Frequency-Based Design of Internal Model Controller Using the Method of Inequalities

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ABSTRACT: *In the traditional frequency domain design of an internal model controller, the design problem is cast in terms of* H_2 *and* H_∞ *norms of sensitivity functions in order to obtain the parameters of a robust controller for an uncertain system. This requires the use of weights, which are often obtained in a cumbersome trial-and-error manner. In this paper, a computer-aided robust internal model control design method which eliminates trial-and-error selection of weights was formulated within the context of the method of inequalities (MoI). Lead, lag and lead-lag networks were used as weights and the model uncertainty was described both in exact and norm-bounded forms. The robust stability and performance criteria of the feedback system were defined as a set of algebraic inequalities. Moving boundaries process (MBP), a search algorithm, was used to automatically and simultaneously obtain the parameters of the controller and the weights which satisfy the performance criteria. An uncertain system from the literature was chosen to illustrate the new technique. The MoI-based method gave rise to internal model controller filter parameters which in most cases are in close agreement with the filter parameters obtained using the traditional trial-and-error method. Furthermore, the uncertainty weights obtained via the MoI-based method are of lower order in comparison with those obtained via the tedious trial-and-error method. It is concluded that the MoI-based method can effectively replace the trial-and-error method for the frequency-based design of internal model controller.*

Keywords: Internal Model Controller; Weights; Robust Controller; Method of Inequalities
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INTRODUCTION

The popular internal model control consists mainly, the controller and the model used in the controller design. It is found to be simple and practical control for stable and unstable plants [1]. It requires a few numbers of parameters for tuning for robustness [2]. A promising way of obtaining a robust internal model controller for uncertain systems is to cast the design problem within H_∞ framework. The H_∞ framework is one of the best methods available today for robust control design. It is an effective method for attenuating noise and disturbances that appear in the system. It is designed to accomplish minima restriction in frequency domain [3]. It has been widely used to address different practical and theoretical problems [4]. However, the selection of suitable weights for controller design in H_∞ framework is not a straightforward task. Traditionally, the designer is required to find such suitable weights by a long and tedious trial-and-error process using his engineering experience and intuition [5]. This is not a trivial task, in that many factors such as the desired performance, specified robustness requirement and/or fundamental performance limitations due to plant dynamics, have to be considered [6].

Several methods for selecting suitable weights have been proposed.

Lundström *et al.,* [7] proposed a first order performance weight as,

$$W_p(s)=\frac{1}{M}\frac{\tau_{cl}s+M}{\tau_{cl}s+A} \qquad \text{……………...…… (1)}$$

Where; $\tau_{cl} = 1/W_B^*$,

W_B^*	=	Approximate closed-loop bandwidth,
A	=	Steady state offset upper bound,
M	=	Upper bound on amplification of high-frequency noise.

The disadvantage of this technique is that tedious trial-and-error attempts have to be made before the parameters of suitable weight can be found. Franchek [8] argued that the frequency domain specifications were addressed directly in the work of Lundstrom and Co-workers [7] while the influence of the system's uncertainty on the transient output performance was not addressed. Thus, he proposed performance weights selection method which can directly enforce hard time domain constraints. However, Khow and Banjerdpongchai [9] argued that both Lundström *et al.,* [7] and Franchek's [8] approaches only provide a simple method of initially choosing weights and that the two approaches did not explicitly include a robust guarantee of time domain specifications. Hence, they proposed an approach for guaranteeing both frequency and time-domain specifications. However, their proposed technique still rely directly on the original trial-and-error formulation of Lundström and Co-workers [7].

In order to overcome the difficulties of trial-and-error processes of selecting weights and that of the sequential design of weights and controllers, some systematic and automatic optimization algorithms which lead to simultaneous design of weights and the controllers have been developed (see [5, 6, 10, 11]). Lanzon's and Richards' [5] and Lanzon's and Cantoni's [6] algorithms were developed using state-space formulations, and as a result, may not be directly applicable to systems with irrational transfer functions; e.g., a system with time delay does not have state-space realization except the delay term is approximated [12] by its rational equivalent. By utilizing lead, lag and lead-lag networks as weights, Whidborne *et al.,* [10] applied the method of inequalities (MoI) for the direct design of robust controllers such as PI/PID, linear quadratic Gaussian, mixed sensitivity, etc. Its application for the direct design of internal model controller has not been reported. Furthermore, the simplicity/complexity of the MoI-based selected weights in comparison with the weights selected via the traditional trial-and-error procedures has yet to be discussed.

This paper describes the procedures for designing an internal model controller within H_∞ framework. Here, the control problem is formulated within the context of the method of inequalities (MoI) which facilitates automatic and simultaneous design of weights and controllers parameters. The simplicity of the resulting MoI-based weights is compared with the weights obtained via trial-and-error method reported in Lundström *et al.,*[7] and Skogestad's and Postlethwaite's [12] works. The framework for incorporating model uncertainties both in the exact and norm-bounded forms within the context of MoI is also presented.

