A REVIEW OF ALL RISKS YIELD AND IMPLIED RENTAL GROWTH RATE EMBEDDED IN THE EQUATED YIELD HYBRID MODEL OF PROPERTY INVESTMENT VALUATION

Ataguba, Joseph Obaje and Tinufa, Anthony Abbey
Department of Estate Management and Valuation
School of Environmental Studies, The Federal Polytechnic Idah, Nigeria
E-mail: josephtgb81@gmail.com

ABSTRACT

The real value/equated yield hybrid model otherwise known as the Crosby's 3-YPs model is a contemporary value model which deploys nominal rate of interest (equated yield), rent review, and inflation risk free yield to the discounting of cash flows of property investments. Notwithstanding its robust features in the valuation of incomes with growth potentials, this model has been observed to be implicit about all risks yield and implied rental growth rate per annum such that they might only be known to the valuer who prepared the valuation; unless additional information on these parameters are provided with the valuation in question. This article evaluates an alternative perspective of how implied rental growth rate per annum and all risks yield are embedded in the Crosby's real value/equated yield hybrid model. An analytical framework which culminated into the derivation of all risks yield and implied rental growth rate per annum from the real value/equated yield model was designed. Thereafter, the synergy between the 3-YPs model and the derived formulas were evaluated with recourse to the valuation of fully let- and reversionary freehold interests respectively. Results indicate that the all risks yield and implied rental growth rate per annum are embedded in the 3-YPs model. It also was observed that this phenomenon was facilitate by equated yield and rent review period which are the variables commonly found in the formula for all risks yield, implied rental growth rate and the 3-YPs model. The formula derivation process and results from the individual valuation cases revealed that all risks yield and implied rental growth rate are adequately captured in the real value/equated yield hybrid model such that valuations ensuing from this model would not deviate from those produced by the growth explicit discounted cash flow (DCF) technique.

Keywords: Property Investment Valuation, 3-YPs Model, All Risks yield, Implied Rental Growth Rate, Equated Yield

INTRODUCTION

Contemporary techniques for the valuation of property investments evolved as a result of the pitfalls identified in the various conventional techniques used by property valuers over the years (Ajayi, 1998; Baum & Crosby, 2007; Bello & Bello, 2007; Crosby, 1983, 1984; Sykes, 1981). Among these pitfalls include the implicit manner in which these conventional techniques treat rent review and rental growth phenomena in freehold and leasehold investment properties (Baum & Crosby, 2007).

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As an improvement over the conventional techniques, all the variants of the contemporary techniques otherwise referred to as growth-explicit valuation models account for specific parameters such as equated yield, implied rental growth rate, rent review period, and in some instances, inflation risk free yield (real return) or all risks yield depending on whether cash flows are expressed in real- or nominal terms. Contrary to nominal cash flow which connotes the current monetary equivalent of income, real cash flow is the purchasing power of money or income at a given period of time (Hoesli & MacGregor, 2000). Related to this is the need to value nominal cash flows using nominal discount rates while real cash flows are valued using real discount rates (Brown & Matysiak, 2000).

One of the contemporary value models which discount nominal cash flows using nominal rate of interest (equated yield) is the real value/equated yield hybrid model otherwise known as the Crosby's 3-YPs model. Specifically, this valuation model incorporates parameters like nominal rate of interest (equated yield), rent review, and inflation risk free yield, and has been adjudged to be robust in the valuation of incomes with growth prospects (Baum & Crosby, 2007; Crosby, 1983, 1986a, 1986b). The model is however implicit about all risks yield and implied rental growth which actually mirror this robust characteristic. Hence, the need to examine how these two parameters (all risks yield and implied rental growth) are embedded in the real value/equated yield hybrid model.

STATEMENT OF PROBLEM

The three major contemporary models of property investment valuation include the rational model (McIntosh, 1983; Sykes, 1981), real value/equated yield hybrid model (Baum & Crosby, 2007; Crosby, 1983, 1986a, 1986b), and the explicit discounted cash flow (DCF) techniques. Besides explaining the conceptual meaning of parameters in each contemporary value model, Ajayi (1998), Baum and Crosby (2007), Brown and Matysiak (2000), Butler and Richmond (1990), Crosby (1996), Crosby, French, and Ward (1997), Udoekanem (2012), and Udo (1989) among others, have examined how these variants of contemporary value models are interrelated in terms of common parameters of equated yield, implied rental growth rate and all risks yield such that the valuation figures arising from the use of any of these models tend to reconcile or produce similar results. The only snag however, is that the Crosby's 3-YPs model tend to be implicit about implied rental growth rate per annum and the all risks yield, which are vital indices for comparative investment analysis. In other words, valuation with recourse to the real value/equated yield model tend to be silent over implied rental growth rate per annum and the all risks yield such that they might only be known to the valuer who prepared the valuation unless information on these parameters are provided with the valuation in question. The overarching question which this research seeks to answer is put forward as follows: Using inductive quantitative analysis, can it be concluded that all risks yield and implied rental growth rate are adequately captured in the equated yield hybrid model of property investment valuation?

AIM

This study aims to examine an alternative analytical proof that implied rental growth rate and all risks yield are embedded in the Crosby's real value/equated yield hybrid model such that a synergy between these parameters and the real value/equated yield hybrid model can be deduced.

