head is composed of pressure head and elevation head. Darcy's law is a constitute equation (empirically derived by Henry Darcy, in 1856) that states the amount of groundwater discharging through a given portion of aquifer is proportional to the cross-sectional area of flow, the hydraulic head gradient, and the hydraulic conductivity.

Groundwater Flow Equation

By Darcy's law , if the flow rate Q is doubled , the $h_{\underline{L}}$ is also doubled as well , but if Q is half read , then h_i h_l also become halved , in general Darcy's found that the flow rate is directly proportional to the loss h_i that is

$$Q \, \alpha \, h_1 i \qquad (1)$$



Suppose the length of sand column is halved, and keeping the same head loss between the ends constant, Darcy's found that as the length L increase, Q decrease and vice versa thus:

$$Q \ \alpha \ i / L \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

Combining equation (1) and (2) above, we get

$$Q \, \alpha \, hi / L \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

In another identical cylinder is put beside the first, filled with identical sand and with flow occurring in the same way, we have the twice the flow occurring through twice the cross sectional area. It is therefore reasonable to assume as Darcy's done without performing the experiments that,

$$dv_s = 0 = d[(1 - n)dxdydz] \qquad \ldots\ldots\ldots\ldots \qquad (4)$$

Equation (4) yields

$$dzdn = (1 - n)d(dz) \qquad \ldots\ldots\ldots\ldots \qquad (5)$$

$$dn = (1 - n)d(dz) / dz \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

The negative sign indicate that the flow of water is in the direction of decreasing head. Equation (5) above is called the Darcy's law.

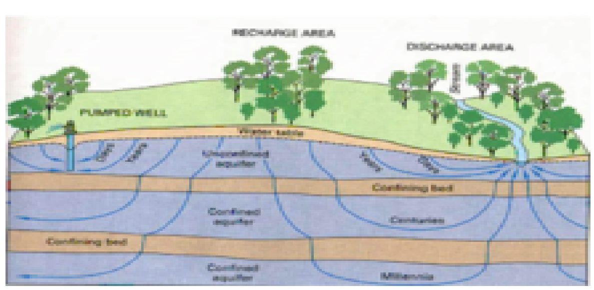
EQUATIONS OF GROUNDWATER FLOW

The derivations of equations used in groundwater applications are based on the conservation principles dealing with mass, momentum, and energy. These principles require that the net quantity (mass , momentum, or energy) leaving or entering a specified volume of aquifer during a given time interval be equal to the change in the amount of that quantity stored in the volume. The derivation of the particular conservation equation involves representing the balance in terms of mathematical expressions. Once the balance equation is developed in mathematical terms, it is necessary to specify additional relationships among variables so that the equations can be solved. These include thermodynamic (e.g. the effect of fluid pressure on density) and constitutive (e.g the effect of fluid pressure on porosity) relationships.

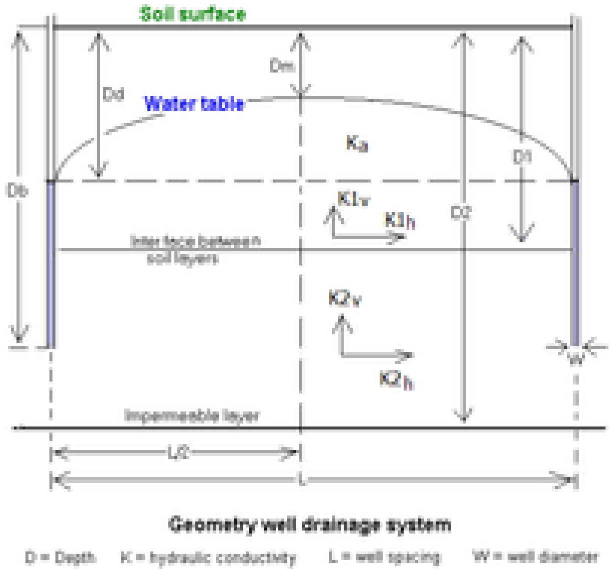
The result of the derivation is usually a set of general partial differential equations in the three-dimensional Cartesian co-ordinate system. As an example of the procedure, a simplified derivation of the groundwater flow equation. General equations for groundwater flow, solute transport and heat transport. It should be noted that these equations were derived based on certain assumptions and even more general (and complex) equations could be presented. The general equations are the ones most commonly used.