The remainder of the paper is organized as follows: section 2 is concerned with the descriptions of various theoretical principles utilized in this work, section 3 is devoted to the description of the proposed method, section 4 presents the application of the existing trial-and-error and the proposed automatic method to a literature example while the results are discussed in section 5. Finally, relevant conclusions are drawn in section 6.

THEORY

Performance Weight Selection

In using trial-and-error approach, standard weights, which are selected by referring to the frequency-domain specifications: bandwidth frequency, steady-state tracking error and maximum peak magnitude of sensitivity function are given by Skogestad and Postlethwaite [12]. They are as follows: first order weight is given as,

$$W_p(s) = \frac{s/M + W_B^*}{s + W_B^* A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

A is the upper bound, $1/|W_p(jw)|$ on the magnitude of sensitivity function, $|\varepsilon|$ at low frequencies, usually, A ≤ 1. A is actually the maximum value of the allowable steady-state error. M is the upper bound, $\frac{1}{|W_p(jw)|}$ on $|\varepsilon|$ at high frequencies, usually, M ≥ 1, W_B^* is the frequency at which $1/|W_p(jw)|$ crosses 1, and this is approximately the bandwidth requirement. In some cases, when there is need to improve performance, a higher-order (such as 2nd order) weight is proposed and is given by

$$W_p(s) = \left(\frac{s}{M^{1/2}} + W_B^*\right)^2 \Big/ \left(s + W_B^* A^{1/2}\right)^2 \quad \ldots \ (3)$$

When the plant transfer function contains one or more of the following: RHP zero, RHP pole, and a pure time delay, the choice of $W_p(s)$ is not just based on M, A and W_B^* but depends on the values of the zero or/and the pole or/and the time delay. According to Skogestad and Postlethwaite [12], e.g., the restrictions on the choice of $W_p(s)$ due to the presence of RHP zero is given as:

$$\|W_p \varepsilon\|_\infty \geq |W_p(z)| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (4)$$

Eqn. (4) must hold before the closed loop stability of the control system can be guaranteed; z is the location of RHP zero. For H_∞ design, performance condition is defined as

$$\|W_p \varepsilon\|_\infty < 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (5)$$

Hence,

$$|W_p(z)| < 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (6)$$

Uncertainty Region Description

Model uncertainty can be described in the exact or norm-bounded form. In the former, the uncertainty regions π(w), $\forall G_p^i \in \Pi$ are generated. Usually, they are complex in shapes. However, non-conservative controllers are eventually obtained. In order to use this approach for controller design, exact region mapping technique [2] is utilized. In the latter, the uncertainties in the model are represented as disks; they are bounded by scalar or complex perturbation whose infinity norm must not exceed unity at all frequencies. It lends itself easily to mathematical evaluations. Using multiplicative weight, a family of uncertain plants is defined as ∏ as follows:

$$\Pi : \left\{ G_p^i(s) = G(s)\left(1 + W_I(s)\Delta_I(s)\right) \right\} \quad \ldots\ldots\ldots \ (7)$$

Where $W_I(s)$ is the uncertainty or multiplicative or relative weight; $G(s)$ is the nominal model transfer function, while G_p^i is the i_{th} perturbed plant and Δ_I is normalized perturbation. Fig.1 is the representation of a plant with multiplicative uncertainty.

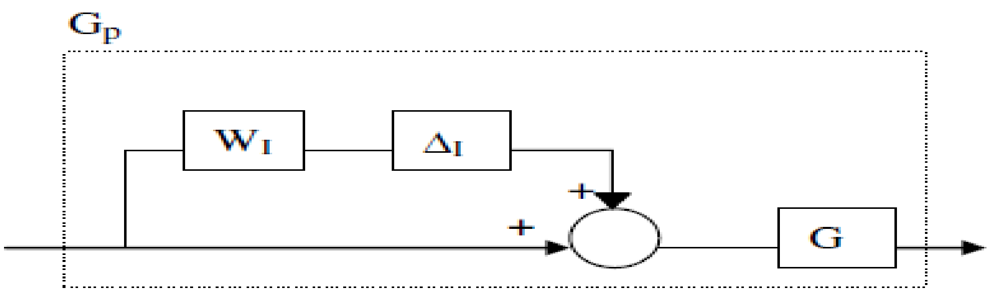

Figure 1: Plant with Multiplicative Uncertainty

Uncertainty Weight, W_I

Given a set $\prod$ of possible perturbed plants G_p^i due to parameter variations in the model, G

$$L_I(w) = \max_{G_p^i \in \Pi} \left| \frac{G_p^i(iw) - G(iw)}{G(iw)} \right|, \quad \forall w, \forall G_p^i \in \Pi \quad \text{.....} \quad (8)$$

Where iw = s, Laplace variable. Based on eqn. (8), a rational weight W_I can be obtained to fit into eqn. (7). However, the condition below must be satisfied:

$$W_I(iw) \geq L_I, \quad \forall w \quad \text{.................. ....} (9)$$

Usually, W_I is obtained in a trial-and-error manner. A first order weight is tested; if it is unsuccessful, higher order weights are tested [12]. A first order weight is given as

$$W_I(s) = \frac{\tau s + r_o}{\left(\tau / r_\infty\right) s + 1} \quad \text{........................} (10)$$

r_o is the magnitude of the relative uncertainty at steady-state i.e. low frequencies, $w \to 0$, $1/\tau$ is the approximate frequency at which the relative uncertainty reaches 100%, and r_∞ is the magnitude of the weight at high frequencies (typically, $r_\infty \geq 2$ is chosen).