OBJECTIVES

Specific objectives of this study include to:

(a) Derive implied rental growth rate from the real value model;
(b) Derive all risks yield from the real value model;
(c) Evaluate the synergy between implied rental growth rate and the real value model; and
(d) Evaluate the synergy between all risks yield and the real value model.

SIGNIFICANCE OF STUDY

While all risks yield have been criticized as being too implicit and "backward looking" on investors' expectations (Baum & Crosby, 2007), its use in both conventional and contemporary models of property investment valuation and analysis still underscores its relevance just as the valuation of equities still require inputs from the implicit price-earning (PE) ratio (French, 1997). Albeit, Brown and Matysiak (2000) settled the furore surrounding the rejection of simple yield capitalization in growth explicit DCF appraisal by establishing its link with discounted cash flow models, the choice of variants of contemporary valuation models is informed by investor's requirement and rational consideration of property value indicators. In addition to addressing the analytical gaps in the synergy between real value model, all risks yield, and implied rental growth rates, the uniqueness of this article stems from the deployment of inductive approach of working from the real value model to derive these two parameters as against the conventional framework of discounting streams of cash flows.

REVIEW OF LITERATURE

Implied Rental Growth Rate

Expressed in percentage, implied rental growth rate is an annual rate at which the rent derived from a rack-rented property investment increases in order to retain its real value and produce the appropriate equated yield which justifies the exit yield (Baum & Crosby, 2007; Parsons, 2003). In property investment appraisal, the implied rental growth rate is used to revise cash inflows upward and pave the way for valuation at the appropriate nominal discount rate (equated yield). Ajayi (1998), Baum and Crosby (2007), Brown and Matysiak (2000), Ifediora (2005), and Wyatt (2007) among others exemplified the following formulas for determining implied rental growth per annum:

$$g = \left[\sqrt[t]{(e-k)\left[\frac{(1+e)^t - 1}{e}\right] + 1} \right] - 1 \qquad (1)$$

Some authors of valuation texts and articles have even attempted to rationalize equation 1 to derive variants in the form of equations 2, and 3 as follows:

$$g = \left(\frac{(e-k)(1+e)^t + k}{e} \right)^{\frac{1}{t}} - 1 \qquad (2)$$

$$g = \left(\frac{(e-k)}{e} \left\{ (1+e)^t - 1 \right\} + 1 \right)^{1/t} - 1 \qquad (3)$$

Within the context of the real value model as captured in works of Baum, Crosby, and MacGregor (1996), Baum and Crosby (2007), and Crosby (1986a), implied rental growth can be expressed as a function of equated yield and inflation risk free yield:

$$g = \frac{1+e}{1+i} - 1 \qquad (4)$$

Alternatively, the implied rental growth rate can be expressed in the usual manner understood by valuers as:

$$(1+g)^t = \frac{YP \text{ in Perp. @ } k - YP \text{ for } t \text{ years @ } e}{YP \text{ in Perp. @ } k \times PV \text{ in } t \text{ years @ } e} \qquad (5)$$

Notwithstanding the variation in the formula for implied rental growth above, the common determinant have been established to be the equated yield of property investment, which shall be examined in due course.

All Risks Yield

All Risks Yield (ARY) is that rate of interest which implicitly reflects all risks inherent in an investment while allowing for value changes throughout its tenor (Ifediora, 2005). In conventional valuation of rack rented freehold properties, it represents the interest rate at which the annual net income is capitalized to ascertain capital value at the valuation date. Fraser (1993) describes it as the ratio of rent and capital value (price) of an investment property, reciprocal of which is the net income multiplier or years' purchase of an ordinary annuity in perpetuity. Fundamentally, all risks yield, k_o for a fully let freehold property is expressed as:

$$k_o = \frac{r_o}{p_o} \qquad (6)$$

Where r_o = rent passing and p_o = market capital value.

Brown and Matysiak (2000), McGough and Tsolacos (2001), and Wyatt (2007) reiterated that the incorporation of rent review adds a new dimension to the determination of all risks yield. They argued that if "*g*" represents the constant growth rate in the rental income per annum; "*e*" represents the discount rate (equated yield), and "*t*" connotes the period between each rent review, a cash flow model indicated in equation 7 will ensue:

$$p_o = \sum_{m=1}^{t} \frac{r_o}{(1+e)^m} + \sum_{m=1}^{t} \frac{r_o(1+g)^t}{(1+e)^{t+m}} + \sum_{m=1}^{t} \frac{r_o(1+g)^{2t}}{(1+e)^{2t+m}} + \ldots\ldots \quad (7)$$

Equation 7 is valid if $e > g$ and can be simplified as:

$$p_o = \frac{r_o}{e - e\left(\frac{(1+g)^t - 1}{(1+e)^t - 1}\right)} \quad (8)$$

Equating (6) and (8) results in the all risks yield formula:

$$k_o = e - e\left(\frac{(1+g)^t - 1}{(1+e)^t - 1}\right) \quad (9)$$

Brown and Matysiak (2000) expressed the relationship between real and nominal rates of interest as:

$$\frac{1}{1+r_r} = \left(\frac{1+\Delta}{1+r_n}\right) \quad (10)$$

Such that $1 + r_n = (1 + r_r)(1 + \Delta)$ (11)

Where the equated yield (nominal rate of interest) is expressed as "r_n"; "r_r" represents the real rate of return (inflation risk free yield) and the symbol "Δ" connotes the expected rate of inflation or the implied rental growth rate. Equation 11 is valid provided the rental growth rate equals inflation rate such that the relationship between equated yield, e and implied rental growth rate, g, and the inflation risk free yield, i is captured in equation 12 as:

$$\frac{1}{1+i} = \frac{1+g}{1+e} \quad (12)$$

It would be recalled that equation 5 for implied rental growth rate was derived from equation 12 above. Ifediora (2005) and Udo (2003) provided an analogy that equation 9 can be simplified in the terminologies understood by valuers as annual sinking fund, ASF in t years @ e and the percentage change in rental value.

