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# SOME APPLICATIONS OF ENUMERATIVE TECHNIQUES FOR k-SEPARABLE AND k-INSEPARABLE ELEMENTS OF SET X

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**Abstract:** We considered applications of r-arrangements of elements of the set X, applied various techniques (Listing, counting and mathematical methods) to obtain the total number of the given r-arrangement. Listing become difficult, if the number of elements of the set X are large ($n \geq r \geq r_0$ ) as in some applications that we shall solve by applying the results obtain by Moore and *et al.* We applied listing, counting and the Enumerative techniques by Moore, and *et al* to solve the applications.

**Keywords**: *k- inclusion, k- non-inclusion, k-inseparable inclusion, k-inseparable non- inclusion*

## INTRODUCTION

We consider various arrangements of the elements in X, such that, a fixed group k of elements is given different restrictions. If n is small (say 2, 3 or 4) it is easy to exhaustively list and count all the possible outcomes in this arrangements for either the inclusion case or non-inclusion case. Furthermore, for any given collections of r-arrangements, observe that for sufficiently large value of n there are various sub-collections of arrangements, which are of interest but without any well-known mathematical formula in literature up to the twentieth century. The mathematical formula to some of these sub-classes of r-arrangements has been provided by Moore and *et al* in the twenty first century. In fact, it is obvious that as n increases we can define many more of these sub-collections of r-arrangements. So far, to the best of our knowledge most of the standard Text in Combinatorics has failed to address this concept. However, even when it is introduced it is left with a vacuum that relegate this concept to the background of no important (see [1; 2; 3; 4; 6; 12 ; 13]), since most of this applications were solved by the time consuming Listing and counting method. For this reason, in this research work we shall consider more than one sub-collections of X with a prescribe restrictions on the elements of these sub-collections of X with the aim of providing some applications associated with the r-arrangements.

Let $X = \{x_i : i = 1,2, \dots, n\}$ be a finite collection and let $K = \{x_{ij} : j = 1,2, \dots, k\}$ be a sub-collection of X. We consider the problem of selecting $r(r \leq n)$ elements from X in such a way that each selection;

(i) Contains the entire k-elements but in such a way that the k-elements are always together; we call this the k-inseparable inclusion.
(ii) Contains the entire k-elements but in such a way that the k-elements are always not together; we call this the k-separable inclusion.
(iii) Contains only some part of K and not the entire k-elements but in such a way that the k-elements are always not together; we call this the k-separable non-inclusion.
(iv) Contains only some part of K and not the entire k-elements but in such a way that the k-elements are always together; we call this the k-inseparable non-inclusion.

We were able to come out with very interesting application to see how we can apply Moore and *et al.* We shall present the following definitions, some basic facts and application in a sequel.

Product Rule [3, 5, 7, 14, 16, 21]
If there are m successive choices to be made, and, for $1 < i < m$, the choice can be made in $n_i$ ways, then the total number of ways of making these choices is

$$\prod_{i=1}^{m} n_i = n_1 \times n_2 \times n_3 \times \ldots \times n_m$$

Sum Rule [3, 5, 7, 14, 16, 32]
If an event $E_1$ can be done in $n_1$ ways, an event $E_2$ can be done in $n_2$ ways and an event $E_i$ can be done in $n_i$ways and $E_i$ for $1 < i < m$ are mutually exclusive, then the number of ways of the events occurring is

$$\sum_{\mathrm{i}=1}^{m} = n_1 + n_2 + n_3 + \cdots + n_m$$

Double Counting [14, 19, 42, 53, 54]
We have seen by Def. 1.2 that the formula

$$n(\sum_{\mathrm{i}=1}^{\infty} A_i) = n(A_1) + n(A_2) + n(A_3) + \cdots$$

Holds provided that the set $A_i$ for i = 1, 2, 3, ... are mutually exclusive, but not if there is an overlap among them. If there is an overlap then apply Def. 1.3

1.1 Definition: Principle of inclusion-exclusion (PIE). [40, 48]}

If $A_i$ for i = 1, 2, 3, ... n be finite sets, then
If $A_1,\ A_2,\ \ldots\ A_k$ is any sequence of finite sets, then

$$n\left(\bigcup_{i-1}^{k} A_i\right) = \sum_{\substack{I \subset [K] \\ I \neq \emptyset}} (-1)^{n(I)-1}\, n\left(\bigcap_{i \in I} A_i\right)$$

We shall consider some of the result published by Moore and *et al* to solve some problems which one would have only use the time consuming LISTING AND COUNTING method that had begin in place up to the twentieth century. However, in the twenty first century Moore and *et al* solved some of these problems with restriction in the set X of n elements. We shall only site the result used if need arise in our solution.

**Problem 1.1**
First Bank Nigeria PLC is considering changing the customer's account numbers by using the elements {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C} to develop a ten digit Account number for her customers. The management will like to know how many ways the numbers can be arranged, such that {0, 5, 6, 7, 9} must be;

i. Separately and partially contained in each arrangement.

ii. Always together and partially contained in each arrangement.