Lead, Lag and Lead-lag Networks

Rather than searching for design weights in a trial-and-error manner, the weights could be defined as standard lead, lag or lead-lag network whose parameters can be obtained using suitable automatic search algorithm.

Phase-Lead and Phase-Lag Networks

Whidborne *et al.,* [10] define a high-pass (phase-lead) and low-pass high-gain (phase-lag) filters of the form

$$W_\theta = \frac{w_i(s + w_j)}{(s + w_k)}, \qquad w_j < w_k \quad \text{...} (11)$$

Where subscript θ could be p if W is W_p or θ = I, if W is W_I. w_i, w_j, $w_k > 0$. If $w_j < w_k$, then w_θ is called phase-lead network or weight, otherwise it is called phase-lag network. i, j, k could be 1, 2, 3 respectively, they could be 4, 5, 6 depending on the number of weights present in the problem.

Lag-Lead Network

According to Distefano *et al.,* [13], the lag-lead network (weight) is given by equation (12), where subscript θ stands for p in W_p and I in W_I

$$W_\theta = \frac{(s+w_1)(s+w_2)}{(s+w_3)(s+w_4)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Internal Model Control (IMC)

IMC consist mainly the controller and the model used in the controller design. It is found to be simple and practical control for stable plants; however, it is also applicable to open-loop unstable systems [1]. It requires a few numbers of parameters for tuning for robustness [2]. According to Morari and Zafiriou [1], a relationship exists between classical feedback controller, C and internal model controller, q which is stated as follows:

$$C = \frac{q}{1 - Gq} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$q = \bar{q}F \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Where filter, F = f (λ), λ is the filter parameter, $\bar{q}$ is the optimal (based on IAE or ISE) nominal internal model controller. For robustness, $\bar{q}$ must be augmented with a low pass-filter, F as described in eqn. (14). A typical IMC configuration in a feedback loop is as shown in Fig. 2, where G is the process nominal model, G_p is the actual process (plant) transfer function, U is the manipulated input, r is the reference input, y is the output response and d is the disturbance. Interested readers should see references [1, 2] for further details on the existing IMC controller design procedures using H_∞ framework.

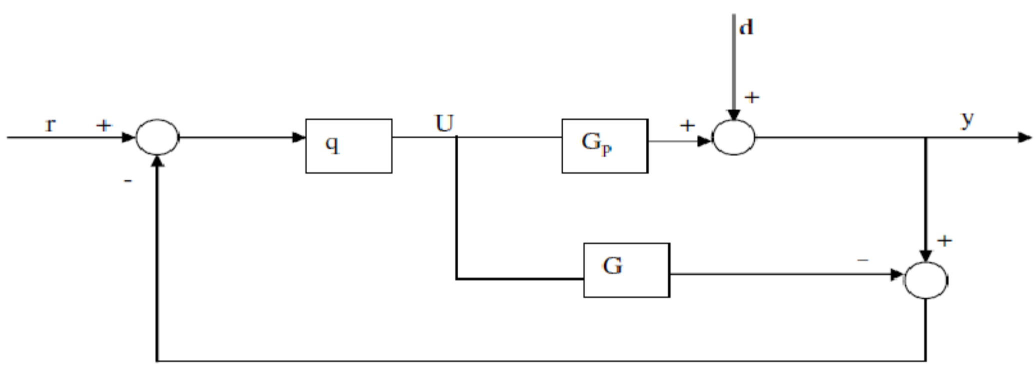

Figure 2: Internal Model Control Configuration

Method of Inequalities (MoI)

MoI [10, 14, 15] is a computer-aided multi-objective design approach, where desired performances are represented by a set of algebraic inequalities. The aim of the design is to simultaneously satisfy these inequalities. Because of the flexibility of MoI, it is also applicable to single-objective multi-parametric problems. The design problem is to find vector **p** such that the inequalities

$$\Phi_i(\mathbf{p}) \le \varepsilon_i, \quad \forall i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

are satisfied ε_i's are real numbers, **p**∈P and it is a real vector **p** = **p**(1), **p** (2), …….**p** (q)) chosen from a given set P; and Φ_i's are real functions of **p**. The design goals ε_i's, which represent the largest tolerable values of the objective functions Φ_i's are chosen by the designer. The aim of the design is to find a vector **p** that simultaneously satisfies the set of the inequalities. The actual solution to the set of inequalities (15) may be obtained by means of numerical search algorithms. The original algorithm used for solving (15) is known as moving boundaries process (MBP) (see ref. [15]). The algorithm was used in the work presented in this paper.