A. **Confined Aquifers:** In a confined aquifer, the flow of fluids through a porous media is governed by the laws of physics; hence the laws of conservation of mass and energy are employed. Since the flow is a function of several variable, it is usually describe by partial differential equation in which the co-ordinates x, y, z, and time t are made independent variables. In deriving the equations of groundwater, the two laws of physics are used and are stated as follows:

   i. The law of mass conservation or continuity principle, state that "there can be no net change in the mass of a fluid contained in a small volume of an aquifer must be balanced by corresponding changes in the mass stored in the volume or both".
   ii. Law of conservation of energy also known as the first law of thermodynamics states that "within any closed system, there is a constant amount of energy which can be neither lost or increased, it can only however changes from one form to another".
   iii. The second law of thermodynamics implies that "when energy changes from one form, it tend to go from more useful form, such as mechanical energy too less useful from such as heat". Based upon these principles and Darcy's law , the main equation of groundwater flow have been derived by (Jacob, 1940-1950, Domenico 1972, Cooper, 1966).



A picture above depicting how the relative travel times flow takes place



The picture above depict how calculation of groundwater flow is taken place

The volume of water in the control volume is equal to ndxdydz, where n is the porosity, therefore the initial mass of the water is wdxdydz. Any change in the mass of water, M, with respect to time (t) is given by

$$\frac{dM}{dt} = \frac{d}{dt}(p_w ndxdydz) \qquad (7)$$

The compressibility of water, β, is defined as the rate of change in density with a change in pressure

$$\beta dp = \frac{dpw}{p_w} \qquad (8)$$

The aquifer compressibility, is given by

$$\alpha dp = \frac{d(dz)}{dz} \qquad (9)$$

As the aquifer compresses or expands, n will change. But the volume of solids V_S will be constant if the only deformation is in the z – direction, d(dx) and d(dy) is equal to zero.

$$dv_s = 0 = d[(1 - n)dxdydz] \qquad (10)$$

Equation (10) yields

$$dzdn = (1 - n)\, d(dz) \qquad (11)$$

$$dn = \frac{(1-n)d(dz)}{dz} \qquad (12)$$

The pressure p, at a point in the aquifer is equal where p_0 the atmospheric pressure is, and h is the height of a column of water above the point, therefore

$$dfW \ = fW\beta(fWgdh) \qquad (13)$$

$$d(dz) = dz \propto (fWgdh\ ) \qquad (14)$$

If $d(dz)$ in equation (12) is replaced by equation (14), yield

$$dn = (1-n) \propto fWgdh \qquad (15)$$

The equation for change of mass with time in the control volume can be expressed as

$$\frac{dm}{dt} = \left[f_W n \frac{d(dz)}{dt} + f_W dz \frac{dn}{dt} + ndz \frac{df_W}{dt} \right] dxdy \qquad (16)$$

Substituting (13), (14), (15) into equation (16), yields

$$\frac{dm}{dt} = (\alpha f_W g + n\beta f_W g) f_W dxdydz \frac{dh}{dt} \qquad (17)$$

$$\left[\frac{d(qx)}{dt} + \frac{d(\sum y)}{dy} + \frac{dqx}{dz} \right] f_W dxdy = (\alpha f_W g + n\beta f_W g) f_W dxdydz \frac{dh}{dt} \qquad (18)$$

Which yield equation

$$qx = \frac{kdh}{dx}\ ,\ \text{qy} = \frac{kdh}{dy}\ ,\ \text{qz} = \frac{kdh}{dz} \qquad (19)$$

Substituting equation (19) into (18) it yields the main equation of flow for a confined aquifer.

$$k(\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} + \frac{d^2h}{dz^2}) \ = \ (\alpha f_W g + b\beta fg)\frac{dh}{dt} \qquad (20)$$

Which is a general equation in three dimensions for an isotropic, homogenous, porous predium.

If S is the storability and T the transmissivity, then

$$\frac{d^2h}{dx^2}+\frac{d^2h}{dy^2}=\frac{S}{T}\frac{dh}{dt} \qquad (21)$$

In steady-state flow, there are no changes in head with time, thus the differential equation is known as Laplace equation.

$$\frac{d^2h}{dx^2}+\frac{d^2h}{dy^2}+\frac{d^2h}{dz^2}=0 \qquad (22)$$

the leakagerate or the rate of accumulation is designated as ℓ, the general equation of flow assume horizontal flow in two dimensions is given by

$$\frac{d^2h}{dx^2}+\frac{d^2h}{dy^2}+\frac{\ell}{T}=\frac{S}{T}\frac{dh}{dt} \qquad (23)$$

The leakage rate can be defined from Darcy's law as

$\ell = k\frac{(h_o-h)}{b}$, where h_o is the head at top of aquitard, h is the head in the aquifer just below aquitard, b is the aquitard thickness and vertical conductivity of K.

B. Unconfined Aquifer

This is derived from storage in water-table aquifers by vertical drainage of water in the drainage results in a decline in the position of the water-table near a pumping drainage progresses. For a confined aquifer the saturated thickness of the aquifer remains constant, thus for an unconfined aquifer the saturated thickness of the aquifer can change with time.