Where the change represents percentage increase in rent, equation 9 translates into annual sinking fund, ASF in t years @ e times the percentage increase in rent expressed as $\{(1+g)^t - 1\}$:

$K_o = e - (ASF \text{ in } t \text{ years } @ \, e) \times (\%age \text{ rise in income in } t \text{ years})$ (13)

Equation 9 and 13 suggest that when there is income or capital appreciation, the capitalization rate, k_o is less than equated yield, e; that is, $k_o < e$. Drawing two interesting conclusions from this analogy, Ifediora (2005) explained that a phenomenon of income or capital loss implies that:

K_o = *e – (ASF in t years @ e) × (%age loss in income in t years)* (14)

On the other hand, equation 15 applies when the income remains stationary as in the case of the valuation of the term income for most contemporary value models.

$K_o = e$ (15)

Other Yield Parameters and their Relationships

In addition to all risks yield, other yield parameters used in property investment valuation include initial yield, reversionary yield, equivalent yield, equated yield, and inflation risk free yield (real return).

Initial Yield

A technical definition of initial yield is the ratio of rent passing to the capital value or price achieved during a transaction, which is represented in equation 6. Hoesli and MacGregor (2000) referred to this yield as income yield or *the all risk yield*. While initial yield is among the litany of income yields, it is a misnomer to conclude that it is synonymous to all risks yield. Illustrative valuations in exhibits 1 and 2 confirm the fact that the only condition where initial yield equals the all risks yield is during the valuation of fully let freeholds. Concerning the divergence between initial yield and all risks yield, Wyatt (2007) explained that the ARY (exit yield) used in discounting estimated rent at the end of the holding period is usually higher than initial yields on recently let comparable property investments since it must reflect a decline in the residual economic life of the property and enormity of risk inherent in estimating the exit cash-flow. In other words, initial yield may be equal to all risks yield but cannot be greater than all risks yield.

Yield on Reversion

Yield on reversion (reversionary yield) as exemplified in exhibit 2 is the capitalization rate used in converting income into the anticipated value of the property at the end of the term (Ifediora, 2005). It seeks to ascertain the rate at which the reversionary income (anticipated income) is secure in relation to the capital value. Equation 6 explains how yields on reversion are determined, but in this context, r_o represents rent at reversion.

Equivalent Yield

Equivalent yield is that growth implicit internal rate of return (IRR) which is used to capitalise both the current and reversionary cash inflows. One of the methods of calculating equivalent yield entails summing up the term rent and annual equivalent of gain on reversion and expressing it as a ratio of capital value (Enever & Isaac, 2002; Isaac & Steley, 1999; Wyatt, 2007). Other techniques of equivalent yield calculation include the use of DCF, spreadsheet iteration and solving the roots of polynomial equations ensuing from term and reversion valuation in order to determine the unknown IRR. Among these methods, spreadsheet iteration and solving the rational positive roots of a DCF polynomial equation produce accurate equivalent yield. While both methods are best tackled using software packages, problem of ensuing multiple roots can be ameliorated if the valuer deploys commonsense.

Equated Yield

Also referred to as the growth explicit internal rate of return (IRR) or inflation prone yield of an investment property, Wyatt (2007) defines equated yield as that discount rate which should adequately compensate an investor for the opportunity cost of capital and exposure to anticipated risk inherent in same investment. According to Ifediora (2005), this yield parameter explicitly reflects all risks including inflation risk, value changes (appreciation or depreciation) and the redemption price of the property. Scholarly debates indicated four methodologies for calculating equated yield. The first, being with recourse to the capital asset pricing model (CAPM) which adds a risk premium to the redemption yield on long-dated gilts to allow for risk differential between property and federal government securities.

$$e = R_f + R_p \qquad (16)$$

Where R_f is the risk free yield (Yield on long dated-gilts), and R_p is the risk premium which is a function of the beta coefficient of an investment property, β; and the expected market return, $E(R_m)$. One of the limitations of this method is the difficulty inherent in determining risk premium of direct property investment due to its relative illiquidity (Ifediora, 2005). Another limitation stems from the choice of risk premium. While 2% risk premium may be argued as the rule of thumb, owing to the relationship between prime property yields and gilt yields prior to reverse yield gap in the UK economy, scholars like Hargitay and Yu (1993) have warned that risk premium could vary over time and differ among property sectors. Related to this assertion is the possibility of a negative risk premium for certain classes of property investments in certain economies thereby violating that rule of thumb.

The second method for determining equated yield is with reference to statistical computation of the volatility of return, β which is a major determining factor for risk premium, R_p.