Robust Stability and Performance

Robust Stability Criterion

For an uncertain plant under feedback control, the condition for which the controlled system remains stable in the presence of all uncertainties for all the perturbed plants is called robust stability [12]. Nyquist stability criterion is found to be very effective. A closed-loop system is stable if the Nyquist plot of its open loop transfer function $G_pG_c(s)$ does not encircle (-1, 0) coordinate in a counter clockwise manner. To ensure robust stability, Laughlin *et al.,* [2] stated the condition for robust stability of a family Π of perturbed plants as follows: when $G_pG_c(s)$ is represented by regions $\pi(w)G_c(iw)$ at each frequency where $\pi(w)$ is the complex uncertainty region mapped out by all $G_p^i \in \Pi$ at each w(where $G_p^i(iw) \in \pi(w),\ \forall w$), then no region $\pi(w)G_c(iw)$ must encircle (-1,0) coordinate. Π is the set of all perturbed plants, $\pi(w)$ is the uncertainty region at frequency, w. The simple summary of this criterion is that at each frequency, the magnitude of $G_p^iG_c(iw)$ is evaluated for all $G_p^i \in \Pi$, and then plotted on Nyquist plane, and as such, Nyquist bands are obtained. Hence, at all frequencies, none of the Nyquist bands should contain, (-1, 0) coordinate. This is better illustrated graphically; e.g., in Fig.3, some of the bands contain (-1,0) coordinate, which indicates non-robust stability.

For the case when the uncertainty in the model is represented as norm-bounded type, then according to Morari and Zafiriou [1], by assuming that all the plants G_p^i in the family, Π

$$\Pi = \left\{ G_p^i : G_p^i \in \Pi \left| \frac{G_p^i(iw) - G(iw)}{G(iw)} \right| \le W_I(w) \right\},$$

$$\forall G_p^i \in \Pi, \forall w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

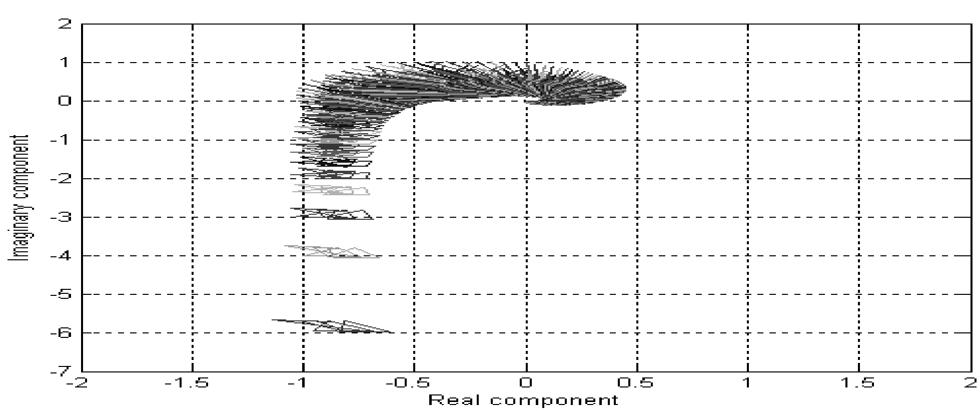

Figure 3: Bands for $\pi G_c(iw)$ on Nyquist Plane

The same number of RHP poles and that a particular controller, G_c stabilizes the nominal plant G, then the system is robustly stable with controller G_c if the complementary sensitivity function η(s) for the nominal plant G satisfies the following bound,

$$\left\| \bar{\eta} W_I \right\|_\infty \triangleq \mathrm{Sup} \left| \bar{\eta} W_I(w) \right| < 1 \quad \ldots\ldots\ldots (17)$$

Note that: For unity feedback control system,

$$\bar{\eta} = \frac{GG_c}{1 + GG_c} \quad \ldots\ldots\ldots (18)$$

G_c is the transfer function of the controller, W_I is the uncertainty weight, G_p^i is the ith perturbed plant, and $\bar{\eta}$ is the nominal complementary sensitivity function.

Robust Performance Condition

For eqn. (5) to be satisfied for the worst-case plant,

$$\left\| \varepsilon W_p \right\|_\infty = \sup \left| \varepsilon W_p(iw) \right| < 1, \ \forall G_p^i \in \Pi \quad \ldots (19)$$

Must hold if the uncertainty in the model is described in the norm-bounded form, then eqn. (19) can be written in terms of W_I and such expression is used directly to guarantee robust performance for all the plants. Therefore, according to Morari and Zafiriou [1],

$$\left\| \left| \bar{\eta} W_I \right| + \left| \bar{\varepsilon} W_p \right| \right\|_\infty < 1 \quad \ldots\ldots\ldots (20)$$

Where; $\bar{\varepsilon}$ is the nominal sensitivity function.