The general flow equation for two-dimensional unconfined flow is known as the Boussinesq equation (Boussinesq, 1904) is given by,

$$\frac{d}{dx}\left(\frac{hdh}{dx}\right)+\frac{d}{dy}\left(\frac{hdh}{dy}\right)=\frac{Sy}{K}\frac{dh}{dt} \qquad (24)$$

Where h is the thickness, K is the conductivity, equation (24) is a non-linear equation, thus the thickness can be replaced by average thickness, b, thus the Boussinesq equation can be linearized as

$$\frac{d^2h}{dx^2}+\frac{d^2h}{dy^2}=\frac{Sy}{Kb}\frac{dh}{dt} \qquad (25)$$

Where Sy is the specific yield.

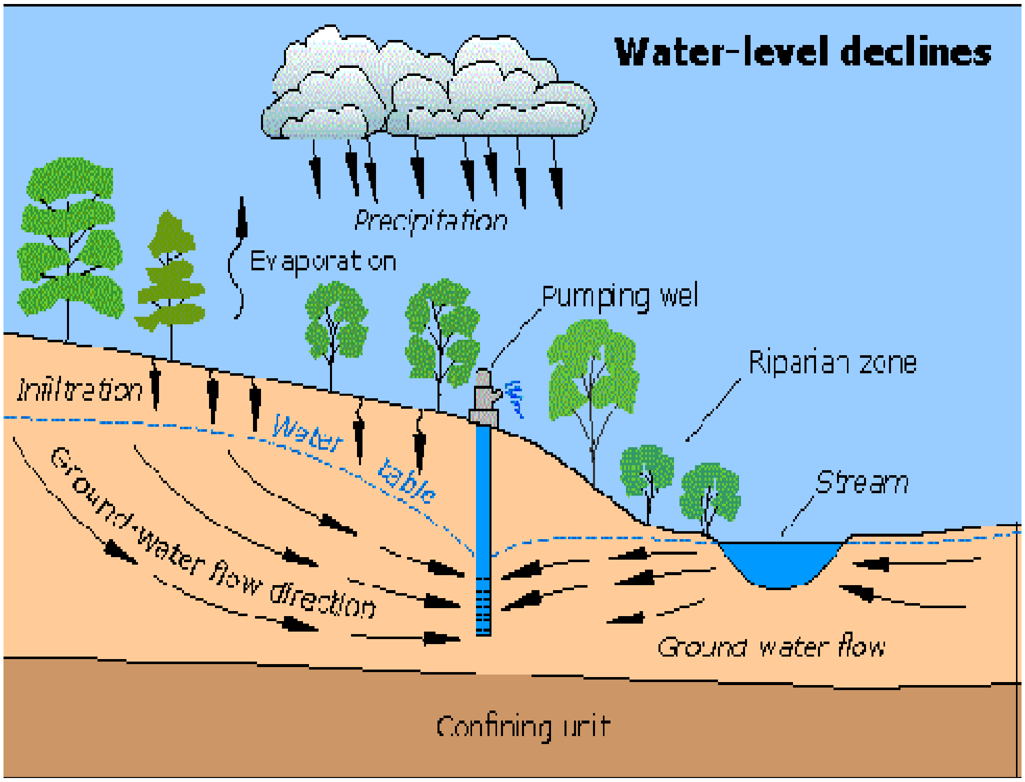
Thus in general, the equation of groundwater is based on the laws of physics and Darcy's law. These equations are expressed in mathematical terms for both confined and unconfined aquifers, which are Laplace's equation and Boussinesq equation.