If order is important

Solution

i) The management requested that each arrangement should contain $\{ 0, 5, 6, 7, 9 \}$ separately and partially and any of the following numbers $\{1, 2, 3, 4, 8, A, B, C \}$ in the customer's account numbers.

By listing and counting, we have the following;

{102536478A, 152637408A, 162730458A, 172035468A, 015263748A, 516273048A, 617203548A, 710253648A, 12304586A7, 12354687A0, 12364780A5, 12374085A6, 120354687A, 125364780A, 126374085A, 127304586A, 12035468A7, 12536478A0, 12637408A5, 12730458A6, 10253648A7, 15263748A0, 16273048A5, 17203548A6, 01526348A7, 51627348A0, 61720348A5, 71025348A6, 01523486A7, 51623487A0, 61723480A6, 71023485A6, 01234586A7, 51234687A0, 61234780A5, 71234085A6, 102536498A, 152639408A, 162930458A, 192035468A, 015263948A, 516293048A, 619203548A, 910253648A, 12304586A9, 12354689A0, 12364980A5, 12394085A6, 120354689A, 125364980A, 126394085A, 129304586A, 12035468A9, 12536498A0, 12639408A5, 12930458A6, 10253648A9, 15263948A0, 16293048A5, 19203548A6, 01526348A9, 51629348A0, 61920348A5, 91025348A6, 01523486A9, 51623489A0, 61923480A6, 91023485A6, 01234586A9, 51234689A0, 61234980A5, 91234085A6,
…
61729348AB, 71926348AB, 91627348AB, 16273948AB, 17293648AB, 19263748AB, 12637498AB, 12739468AB, 12936478AB, 12364789AB, 12374986AB, 12394687AB, 1234789A6B, 1234986A7B, 123486A7B9, 123487A9B6, 123489A6B7, 1234687AB9, 1234789AB6, 1234986AB7, 1236478AB9, 1237498AB6, 1239468AB7, 1263748AB9, 1273948AB6, 1293648AB7, 1627348AB9, 1729348AB6, 1926348AB7, 123648A7B9, 123748A9B6, 123948A6B7, …

7192348ABC, 9172348ABC, 1729348ABC, 1927348ABC, 1273948ABC, 1293748ABC, 1237498ABC, 1239478ABC, 1234789ABC, 1234987ABC, 123487A9BC, 123489A7BC, 12348A7B9C, 12348A9B7C, 12348AB7C9, 12348AB9C7, …}

Thus, we have a $total = \mathbf{249,177,600}\ \boldsymbol{ways}$.

OR

i) Solution

Consider the First Bank Nigeria PLC. Problem for changing the customer's account numbers

With $n = 13, r = 10\ and\ k = 5$ Clearly $2k - 1 = 9 < r$

We now applied the Moore and *et al*[23] in solving Problem 1.1 (i) , we have;

$$P_{sni(n,r,k)} = \sum_{i=2}^{4} \frac{P_{(8,\ 10-i)} P_{(5\ ,\ ii)} P_{(11-i,\ i)}}{i!} = \mathbf{249,177,600\ ways}$$

Problem 1.1 (ii) the management requested that each arrangement should contain $\{ 0, 5, 6, 7, 9 \}$ always together and partially and any of the following numbers $\{1, 2, 3, 4, 8, A, B, C \}$ in the customer's account numbers.

By listing and counting, we have the following;