INTERNAL MODEL CONTROL (IMC) DESIGN USING MoI

The proposed method formulates frequency-based IMC design procedures using the method of inequalities framework. Given an uncertain plant G_p, the steps below are followed:

a. Using simple principles of permutations and combinations, all possible distinguished extreme (perturbed) plants are identified and defined as $G_p^i \in \Pi, \quad \forall i$.

b. IMC structure is defined; the controller transfer function is $q = \bar{q}F$.

c. Obtain $G_p^i C(iw), \ \forall i$. C is given by eqn. (13), $\bar{q}$ is obtained via the existing IMC design approach [1].

d. Appropriate form(s) of weight(s) is/are defined as lead, lag or lag-lead networks as described in section 2.4 depending on the form of the uncertainty description (exact or norm-bounded) and the objective functions $\Phi_j(\mathbf{w}, s,) \le \varepsilon_j, \ j = 1, \ldots\ldots n$ are defined.

e. Necessary performance criteria for optimization are defined. For exact region mapping, the performance criterion is robust performance factor (RP_F) i.e. eqn. (19); for norm-bounded uncertainty representation, the criteria are RP_F [eqn. (20)] and uncertainty weight factor, UW_F. Also, appropriate stability margin Φ_{sm}, is defined. The optimization problem is set up as algebraic inequalities as described in section 2.6; RP_F is infinity norm of the weighted sensitivity function.

f. The values of the upper bounds, ε_j's on the objectives are specified. Also, upper bound ε_{sm} on Φ_{sm} is also specified. Also, necessary bounds are placed on the parameters.

g. Initial values of the parameters are chosen at designer's discretion and hence a search algorithm such as MBP is implemented. If solution is found, the design is accomplished, if no solution is found, step ‘b’ may be returned to, to change controller form, or step ‘d’ to change weights’ forms or orders, or step ‘f’ to change the values of ε_j or ε_{sm}.

Comment

In this work, a new index, uncertainty weight factor, UW_F is introduced to incorporate multiplicative weight into MoI framework when the uncertainty is described as norm-bounded form. From eqn. (9),

$$|W_I(iw)| \geq L_I(w), \qquad \forall w$$

A single factor is required to serve as a representative over all the frequencies range; rearranging the expression leads to

$$\frac{L_I(w)}{|W_I(iw)|} \leq 1, \forall w$$

Let $$UW_F = \left\| \frac{L_I(w)}{W_I(iw)} \right\|_\infty \leq 1 \qquad \text{………… (21)}$$

This offers an advantage over the earlier stage-wise, graphical and trial-and-error approach since this will be found numerically, automatically and simultaneously with the controller and weights’ parameters.

ILLUSTRATIVE EXAMPLE

The existing closed loop transfer function shaping (CLTFS) approach when using IMC configuration is applied to the uncertain RHP-zero plant described by Whidborne et al., [10]. The nominal plant model is defined as,

$$G(s) = \frac{K(-T_2 s+1)W_0^2}{\left(s^2+2\xi W_0 s+W_0^2\right)(T_1 s+1)} \qquad \text{…...... (22)}$$

Where $T_1 = 5$, $T_2 = 0.4$, $W_0 = 5$, $\xi = 0.3$, $K = 1$. There are three stress levels of operation such that the transfer function of the plant changes at each stress level due to parameters’ variations, hence the complete transfer function for each level is given as,

$$G_p(s) = \frac{K(-T_2 s+1)W_0^2}{\left(s^2+2\xi W_0 s+W_0^2\right)(T_1 s+1)} \frac{W_\delta^2}{\left(s^2+2\xi_\delta W_\delta s+w_\delta^2\right)\left(T_1^\delta s+1\right)\left(T_2^\delta s+1\right)} \qquad \text{….. (23)}$$

Where; $T_1^\delta = 1/8,\ T_2^\delta = 1/12,\ W_\delta = 15, \xi_\delta = 0.6$

The variations in the parameters at each stress level are given in Table 1.

Table 1: Parameters’ Variations at Each Stress Level

Stress Level	δT_1	δT_2	δW_0	$\delta\xi$	δK
1	±0.20	±0.05	±1.50	±0.10	±0.00
2	±0.30	±0.1	±2.50	±0.15	±0.15
3	±0.30	±0.15	±3.00	±0.15	±0.50

The aim is to design for each stress level, a controller to achieve rise time as fast as possible, subject to the following conditions:

a. The plant output must be within -1.5 and +1.5 at all times,
b. Zero steady-state tracking error,

c. It is preferable if the under/overshoot is around 0.2 most of the time (occasional large
d. over/under shoots are acceptable as long as the output is within ± 1.5),
e. Fast settling time,
f. Plant input saturates at -5.0 and +5.0.

The set-point may be pre-filtered. **N.B:** The reference input to the loop (control) is square-wave with period of 20seconds.

