Journal of Engineering and Applied Scientific Research
ISSN: 2384-6569
Volume 9, Number 1, 2017

PERFORMANCE OF ALL NIGERIA BANKS' SHARES USING STUDENT-*T* MIXTURE AUTOREGRESSIVE MODEL

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Abstract: This study attempts to model the all Nigeria banks' volume of shares using the Student-t Mixture Autoregressive (TMAR) and the Mixture Autoregressive (MAR) models because the series has been characterized by fluctuation, excessive kurtosis and excessive skewness that was justified by the Degree of Freedom (DF)that was greater than four, that is, $(v_t > 4, t = 1, \cdots, 5)$ in each regime of the TMAR model. The TMAR model substituted the Student-t Probability Density Function (PDF) for the error term in contrast to Gaussian used by MAR in order to cater for positive or negative excess kurtosis that might have distorted the parameters' estimation in MAR. Though the stylized traits of the shape changing means, variances (volatilities), and conditional distributions of the two models were still maintained. The approach adopted the E-step and M-step (Expected and Maximization) iterative procedure in parameters' estimation and detection of kurtosis whenever the DF is greater than four. The all Nigeria banks' shares were fitted using TMAR and MAR models, the density function plot of all Nigeria banks' shares between 4 January, 2007 and 20 April, 2015 revealed a 5-regime shift and the best model was recorded at MAR (5: 2, 1, 6, 2, 2) and TMAR (5: 2, 1, 6, 2, 2). Apart from the fact that the standard errors of estimates from TMAR model were smaller compared to standard errors of estimates from MAR model, the TMAR model out-performed the MAR model with minimum Akaike Information Criterion and Bayesian Information Criterion of (438.98 and 454.54) and (445.84 and 459), respectively. In addition, the TMAR model recorded a lesser predictive error of Residual Mean Square Error (RSME) of 8.2960 compared

to10.6061 recorded for MAR. Lastly, the banks' shares recorded a rapid change and unpredictably higher volatility (risk) of 10.4370 for TMAR and 20.6902 for MAR in the third and fifth regimes compared to other three regimes with lesser risk.

Keywords: Nigeria All Banks' Shares, Student-t Mixture Autoregressive (TMAR), Mixture Autoregressive (MAR), Probability Density Function (PDF) and Residual Mean Square Error (RMSE)

Received for Publication on 5 April 2017 and Accepted in Final Form 11 April 2017

INTRODUCTION

Financial return series are suitably described by non-linear time series models because of their ability to capture complex stylized traits in financial market data such as exchange rates, interest rates, crude oil prices, bank shares, stock indices, and so on, while linear time series models such as Autoregressive (AR), Moving Average (MA) Autoregressive Moving Average (ARMA) lacked the ability to capture traits like conditional distributions with time-varying dependency variances. Although, some non-linear time series models such as Generalized Autoregressive Conditional Heteroscedasticity (GARCH), Self-Exciting Threshold Autoregressive (SETAR) relaxed these properties the linear time aforesaid failed to treat but not all, among the stylized properties entailed by regime-switching models that some non-linear time series models lacks its acumen are outburst, change points like behavior, time-varying volatilities (conditional variances), time-varying mixing weights, full range of shape changing predictive distributions (multimodalities) and ability to handle cycles (Wong & Li, 2000). In other words, each of the regimes has its own marginal conditional distribution (f_t) with its corresponding Cumulative Distribution Function (CDF) (F_t) which is a replica of the immediate past distribution such that both the conditional mean and conditional variance in each component depends on the immediate past time. Wong (2000) proposed a regime-switching model called Mixture Autoregressive (MAR) model that relax all these mentioned stylized properties while Boshnakov (2006), as well as Ojo& Olanrewaju (2016) had scholarly worked on the application and modification of MAR model but took less cognizance about the time-varying of either the long-tailed or short-tailed of the conditional distributions that would definitely have distorted the ability of the model to capture serial correlation, time-varying means and volatilities via the normal distribution. Moreover, financial data are always characterized with excessive kurtosis and excessive skewness that will not only distort the normality assumption of the white noise but also affect the time-varying leptokurtic or time-varying platykurtic marginal conditional distributions (Wong, 1998). A heavy-tailed distribution is needed to capture excessive kurtosis and skewness that is being characterized by financial time series data that do affect the normality assumption of the Gaussian distribution such that the tails of each component (modal) will be adjusted for a more flexibility model than that of MAR model.

Wong, et al (2009) extended the MAR frame work by replacing the white noise (error term) with Student-t distribution such that the mixture of each components was governed by Student-t distribution, that is Student-t Mixture Autoregressive Model (TMAR). They stressed further that the MAR model will not only underestimate but also overestimate the mixing probabilities of the component membership of the mixture model. They maintained that the TMAR model will give a more robust estimation of parameters in contrast to MAR model. In light of their research, it was deduced that the MAR model is a limiting case of the TMAR model because the estimation of component variances and degrees of freedom will be separated which corresponds to the modeling of variability and shapes of conditional distribution respectively.

Both linear and non-linear time series models have been applied to the Nigeria banks' volume of shares in the last decade but it was gathered that the flexibility and the extent to which conditional variability exist over few trial (Recession and blooming periods) periods have not been ascertained. So, the TMAR and MAR model will be applied to the Nigeria Banks' volume of shares in order to know the numbers of regime shift the financial series has maneuvered with the corresponding conditional variability of each cycle.