If $R_p = \beta[E(R_m) - R_f]$ (17)

From statistical perspective, $\beta = \frac{Cov_{jm}}{\sigma_m^2}$ (18)

Where j is investment class, m is market rate of return, Cov_{jm} is the covariance of the observed investment property j with the market rate of return m, and σ^2_m is the variance of the market rate of return; so that equated yield is determined using the model for security market line:

$$e = R_f + \beta[E(R_m) - R_f] \qquad (19)$$

$$e = R_f + \frac{Cov_{jm}}{\sigma_m^2}\left[E(R_m) - R_f\right] \qquad (20)$$

The third method for determining equated yield is with recourse to DCF appraisals and computation of IRR using linear interpolation:

$$e = R_f + \left[\frac{NPV @ R_f}{NPV @ E(R_m) + NPV @ R_f} \left(E(R_m) - R_f \right) \right] \quad (21)$$

In equation 21, *e* = equated yield, R_f and $E(R_m)$ retain their previous definitions. *NPV @* R_f connotes net present value at the risk free yield (gilt yield) while the negative sign in net present value churned out using the market rate of return (*NPV @* $E(R_m)$) is ignored to warrant its treatment as positive real number. The fourth method for determining equated yield is similar to the growth explicit DCF technique for calculating IRR; howbeit, cash flow modelling dovetails into determining a positive rational root of a polynomial equation which represents the investor's equated yield.

Analogy of equations 16 to 21 indicates interesting relationships that might be used to determine proximate measures of certain investment parameters:

$\beta = \frac{Cov_{jm}}{\sigma_m^2}$ Hence, implying that beta coefficient can be approximated using the formula - $\beta = \frac{NPV @ R_f}{NPV @ E(R_m) + NPV @ R_f}$ (22)

Likewise, Risk premium which is originally expressed as $R_p = \frac{Cov_{jm}}{\sigma_m^2} \left[E(R_m) - R_f \right]$ can be approximated using the expression:

$$R_p = \left[\frac{NPV @ R_f}{NPV @ E(R_m) + NPV @ R_f} \left(E(R_m) - R_f \right) \right] \quad (23)$$

Contrary to the assertion of Banfield (2005), Hoesli and MacGregor (2000), and Ifediora (2005) who argued that the CAPM has limited application to direct property media owing to data constraint, data inconsistency and imperfections of direct property investments, Brown and Matysiak (2000) opined that the CAPM is highly applicable to direct property investments in advanced markets save for the abuse of data requirements necessary to engender its workability. Based on the arguments of Brown and Matysiak (2000), it is recommended in this article that these interesting relationships identified in equations 22 and 23 be subject to further empirical research taking into cognizance the divergence between true IRR and IRR estimate as put forward by Wyatt (2007). For emerging markets where investment decisions are predominantly motivated by subjective techniques as against transaction-based data and valuations, it is obvious that dearth of data shall constitute bottleneck to the application of CAPM in property valuation.

Inflation Risk Free Yield

Inflation is a systematic risk factor which describes the decline over time in the purchasing power of the monetary equivalent of an investment. To clarify this assertion, Adams, Booth, Bowie et al. (2003) opined that the value of an investment will increase in terms of local currency and simultaneously declines in real terms as a result of inflation. Therefore, it is imperative to hedge against inflation and avert depreciation in rental or capital value of investments. Udo (2003) affirmed that the degree of proof against inflation is measured in terms of the purchasing power of the indicated capital value. This implies that the future value of an interest in property bought today is capable of purchasing an equivalent of that interest at the stated future date provided the implied rental growth rate is sustained. On this premise, Ifediora (2005), defined inflation risk free yield (IRFY) or real return as the capitalization rate when the rate of inflation is zero or equals the implied rental growth rate. Inflation risk free yield, *i* can be conveniently derived from equation 12 to yield equation 24 below:

$$i = \frac{1+e}{1+g} - 1 \tag{24}$$

The rationale for this yield parameter is to enable upward revised income to cancel out the effect of inflation and produce its real value at the appropriate review period.

Significance of implied rental growth rate and all risks yield in property analysis

Besides estimating property value, indices of implied rental growth rate and equated yield are crucial inputs for the formulation and implementation of property portfolio strategies. Brown and Matysiak (2000) reiterated that these parameters are utilized in tracking underpriced or overpriced property. For instance, all risks yield (ARY) is primarily a measure of income return, risk, profitability and implicit measure of income growth (Ajayi, 1998; Hoesli & MacGregor, 2000; Ifediora, 2005). Ifediora (2005) further affirms that a higher (lower) ARY signifies increased (decreased) earnings to an investor and also a higher (lower) risk of default in rent payment. Another significant application of all risks yield is in the construction and interpretation of property cycles (Sayce, Smith, Cooper et al., 2006).

Besides the ARY, investors might be concerned with the real return (inflation risk free yield) on property assets which is a function of equated yield and implied rental growth rate. According to Ifediora (2005), yield on inflation prone investment must be higher than what it would have been in the absence of inflation. Hence, a higher (lower) rate of inflation engenders higher (lower) equated yield. In concluding this review of yield parameters, it is imperative to note that yields may not always represent capitalization rates. In agreement with Brown and Matysiak (2000) and Ifediora (2005), the adoption of yields as capitalization rates should be anchored on facts and circumstances surrounding the valuation cash flows which might be expressed in nominal or real terms.

ANALYTICAL FRAMEWORK

Objectives of this study were achieved by drawing upon the works of Baum and Crosby (2007), Brown and Matysiak (2000), Crosby (1983), Crosby (1986a), Fraser (1993) and Ifediora (2005). Brown and Matysiak (2000) opined that the application of yield in a valuation model implies a growth in streams of cash inflows. They approached this growth in cash inflow from two perspectives comprising the

Gordon growth model which allows for review of cash inflow once every year and the periodic income growth model which allows for a periodic review of cash inflow.