APPLICATION OF THE EXISTING TRIAL-AND-ERROR APPROACH

Here, the existing trial-and-error method for the design of internal model controller was utilized. The results would be compared with those obtained via the method of inequalities.

Trial-and-Error Method When the Uncertainty is Described in the Exact Form

The trial-and-error procedures summarized in Morari and Zafiriou [1] are followed:

a. The process model, G is obtained as;

$$G = \frac{10(-s+2.5)}{\left(s^2+3s+25\right)(5s+1)} \quad \ldots\ldots\ldots\ldots\ldots (24)$$

b. **Input Type Specification:** The reference input in this case is square-wave with a period of 20 seconds.

$$\begin{Bmatrix} r(t)=1, & 0<t<10\text{sec} \\ r(t)=-1, & 10<t<20\text{sec} \end{Bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (25)$$

c. **The family, Π of perturbed plants is defined**: Substituting the values of constant parameters into eqn. (23) and also noting that the following parameters: W_o, K, T_1, T_2 and ξ are uncertain,

$$G_p = \frac{21600KW_0^2(-T_2s+1)}{\left\{T_1s^3+\left(1+2\xi W_0T_1s^2+\left(T_1W_0^{\ 2}+2\xi W_0\right)s+W_0^2\right)\right\}}\left\{\frac{1}{s^4+38s^3+681s^2+6228s+21600}\right\} \quad \ldots (26)$$

Using the principles of permutations and combinations for stress levels 1, 2, and 3 (refer to Table 1), $2^4 = 16$, $2^5 = 32$ and $2^5 = 32$ extreme plants and their corresponding parameters' values were identified.

d. Design Stage:

(i) q is obtained [1] as,

$$q = \frac{\left(s^2+3s+25\right)(5s+1)}{10(s+2.5)(\lambda s+1)^2} \quad \ldots\ldots\ldots\ldots\ldots (27)$$

(ii) Robust Stability Analysis, From eqn. (13),

$$C = \frac{\left(s^2+3s+25\right)(5s+1)}{10\left(\lambda^2s^3+\left(2\lambda+2.5\lambda^2\right)s^2+(2+5\lambda)s\right)} \quad \ldots (28)$$

Applying Nyquist stability criterion as discussed in section 2.7, the minimum λ required for robust stability was found as λ = 0.021, 0.31, 0.70 for stress levels 1, 2 and 3 respectively.

(iii) Robust Performance Analysis: $w_p(s)$ was selected in a trial-and-error manner by bearing in mind the bandwidth limitations due to RHP zero in the plant dynamics. A first order weight [eqn. (2)] was tested. A

≈ 0 and $M \approx 2.5$ were used. After a series of trials, bandwidths (w_B^*) were found for levels 1, 2 and 3 as 0.301, 0.185 and 0.021, respectively. The bandwidth limitation imposed on $w_P(s)$ due to RHP-zero present in plant dynamics was found to be w_B^* < 1.5, which holds for the 3 levels. The value of λ was adjusted until expression (19) was slightly less than unity (maximized). For levels 1, 2 and 3, λs were found to be 0.646, 1.00 and 1.50 respectively.

Trial-and-error Method When the Uncertainty is Described in Norm-Bounded Form

(i) Robust Stability and Performance Analysis: In this case, satisfying robust performance condition satisfies robust stability condition automatically. $w_p(s)$ and $w_I(s)$ were selected sequentially in a trial-and-error manner. $w_p(s)$ for all the 3 levels are the same as those obtained in section 4.1.1. The concepts described in section 2.3 were utilized and the uncertainty weights for levels 1, 2, and 3 were obtained as follows after a series of trial-and-error attempts. The first order uncertainty weight given by eqn. (10) or eqn.10) augmented with a lag-lead network was searched. The results are as presented in Tables 2 through 4. Note, '×' is multiplication sign, while '+' is addition sign.

Table 2: Weight, Filter Parameter and Uncertainty Weight Factor for Level 1

	Exact Region Mapping & Trial- and-error Approach	Exact region Mapping & The New Method	Norm-bounded Uncertainty Rep & Trial-and-error Approach	Norm-bounded Uncertainty Rep. & New Method
Filter param, λ	0.646	0.632	1.02	0.99
W_p	$\frac{s+0.7525}{2.5s}$	$\frac{0.4275(s+0.62)}{\left(s+5.83\times10^{-9}\right)}$	$\frac{s+0.7525}{2.5s}$	$\frac{0.5424\ (s+0.327\ )}{(s+1.1693\ )}$
W_I	-	-	$\frac{(0.0065+0.4739\,s)}{(1+0.1994\,s)}\times\frac{\left(s^2+7.6s+1\right)}{\left(s^2+4s+1\right)}$	$\frac{6.379(s+0.102)}{(s+3.416)}$
UW_F	-	-	-	0.49