{7912348ABC, 1792348ABC, 1279348ABC, 1237948ABC, 1234798ABC, 1234879ABC, 12348A79BC, 12348AB79C, 12348ABC79, 9712348ABC, 1972348ABC, 1297348ABC, 1239748ABC, 1234978ABC, 1234897ABC, 12348A97BC, 12348AB97C, 12348ABC97, 6912348ABC, 1692348ABC, 1269348ABC, 1236948ABC, 1234698ABC, 1234869ABC, 12348A69BC, 12348AB69C, 12348ABC69, 9612348ABC, 1962348ABC, 1296348ABC, 1239648ABC, 1234968ABC, 1234896ABC, 12348A96BC, 12348AB96C, 12348ABC96, 6712348ABC, 1672348ABC, 1267348ABC, 1236748ABC, 1234678ABC, 1234867ABC, 12348A67BC, 12348AB67C, 12348ABC67, 7612348ABC, 1762348ABC, 1276348ABC, 1237648ABC, 1234768ABC, 1234876ABC, 12348A76BC, 12348AB76C, 12348ABC76, 5712348ABC, 1572348ABC, 1257348ABC, 1235748ABC, 1234578ABC, 1234857ABC, 12348A57BC, 12348AB57C, 12348ABC57, 7512348ABC, 1752348ABC, 1275348ABC, 1237548ABC, 1234758ABC, 1234875ABC, 12348A75BC, 12348AB75C, 12348ABC75, 5912348ABC, 1592348ABC, 1259348ABC, 1235948ABC, 1234598ABC, 1234859ABC, 12348A59BC, 12348AB59C, 12348ABC59, 9512348ABC, 1952348ABC, 1295348ABC, 1239548ABC, 1234958ABC, 1234895ABC, 12348A95BC, 12348AB95C, 12348ABC95, 5612348ABC, 1562348ABC, 1256348ABC, 1235648ABC, 1234568ABC, 1234856ABC, 12348A56BC, 12348AB56C, 12348ABC56, 6512348ABC, 1652348ABC, 1265348ABC, 1236548ABC, 1234658ABC, 1234865ABC, 12348A65BC, 12348AB65C, 12348ABC65, 0512348ABC, 1052348ABC, 1205348ABC, 1230548ABC, 1234058ABC, 1234805ABC, 12348A05BC, 12348AB05C, 12348ABC05, 5012348ABC, 1502348ABC, 1250348ABC, 1235048ABC, 1234508ABC, 1234850ABC, 12348A50BC, 12348AB50C, 12348ABC50, 0612348ABC, 1062348ABC, 1206348ABC, 1230648ABC, 1234068ABC, 1234806ABC, 12348A06BC, 12348AB06C, 12348ABC06, 6012348ABC, 1602348ABC, 1260348ABC, 1236048ABC, 1234608ABC, 1234860ABC, 12348A60BC, 12348AB60C, 12348ABC60, 0712348ABC, 1072348ABC, 1207348ABC, 1230748ABC, 1234078ABC, 1234807ABC, 12348A07BC, 12348AB07C, 12348ABC07, 7012348ABC, 1702348ABC, 1270348ABC, 1237048ABC, 1234708ABC, 1234870ABC, 12348A70BC, 12348AB70C, 12348ABC70, 0912348ABC, 1092348ABC, 1209348ABC, 1230948ABC, 1234098ABC, 1234809ABC, 12348A09BC, 12348AB09C, 12348ABC09, 9012348ABC, 1902348ABC, 1290348ABC, 1239048ABC, 1234908ABC, 1234890ABC, 12348A90BC, 12348AB90C, 12348ABC90, ...

67912348AB, 16792348AB, 12679348AB, 12367948AB, 12346798AB, 12348679AB, 12348A679B, 12348AB679, 79612348AB, 17962348AB, 12796348AB, 12379648AB, 12347968AB, 12348796AB, 12348A796B, 12348AB796, 96712348AB, 19672348AB, 12967348AB, 12396748AB, 12349678AB, 12348967AB, 12348A967B, 12348AB967,...

567912348A, 156792348A, 125679348A, 123567948A, 123456798A, 123485679A, 12348A5679, 679512348A, 167952348A, 126795348A, 123679548A, 123467958A, 123486795A, 12348A6795, 795612348A, 179562348A, 127956348A, 123795648A, 123479568A, 123487956A, 12348A7956, 956712348A, 195672348A, 129567348A, 123956748A, 123495678A, 123489567A, 12348A9567}

Thus, we have a $total = \mathbf{43,545,600}\ \boldsymbol{ways}$.

OR

ii. Solution

Consider the First Bank Nigeria PLC. problem for changing the customer's account numbers with n = 13, r = 10, and k = 5
We now applied the Moore and *et al* [22] in solving Problem 1.1 (ii), we have
Clearly 2k-1 = 9 < r

$$P_{isni(n,r,k)} = \sum_{i=2}^{4} \frac{P_{(8,\ 10-ii))}P_{((5\ ,,i))}(11-\ i)!}{(10-\ i)!} = \mathbf{43,545,600\ ways}$$

Problem 1.2
In a trade-fair, a Toy company with Toys of types $t_i$ $\{i = 1,\ 2,\ \ldots\ ,8\}$ wish to display in row five of her products so that each display is ordered uniquely. However, they required that type $t_1, t_2, and\ t_3$ Toys must not be next to each other and will not be included all at once in any display, finally $t_4, t_5, and\ t_6$ Toys will not be included all at once. For effective cost projection, the firm requires to know the number of possible display at her disposal since each display attracts a cost.

Solution
To solve this problem, it is possible to use the manual listing method of all possible display, that is the permutation of eight toys taken five at a time satisfying the prescribed conditions on $t_1, t_2, and\ t_3$ and $t_4, t_5, and\ t_6$. From the computation, we did using the manual list and count approach we got 3024 displays. We do not intend generating a pictorial display of the 3024 permutations of the toys satisfying the prescribed conditions, however we leave it as an exercise for the reader to generate.

We now applied the Moore and *et al* [26] in solving Problem1. 2, we have
Now by the formula method if we take $M = \{t_4, t_5,\ t_6\}\ and\ \ K = \{t_1, t_2, and\ t_3\}$ , hence we have;

$$P_{snni(n,r,k)} = \sum_{i=1}^{2}\sum_{j=0}^{2} i!\, j!\, (5-i-j)! \binom{2}{5-i-j}\binom{3}{i}\binom{3}{j}\binom{5-i}{j}\binom{6-j}{i} = 3024$$

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