With recourse to the valuation of cash flows, the Gordon growth model is expressed as:

$$P_0 = \frac{a}{e-g} \tag{25}$$

Equation 25 is valid on the condition that *e > g*, where g is the annual income growth rate; e is the equated yield; *a* is the initial cash inflow and P_0 equals the capital value or price of an investment. Baum and Crosby (2007), Brown and Matysiak (2000), and Hoesli and MacGregor (2000) observed that the Gordon growth model is used in the valuation and analysis of equities. Hence, if *e – g* equals the capitalization rate, *k*, then equation 25 bears some synergy with equation 6 in this paper. Although applicable to valuation of growth incomes, Equation 25 is not suitable for contemporary valuation of property asset characterized by periodic cash flow reviews. Hence, it had to be streamlined to suit contemporary property investment appraisals.

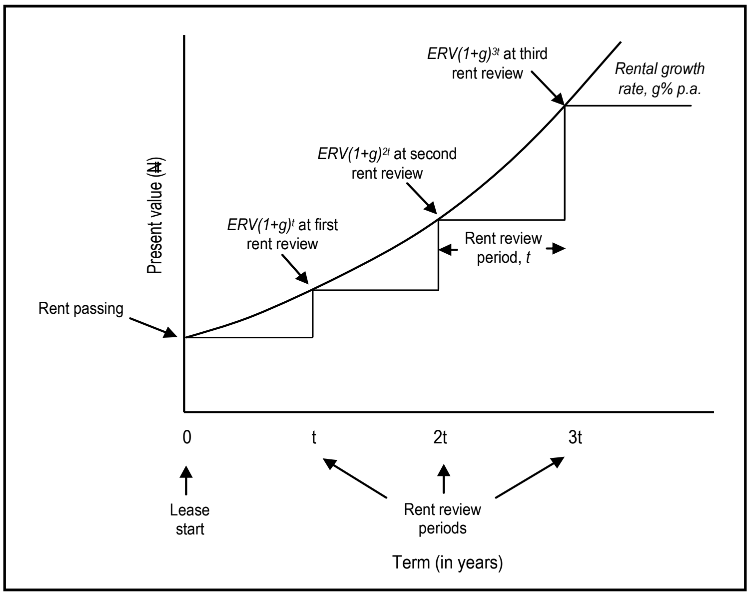

Figure 1: Growth profile of property cash flows

Another dimension for assessment of growth incomes is the periodic growth model which is the focus of this paper. Brown and Matysiak (2000) reiterated that periodic reviews of cash inflows is characterized by progressive step movements such that the present value of these perpetually stepped income profile represents the value of a property as depicted in Figure 1.

Valuation of the cash flow streams in Figure1 can be approached using equation 26

$$P_0 = \frac{a}{e}\left(\frac{(1+e)^t - 1}{(1+e)^t - (1+g)^t}\right) \text{ where } e > g \tag{26}$$

The equated yield model in equation 26 can be expanded as follows:

$$P_0 = \left(\frac{a}{e} - \frac{a}{(1+e)^t}\right) + \frac{a}{k}\left(\frac{1+g}{1+e}\right)^t \tag{27}$$

Baum and Crosby (2007), Crosby (1983), and Crosby (1986a) demonstrated three major approaches for the calculation of implied rental growth rate per annum. The first approach incorporates the explicit determination of all risks yield by setting up an equation such that the equated yield comprises the sum of all risk yield and annual sinking fund to recoup capital gain at the equated yield over the review period for property cash inflows:

$$e = k + \left(\frac{e}{(1+e)^t - 1}\left((1+g)^t - 1\right)\right) \tag{28}$$

Therefore, the synergy in equations 9, 13, and 28 can be attributed to their ability to determine the all risks yield of a given property investment.

With respect to determination of implied rental growth, Baum *et al.* (1996) and Baum and Crosby (2007) rearranged equation 9 such that $k = e - (SF \times p)$ where k and e retain their original definitions, SF equals annual sinking fund to replace the capital gain and p connotes the total rental growth over the rent review periods.

With t representing the standard rent review period, implied rental growth rate, g is defined as the tth root of $(1 + p)$ less unity:

$$g = \sqrt[t]{(1+p)} - 1 \tag{29}$$

The second approach for the calculation of implied rental growth rate is such that the ensuing equation is derived from equation 28 by subtracting k from the Right- and Left hand sides and then dividing both sides by the annual sinking fund factor to arrive at either equation 1 or 4 as presented earlier in this paper.

The third approach for the calculation of implied rental growth rate which they demonstrated is anchored on the DCF-based equated yield model in equation 27. Rationalizing that equation churns out equation 2 which many valuers are conversant with. In a related development, Fraser (1993) demonstrated how the Gordon growth model can be transformed into periodic growth model for property investments as follows:

If $P_0 = \frac{a}{e-g}$, on the condition that *e > g.*

Fraser (1993) established that the all risks yield formula (equations 9) ensues from the Gordon growth model provided *g* is replaced with the formula - $e\left(\frac{(1+g)^t - 1}{(1+e)^t - 1}\right)$. In addition, Fraser (1993) derived the formula for implied rental growth rate using the same parameters from the Gordon growth model to accommodate the periodic rent review pattern of property investment as captured in equation 4.