Table 3: Weight, Filter Parameter and Uncertainty Weight Factor for Level 2

	Exact Region Mapping & Trial-and-error Approach	Exact region Mapping & New Method	Norm-bounded Uncertainty Rep. & Trial-and-error Approach	Norm-bounded Uncertainty Rep. & New Method
Filter parameter, λ	1.00	1.03	1.53	1.2
W_p	$\frac{s+0.4628}{2.5s}$	$\frac{0.443(s+0.285)}{\left(s+2.698\times10^{-10}\right)}$	$\frac{s+0.4625}{2.5s}$	$\frac{6.379(s+0.14)}{(s+3)}$
W_I	-	-	$\frac{(0.1504+0.7634s)}{(1+0.1549s)}\times\frac{\left(s^2+8s+1\right)}{\left(s^2+3s+1\right)}$	$\frac{6.379(s+0.14)}{(s+3)}$
UW_F	-	-	-	0.99

Table 4: Weight, Filter Parameter and Uncertainty Weight Factor for Level 3

	Exact Region Mapping Trial-and-error Approach	Exact Region Mapping & New Method	Norm-bounded Uncertainty Rep. Plus Trial-and- error Approach	Norm-bounded Uncertainty Rep. Plus New Method
Filter Parameter, λ	1.5	1.442	1.815	1.90
W_p	$\frac{s+0.0525}{2.5s}$	$\frac{0.3572(s+0.27075)}{\left(s+2.9678\times10^{-9}\right)}$	$\frac{s+0.0525}{2.5s}$	$\frac{1.003(s+0.32832)}{(s+8.7002)}$
W_I	-	-	$\frac{(0.5003+1.4085s)}{(1+0.1766s)}\times\frac{\left(s^2+7.5s+1\right)}{\left(s^2+3.4s+1\right)}$	$\frac{14.6717(s+0.196)}{(s+3.4148)}$
UW_F	-	-	-	0.77

(ii) Robust Performance Analysis: By applying eqn. (20), $\bar{\eta}$ and $\bar{\varepsilon}$ was obtained as;

$$\bar{\eta}=\frac{(-s+2.5)}{(s+2.5)(\lambda s+1)^2} \quad \text{……………………... (29)}$$

$$\bar{\varepsilon}=\frac{\lambda^2 s^3+\left(2\lambda+2.5\lambda^2\right)s^2+(2+5\lambda)s}{(s+2.5)(\lambda s+1)^2} \quad (30)$$

Substituting $\bar{\eta}$, $\bar{\varepsilon}$, $W_I(s)$ and $W_p(s)$ into eqn. (20) i.e. RP_F, and the value of λ was adjusted until RP_F was slightly less than unity in magnitude, hence the search was terminated. At this juncture, the robust performance factor has been maximized and the values of λ obtained for levels 1, 2 and 3 are 1.02, 1.53 and 1.815 respectively. With these values obtained, the design was completed.

APPLICATION OF THE NEW METHOD

In this section, the reported weights' and robust internal model controller's parameters are obtained simultaneously and automatically using the method of inequalities.

Application of the New Method When the Uncertainty is Described in Exact Form.

Following the procedures described in section 3, the design progressed as follows:

a. Distinguished extreme plants were identified as done in section 1.

b. The control structure, IMC was defined as in the preceding sections.

c. Appropriate weighting function (lead, lag or lag-lead) was defined. Here, only Wp(s) was required, hence, it was defined as Wp(s) = $\frac{\mathbf{w}(1)(s+\mathbf{w}(2))}{(s+\mathbf{w}(3))}$

d. Necessary performance criteria for optimization were defined. Here, the objectives were posed as algebraic inequalities i.e. $\Phi_j(\mathbf{w},s) \le \varepsilon_j$

The main objective was RP_F, $\therefore \Phi_1 = RP_F$. Also, necessary stability margin was defined as $\Phi_{sm} < \varepsilon_{sm}$. Here, Nyquist bands (graphical approach) are not suitable for numerical and automatic analysis. Since there is a possibility of obtaining characteristic polynomial in this case, then, $\Phi_{sm} < \delta$; Φ_{sm} = max (real (root (denominator of closed loop transfer function))). Usually, a small value $\delta < 0$ is chosen such as $\delta = -10^{-2}$. This is to ascertain that no pole of the denominator lies on the RHP. Here, for unity feedback control system, the closed loop transfer function is defined as

$$y = \frac{G_p^i G_c}{1+G_p^i G_c} r, \quad \forall G_p^i \in \Pi$$

Therefore,

$$\Phi_{sm} = \max(\text{real}(\text{root}(1+G_p^i G_c))) \ \ldots \ (31)$$

Note that: $G_c = C$. The RP_F is the main objective here, utilizing eqn. (19) i.e.

$$\Phi_1 = RP_F = \|\varepsilon\, W_P\|_\infty < 1, \quad \forall G_p^i \in \Pi$$

Sensitivity function is defined as follows

$$\varepsilon(s) = \frac{1}{1+G_p C(s)}$$

Setting s = iw, and substituting eqns. (26) and (28) into ε (iw) as just defined above,