VARIATION OF GROUNDWATER LEVEL WITH TIME

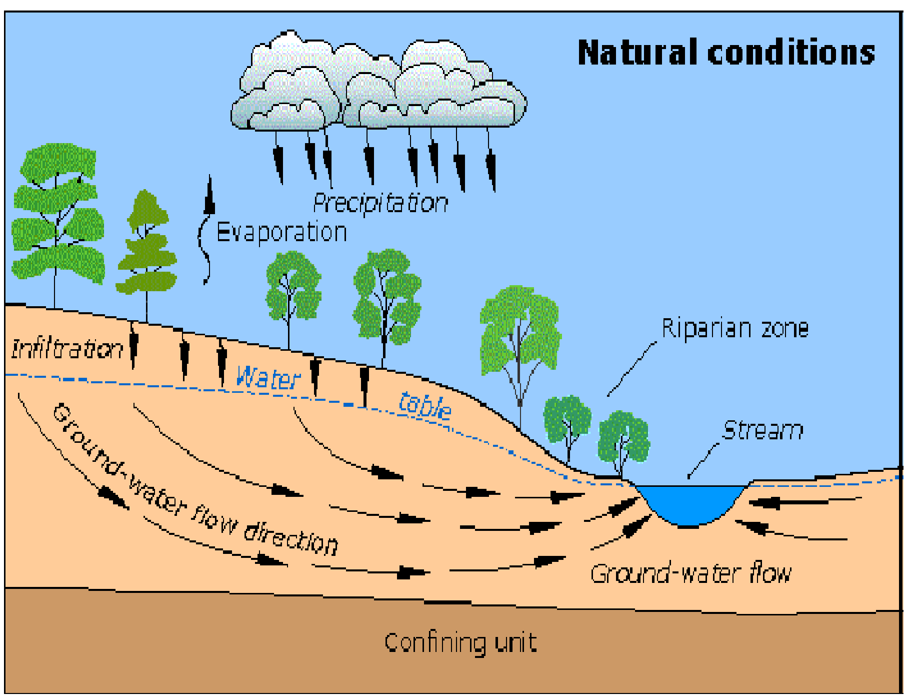
Basic Assumptions

We need to make assumptions about the hydraulic conditions in the aquifer and about the pumping and observation wells, which are:

1. The aquifer is bounded on the bottom by confining layers
2. All geologic formation is horizontal and has infinite horizontal extent.
3. The potentiometric surface of the aquifer is horizontal prior to the start of the pumping.
4. The potentiometric surface of the aquifer is not changing with time prior to the start of the pumping.
5. All changes in the position of the potentiometric surface are due to the effort of the pumping well along.
6. The aquifer is homogenous and isotropic.
7. All flow is radial toward the well.
8. Groundwater flow is horizontal
9. Darcy's law is valid.
10. Groundwater has a constant density and viscosity.
11. The pumping well and the observation well are fully penetrating, that is they are screened over the entire thickness of the aquifer
12. The pumping well has an infinitemal diameter and is 100% efficient.



A picture above shows how water level declines



A picture above depicting natural conditions

GROUNDWATER DEVELOPMENT

Dynamic Equilibrium in Natural Aquifers

Under natural condition an aquifer is usually in a state of dynamic equilibrium (Theis 1838). A volume of water charges the aquifer and equal volume is discharged. The potentiometric surface is steady and the amount of water in storage in the aquifer is a constant. The aquifer transmits the water from the recharge areas to the discharge zones. The maximum amount of water any section can transmit is a function of the transmissivity and the maximum gradient of the potentiometric surface. If the water table is closer to the surface of an unconfined aquifer, the aquifer is full and is transmitting the maximum, amount of water. If however, the water table is far below the surface, the aquifer is not transmitting water at full capacity.

The amount of water that recharges an unconfined aquifer is determined by three factors namely:

i. The amount of precipitation that is not lose by evaporation and run off and is thus available for recharge.
ii. The vertical hydraulic conductivity of surface deposit and other strata in the recharge area of the aquifer, which determines the volume of recharged water capable of moving downward to the aquifer.
iii. The transmissivity of the aquifer and potentiometer gradient, which determine how much water can move away from the recharge area.

Groundwater Budgets

Some knowledge of the amount of natural recharge to an aquifer is mandatory in groundwater development. A method can be used to determine the flow in the aquifer across a vertical plane at the boundary of the recharge area and the discharge area (Walton, 1960). The rate of study flow from recharge area to discharge areas is determined using either Darcy's law for a confined aquifer or the Dupuit equation for an unconfined aquifer. The flow from the recharge area is equal to the rate of recharge for aquifers in dynamic equilibrium. A water budget for the recharge area of an aquifer. An advantage of the water budget method is that the aquifer does not have to be in dynamic equilibrium to use it.

Management Potential of Aquifers

Aquifer can play many roles in the overall development of the water resource of an area; the most obvious use of the water resource of an area. Resources of an aquifer are to supply water to wells-the supply function. Aquifer also transmit water from one location to another, this has been called a pipeline function (Kazmann, 1956). These aquifers are not as sufficient in carrying large volumes of water as are surface canals. However, surface canals can lose large amount of water by evaporation, furthermore, they require capital for construction.

Groundwater can be mined in the same manner as minerals (using seismic method of geophysical investigation for example) whenever groundwater is withdrawn at a rate greater than the rate of replenishment, mining is occurring, under, these conditions the water in storage in the aquifer must be considered a nonrenewable resource. This is the mining function of an aquifer. The unsaturated zone overlying an aquifer can acts as a water table (Sellers, 1973) has been called the 'filter plant' function of aquifer (Hordon, 1977).

Groundwater can also have an energy sources function the groundwater heat pump is a viable alternative to conventional heat pumps in some localities (Gass and Lehr, 1977). As the thermal energy of the aquifer is removed by a heat pump in colder climates, it is another type of mining. Groundwater reservoir sometimes also have a storage function, this is not true for an undeveloped aquifer in dynamic equilibrium if the recharge zone is rejecting potential recharge. Aquifers are also used for the storage of natural gas. The aquifer must be confined, and a structural or stratigraphic feature, such as an anticline, is required to hold the gas in place.

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