The only proximate attempt towards deriving all risks yield from the real value/equated yield model was demonstrated by Ifediora (2005) who commenced by setting up a DCF valuation of annuities subject to upward reviews at a growth factor, *g*. With reference to Figure 1, the ensuing annuities include 1, $(1+g)^t$, $(1+g)^{2t}$, $(1+g)^{3t}$,..........., and $(1+g)^{n-t}$ with DCF valuation presented as:

Annuity	Present value
First	Y.P. for t years @ e
Second	$(1+g)^t$ × Y.P. for t years @ e × P.V. of ₦1 in t years @ e
Third	$(1+g)^{2t}$ × Y.P. for t years @ e × P.V. of ₦1 in 2t years @ e
Fourth	$(1+g)^{3t}$ × Y.P. for t years @ e × P.V. of ₦1 in 3t years @ e
.	. . .
.	. . .
.	. . .
(n/t)th	$(1+g)^{n-t}$ × Y.P. for t years @ e × P.V. of ₦1 in (n – t) years @ e

Capital value of these annuities was expressed as the sum of geometric series with Y.P. for t years @ e as the first term and $(1+g)^t$ × P.V. of ₦1 in t years @ e as the common ratio expressed algebraically as

$\left(\frac{(1+g)}{(1+e)}\right)^t$. Therefore the Y.P. of a series of cash flow is expressed as -

$$Y.P. = Y.P. for\ t\ years\ @\ e \times \left(\frac{1-\left(\frac{1+g}{1+e}\right)^n}{1-\left(\frac{1+g}{1+e}\right)^t} \right) \qquad (30)$$

On condition that $1-\left(\frac{1+g}{1+e}\right)^n$ in equation 30 equals unity during the valuation of perpetual incomes, Ifediora (2005) derived all risks yield as:

$$k=\frac{e\left((1+e)^t-(1+g)^t\right)}{(1+e)^t-1} \tag{31}$$

While insights from these fundamental works indicate a number of approaches for the determination of implied rental growth rates and all risks yield, a cursory examination of equations 12 and 29 reveals algebraic relationships with the real value/equated yield hybrid (Crosby's 3-YPs) model which forms the foundation for achieving the aim of this study.

MODELLING ALL RISKS YIELD AND IMPLIED RENTAL GROWTH USING THE 3-YPS FORMULA

Deriving all risks yield from the 3-YPs formula

Crosby's real value/equated yield hybrid (3-YPs) model is given as:

$$Y.P.=Y.P. for\ n\ years\ @i\times\frac{Y.P. for\ t\ years\ @e}{Y.P. for\ t\ years\ @i} \tag{32}$$

The symbolic translation of equation 32 becomes:

$$Y.P.=\left(\frac{1-\frac{1}{(1+i)^n}}{i}\right)\times\left(\frac{1-\frac{1}{(1+e)^t}}{e}\right)\times\left(\frac{i}{1-\frac{1}{(1+i)^t}}\right) \tag{33}$$

In order to use the real value/equated yield hybrid model to derive the formula for all risk yield and implied rental growth rates for any investment property, it shall be assumed that the investment property in question is a freehold interest with possibility of upward rent reviews as depicted in Figure 1 above. This assumption further validates the notion that market rental growth accrues to the freehold property investor.

In conventional valuation practice, Years purchase in perpetuity is the reciprocal of the all risk yield, *k* such that Y.P. for *n* years @ *i* as expressed in equation 32 is replaced with 1/*i*, while the entire expression on the right hand side (RHS) of equation 33 is equated to 1/*k*

Therefore,

$$\frac{1}{k}=\frac{1}{\cancel{i}}\times\frac{1-\frac{1}{(1+e)^t}}{e}\times\frac{\cancel{i}}{1-\frac{1}{(1+i)^t}}$$ Conveniently reduces to:

$$\frac{1}{k}=\frac{1-\frac{1}{(1+e)^t}}{e}\times\frac{1}{1-\frac{1}{(1+i)^t}}$$

The expression for real return (inflation risk free yield), *i* in relation to *g* and *e* given as $\frac{(1+e)}{(1+g)}-1$ have been substituted for *i* in the equation obtained so that it further reduces to:

$$\frac{1}{k}=\frac{(1+e)^t-1}{\cancel{(1+e)^t}}\times\frac{1}{e}\times\frac{\cancel{(1+e)^t}}{(1+e)^t-(1+g)^t}=\frac{(1+e)^t-1}{e\left((1+e)^t-(1+g)^t\right)} \quad (34)$$

Therefore, the all risks yield is expressed as $k=\frac{e\left((1+e)^t-(1+g)^t\right)}{(1+e)^t-1}$ Which is synonymous to equation 31 in the analytical framework of this study. Alternatively, this all risks yield can be written as:

$$k=\frac{e}{(1+e)^t-1}\left\{(1+e)^t-(1+g)^t\right\} \quad (35)$$

Hence, equation 35 can be expressed in the usual format understood by most valuers as:

k = ASF in t years @ e × {Amount of ₦1 in t years @ e – Amount of ₦1 in t years @ g} (36)

Deriving implied rental growth rate from the 3-YPs formula

Within the existing framework of the 3-YPs model, equation 35 have been deployed to provide an alternative approach towards deriving implied rental growth rate, *g*.