$\varepsilon(iw) = \frac{1}{1+G_p C(iw)}, \forall w, \forall G_p^i \in \Pi$ is defined

Substituting $W_p(s)$ obtained in step 'c' into

$\Phi_1 = RP_F = \|\varepsilon W_p(s)\|_\infty < 1$ by setting s = iw,

Φ_1 ($\mathbf{w}$,iw) was defined ready for implementation by using MoI.

$$\Phi_1 = RP_F = \|\varepsilon W_p(iw)\|_\infty < 1$$

e. The upper bounds were set on the Φ_i and on Φ_{sm} i.e. $\varepsilon_1 = 1$; $\varepsilon_{sm} = -0.01$

f. Initial values of $\mathbf{w}(i)$ $\forall i$ were chosen by the designer. $\mathbf{w}(1)$, …… $\mathbf{w}(3)$ are the parameters of the $W_p(s)$ while $\mathbf{w}(4)$ was set equal to λ the filter parameter ; $\mathbf{w}(i) \ge 0$. With all the necessary steps analyzed properly, the MBP was implemented until satisfactory solution was obtained i.e. when Φ_1 was slightly less than unity (maximized) and $\theta_{sm} \angle -0.01$. The results obtained are reported in Tables 2 through 4.

Application of the New Method When the Uncertainty is Described in Norm-Bounded Form

(a) Here, two weights are required; they are $W_p(s)$ and $W_I(s)$. The proposed weights are

$$W_p(s) = \frac{w(1)(s+w(2))}{(s+w(3))} \text{ and } W_I(s) = \frac{w(4)(s+w(5))}{(s+w(6))}$$

(b) All necessary optimization criteria were defined as in section 4.1.1(d) with the inclusion of UW_F, Φ_2 as stated below: from eqn. (21),

$$\Phi_2 = UW_F = \left\| \frac{L_I(w)}{\|W_I(s)\|} \right\|_\infty \le 1, \quad \forall G_p^i \in \Pi$$

Using eqn. (8), by substituting eqns. (24) and (26) for G and G^i_p respectively, L_I (w) was defined. Substituting $W_I(s)$ and L_I into Φ_2 and setting s = iw in Φ_2 (**w**, s),

$$\Phi_2(\mathbf{w}, iw) = UW_F = \left\| \frac{L_I(w)}{\|W_I(iw)\|} \right\|_\infty \le 1 \ldots\ldots (32)$$

Where;
$\mathbf{w} = [\mathbf{w}(1), \mathbf{w}(2), \ldots. \ \mathbf{w}(7)]$. $\mathbf{w}(1)\ldots\ldots\mathbf{w}(6)$ are the parameters of the weights, while **w**(7) is the filter parameter, λ

(c) Having defined the objectives, $\varepsilon_{j's}$ were specified i.e. $\varepsilon_1 = 1$, $\varepsilon_2 = 1$ and $\varepsilon_{sm} = -0.01$.

(d) Initial values of **w**(i)'s were chosen at the designer discretion and MBP was used to search for the satisfactory parameters.

The results obtained are, as reported in Table 2 through 4.

RESULTS AND DISCUSSION

Based on H_∞ design formalism, robust internal model controllers were designed for an uncertain system which was subjected to square-wave input at three different stress levels. Both the existing trial-and-error procedure and the proposed automatic approach, the method of inequalities (MoI) were applied. Model uncertainty was incorporated into the design in exact and norm-bounded forms.

The results-filter parameter (λ), weights (W_p and W_I) and the uncertainty weight factor (UW_F), are as reported in Tables 2, 3 and 4. Across all the three stress levels, for both the trial-and-error and the new methods, λ values when the model uncertainty is described in the exact form are lower than when the uncertainty is described in the norm-bounded form. This should be expected, as the norm-bounded uncertainty description is a conservative approach. Most of the filter parameters obtained from the new method are in close agreement with those obtained from the existing tedious, trial-and-error method. This indicates that the former can effectively replace the latter. The uncertainty weights obtained from the new method are simple lag and lead (1st order networks), as opposed to 3rd order networks obtained from the existing tedious approach. The uncertainty weight factor (UW_F) is less than 1 at all levels and this satisfies the condition stated in eqn. (21). This demonstrates that the tedious graphical trial-and-error approach of deriving uncertainty weight in the existing technique can be by-passed since only the numerical value of UW_F, which is obtained automatically guarantees satisfactory uncertainty weight in the new method.

CONCLUSIONS

This study developed a frequency-based framework for designing robust internal model controller using the combination of H_∞ formalism and the method of inequalities (MoI). The use of MoI facilitated automatic and simultaneous design of robust controller and simple 1st order uncertainty weights. This is opposed to the existing trial-and-error method which resulted in 3rd order uncertainty weights. It is therefore concluded that MoI can successfully replace the previous tedious graphical trial-and-error approach used in the design of robust internal model controller within H_∞ framework.

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