If $k=\frac{e}{(1+e)^t-1}\left\{(1+e)^t-(1+g)^t\right\}$

By induction, $k=\frac{e\left((1+e)^t-(1+g)^t\right)}{(1+e)^t-1}$

Multiplying both sides of the equation by $(1+e)^t-1$ yields

$k\left\{(1+e)^t-1\right\}=e\left\{(1+e)^t-(1+g)^t\right\}$; so that

$$(1+g)^t = \frac{e(1+e)^t - k\{(1+e)^t - 1\}}{e}$$

Then *g* can be isolated using the formula:

$$g = \left\{ \sqrt[t]{(1+e)^t - k\left[\frac{(1+e)^t - 1}{e}\right]} \right\} - 1 \qquad (37)$$

It can be generally observed that equated yield and rent review period are the two variables that are common to the real value/equated yield hybrid model (equation 32), all risks yield (equation 35) and implied rental growth rate (equation 37).

Furthermore, these analogies have confirmed the relationship between Crosby's 3-YPs model, all risks yield, *k*, and implied rental growth rate, *g* and underscores the interdependence among key variables of contemporary value models which accounts for similarities in valuations ensuing from the application of these models. Attention shall now be turned towards the application of equations 35 and 37 in cases of freehold property investment.

SYNERGY OF VALUE MODEL WITH ALL RISKS YIELD AND IMPLIED RENTAL GROWTH RATE

The synergy between the real value model and the two parameters mentioned above shall be evaluated with recourse to specific valuation cases. Specifically, case 1 evaluates the synergy between implied rental growth rate per annum (equation 37) and the real value model (equation 32), while case 2 evaluates the synergy between all risks yield (equation 35) and the real value model (equation 32).

Case 1

This case pertains to valuation and analysis of a fully let freehold property. Preliminary data and valuation in Exhibit 1 indicates that the property commands a current net rental value of ₦510,000.00 per annum and subject to 2 yearly upward rent reviews. Market evidence shows that all risks yield on similar properties is 4.5%. Although the inflation risk free yield of the property is put at 3.730224591% and the investor's overall return is estimated at 17.8%, the investor wants to know the rental growth rate necessary to achieve these indices as well as the capital value of her interest in the property.

Exhibit 1: Valuation and Analysis of fully let freehold

Data

Current net rental value:		₦510,000.00 p.a.
Equated yield:		17.8%
All risk yield:		4.5%
Rent review period:	2 yearly	
Inflation risk free yield:		3.730224591%

Valuation

The real value/equated yield hybrid

Current net rental value (p.a.) .... ₦510,000.00

$$\frac{YP \text{ for 2 years @ } 17.8\% \times YP \text{ Perp @ } 3.73022459\ 1\%}{YP \text{ for 2 years @ } 3.73022459\ 1\%}$$ 22.22222222

Capital Value .... ₦ 11,333,333.33

Yield analysis

Initial yield:		4.5%
All risks yield:		4.5%
Yield on reversion:	Nil	
Equivalent yield:		4.5%
Equated yield:		17.8%
Inflation risk free yield:		3.730224591%

With recourse to equation 37, and available data showing $t = 2$, $k = 0.045$, and $e = 0.178$

Implied rental growth rate per annum, $g = \left\{ \sqrt{(1.178)^2 - 0.045\left[\frac{(1.178)^2 - 1}{0.178}\right]} \right\} - 1$

$g = 13.56381466\%$ p.a.

Table 1: Growth explicit DCF valuation of a fully let freehold property

Year	Net rent received ₦	Growth factor @ 13.56381466 %	Projected net rent ₦	Y.P. @ 17.8%	P.V. @ 17.8%	Present value of cash flow ₦
1 – 2	510,000	1.00000000	510,000.00	1.569521591	1.00000000	800,456.01
3 – 4	510,000	1.28967400	657,733.74	1.569521591	0.72062516	743,921.03
5 – 6	510,000	1.66325903	848,262.10	1.569521591	0.51930062	691,379.02
7 – 8	510,000	2.14506192	1,093,981.58	1.569521591	0.37422109	642,547.98
9 – Perp	510,000	2.76643059	1,410,879.60	22.22222222*	0.26967313	8,455,029.30
				Capital value		11,333,333.34

*Y.P. in perpetuity @ 4.5%

Commenting on the valuation in Exhibit 1 and the accompanying analysis, the implied rental growth rate necessary to avail the investor with the capital value of ₦11,333,333.33 besides the expected total return and other indices is put at 13.56381466% p.a. By induction, the synergy between the formula for implied rental growth rate (equation 37) and the real value model (equation 32) was evaluated with recourse to a discounted cash flow (DCF) valuation in Table 1.

It is observed from Table 1 that the implied rental growth rate of 13.56381466% p.a. shall produced a capital value of ₦11,333,333.33 in conjunction with the all risks yield and equated yield for Case 1. In other words, there is a synergy between the model of implied rental growth rate (equation 37) and the real value/equated yield hybrid model (equation 32) notwithstanding the errors of approximation in the DCF valuation to the tune of ₦0.01.

Case 2

Case 2 pertains to the valuation and analysis of a reversionary freehold interest. Preliminary data and valuation in Exhibit 2 reveals this property as commanding a net contract rent of ₦702,000.00 per annum subject to 3 yearly upward review, while the current net rental value of the property stands at ₦950,000.00 per annum. Implied rental growth of the subject property is 19.17105781% per annum, inflation risk free yield is 3.212979956%, while the investor expects an overall return of 23%. the investor wants to know the all risks yield which captures his expected total return, rental growth rate, and the capital value of the freehold interest.

Exhibit 2: Valuation of reversionary freehold interest

Data

Net rent passing:	₦ 702,000.00 p.a. subject to 3 yearly rent reviews
Current net rental value:	₦ 950,000.00 p.a. subject to 3 yearly rent reviews
Equated yield:	23%
Implied rental growth rate:	19.17105781%
Inflation risk free yield:	3.212979956%

Valuation

The real value/equated yield hybrid

		₦	₦
Term			
Rent received per annum		702,000.00	
Y.P. for 3 years @ 23%		2.011374268	1,411,984.74
Reversion			
Current Net rental value per annum		950,000.00	
$\frac{YP\ for\ 3\ years\ @\ 23\% \times YP\ Perp.\ @\ 3.21297995\ 6\%}{YP\ for\ 3\ years\ @\ 3.21297995\ 6\%}$ ...	22.22222222		
PV of ₦1 in 3 years @ 3.212979956%	0.909488157	20.2108479	19,200,305.51
Capital Value			20,612,290.25

Yield analysis

Initial yield:	3.405735082%
*All risks yield:	4.5%
Yield on reversion:	4.608900751%
Equivalent yield:	4.461239150%
Equated yield:	23%
Inflation risk free yield:	3.212979956%

*Computed with recourse to equation 35

Using equation 35 and available data showing g = 19.17105781%, e = 23%, and t = 3 years, the all risks yield, k was calculated as follows:

$$k = \frac{0.23}{(1.23)^3 - 1}\left((1.23)^3 - (1.19171057\ 81)^3\right)$$

k = 4.5%

Commenting on the valuation in Exhibit 2 and the accompanying analysis above, the all risks yield necessary to avail the investor with the capital value of ₦20,612,290.25 besides the expected total return and rental growth rate is put at 4.5%.

By induction, a DCF valuation has been carried out in Table 2 to reveal the synergy between the formula for all risks yield (equation 35) and the real value model (equation 32).

Table 2: Growth explicit DCF valuation of reversionary freehold property

Year	Net rent received ₦	Growth factor @ 19.17105781%	Projected rent ₦	Y.P. @ 23%	P.V. @ 23%	Present value of cash flow ₦
1 – 3	702,000	1.00000000	702,000.00	2.011374268	1.00000000	1,411,984.74
4 – 6	950,000	1.69243650	1,607,814.68	2.011374268	0.53738392	1,737,855.02
7 – 9	950,000	2.86434131	2,721,124.24	2.011374268	0.28878148	1,580,558.56
10 - Perp	950,000	4.84771578	4,605,329.99	22.22222222*	0.15518652	15,881,891.97
				Capital value		20,612,290.29

*Y.P. in perpetuity @ 4.5%

Notwithstanding approximation error to the tune of error of ₦0.04, inference drawn from the DCF valuation in Table 2 is that the all risks yield (exit yield) of 4.5% contributed towards determining a capital value of ₦20,612,290.25 when used in conjunction with the implied rental growth rate and equated yield for Case 2. This result reaffirms the synergy between the real value/equated yield hybrid model (equation 32) and the model for the determination of all risks yield (equation 35) which have been derived from the same 3-YPs model in equation 32.

Results from the valuation cases in this paper aligns with previous scholarly works on the synergy among the contemporary value models (Ajayi, 1998; Baum & Crosby, 2007; Brown & Matysiak, 2000; Butler & Richmond, 1990; Crosby, 1986a, 1986b, 1996; Crosby et al., 1997; Udo, 1989). Most importantly, the implied rental growth rate per annum and the all risks yield are embedded in the real value/equated yield hybrid model notwithstanding that this contemporary value model is silent about them. Instead, the model concentrates on the real value of cash inflows through the explicit application of real rate of return (inflation risk free yield) which is derived from an interplay of these two embedded parameters. Finally, it has been observed that the real value/equated yield hybrid model deploys two major variables notably equated yield and rent review period to implicitly capture rental growth rate per annum and the all risks yield of an investment property.

CONCLUSION

This study provided an alternative analysis of how all risks yield and implied rental growth are embedded in the real value/equated yield hybrid model (3-YPs model). This was achieved by deriving these parameters from the 3-YPs model, and showing the synergy among them following the reconciliation of growth explicit valuations of fully let- and reversionary freehold interests involving the use of these parameters. Specifically, the real value/equated yield hybrid model of property investment valuation share two common variables with all risks yield and implied rental growth rate inasmuch as they remain implicit or embedded in the model. These variables include equated yield and rent review period. In view of these results, it can be concluded that all risks yield and implied rental growth rate are embedded in the equated yield hybrid model of property investment valuation. This conclusion is founded on the synergy between these two parameters and the real value/equated yield hybrid model which formed the basis of their derivation in this paper.

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Biographical Notes
Mr. Joseph O. Ataguba is a Lecturer in the Department of Estate Management and Valuation at the Federal Polytechnic, Idah, Nigeria. His research interests encompass real estate investment, contemporary valuation models, and computerized approaches to real estate and land administration practice.

Mr. Anthony A. Tinufa is a Lecturer in the Department of Estate Management and Valuation at the Federal Polytechnic, Idah, Nigeria where he teaches Valuation and Compulsory acquisition and compensation Law.