MATERIALS AND METHODS

Wong et al (2009) defined a g – component student- t Mixture Autoregressive (TMAR) model to be

$$F\left(y_t / f_{t-1}\right) = \sum_{k=1}^{g} \alpha_k F_{vk}\left(\frac{y_t - \phi_{k0} - \phi_{k1} y_{t-1} - \cdots - \phi_{k p_k} y_{t-p_k}}{\sigma_k}\right)$$

Alternatively,

$$y_t = \begin{cases} \phi_{1,0} + \sum_{i=1}^{p_t} \phi_{1,t}\, y_{t-1} + \sigma_1 \varepsilon_1 & w.r.t\ \alpha_1 \\ \phi_{2,0} + \sum_{i=1}^{p_t} \phi_{2,t}\, y_{t-1} + \sigma_2 \varepsilon_2 & w.r.t\ \alpha_2 \\ \vdots \quad \vdots \quad \vdots & \vdots \\ \phi_{p,0} + \sum_{i=1}^{p_t} \phi_{p,t}\, y_{t-1} + \sigma_p \varepsilon_p & w.r.t\ \alpha_g \end{cases}$$

Where the mixing weight $\alpha_1 + \alpha_2 + \cdots + \alpha_g = 1$, $\alpha_k > 0$ and $v_k > 2$ for $k = 1, \cdots, g$. The model is usually denoted by TMAR$(g : p_1, \cdots, p_g)$. Where $F(y_t / f_{t-1})$ is the Cummulative Distribution Function (CDF) of Y_t given the past information, evaluated at y_t ; f_t is the information set up to ; $F_{vk}(.)$ is the CDF of the standardized student- t distribution with v_k degree of freedom. The probability distribution function of the student- t distribution with unit variance and "v" degrees of freedom is

$$f_v(y_t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\sqrt{\left(\frac{v}{2}\right)}}\left\{1+\frac{y_t^2}{v-2}\right\}^{-\frac{v+1}{2}} \qquad y_t \varepsilon$$

Such that $2 < v < \infty, \Gamma(.)$ *gamma function* and $F_v(y_t) = \int_0^\infty f_v(y_t)\,\partial y_t$

Condition Mean and Variance of TMAR

$$E(y_t / f_{t-1}) = \sum_{i=1}^{k} \alpha_k E(y_t) = \sum_{i=1}^{k} \alpha_k E(\phi_k y_{t-k} + \varepsilon_t) = \sum_{i=1}^{k} \alpha_k (\phi_k y_{t-k}) + E(\varepsilon_t)$$

Since $E(\alpha_k) = 0, \quad E(\phi_k y_{t-k}) = \mu_{t-k}$

So,
$$E(y_t / F_{t-1}) = \sum_{i=1}^{k} \alpha_k \mu_{t-k}$$

The variance,

$$Var(y_t / F_{t-1}) = E(y_t / F_{t-1})^2 - \left[E(y_t / F_{t-1})\right]^2$$

But,

$$E(y_t / F_{t-1})^2 = E\left[\sum_{i=1}^{k}\left(\alpha_k(\phi_k y_{t-k} + \varepsilon_t)\right)\right]^2 = E\left[\sum_{i=1}^{k} \alpha_k (\phi_k y_{t-k})^2 + 2\sum_{i=1}^{k} \alpha_k (\phi_k y_{t-k})\varepsilon_t + \sum_{i=1}^{k} \alpha_k \varepsilon_t^2\right]$$

Since $E(\varepsilon_t) = 0, \quad E(y_{t-k}\varepsilon_t) = 0 \ \forall k = 0, \quad E(\varepsilon_t^2) = \sigma^2$

$$= \sum_{i=1}^{k} \alpha_k \mu_{t-k}^2 + \sum_{i=1}^{k} \alpha_k \sigma_k^2$$

So,

$$Var(y_t / F_{t-1}) = \sum_{i=1}^{k} \alpha_k \mu_{t-k}^2 + \sum_{i=1}^{k} \alpha_k \sigma_k^2 - \left[E\left(\sum_{i=1}^{k} \alpha_k \mu_{t-k}\right)\right]^2 = \sum_{i=1}^{k} \alpha_k \sigma_k^2 + \sum_{i=1}^{k} \alpha_k \mu_{t-k}^2 - \sum_{i=1}^{k}\left(\alpha_k \mu_k\right)^2$$

The mean affirmed the shape of the conditional distributions may change from uni-modal to multi-modal while the variance confirmed the ability of the model in changing conditional variance.

Stationary Conditions of TMAR Model

Wong et al. (2009) proposed that

(i) A necessary and sufficient condition for the process y_t to be first-order stationary is that the roots $z_1, \ldots, z_p$ of the equation $1 - \sum_{i=1}^{p}\left(\sum_{k=1}^{g} \alpha_k \phi_{ki}\right) z^{-i} = 0$ all lie inside the unit circle, where $\phi_{ki} = 0$ for $i > p_k$.

(ii) A necessary and sufficient condition of a stationary for the process to be second-order stationary is $\alpha_1\phi_{11}^2+\cdots+\alpha_g\phi_{g1}^2<1$.

(iii) The fourth-order moment of a stationary of TMAR model with $\phi_{k0}=0(k=1,\cdots,g)=0$, exists if and only if $v_k\geq 4$. The existence condition of the fourth-order moment for the model in (1) is the same as those for the student-*t* distribution. If $v_k\geq 4$, the expression for the fourth-order moment is given by

$3\sum_{k=1}^{g}\frac{\alpha_k\sigma_k^4(v_k-2)}{(v_k-4)}$. It implies that the kurtosis of y_t is generally greater than 3.

Parameter Estimation for TMAR Model

Dempster et al. (1997) and McLachlan & Krishnan (1997) introduced a two-step (EM algorithm) procedure for parameter estimation in a non-linear log-likelihood of a model. Wong et al. (2009) also adopted the EM algorithm in solving the parameters in model in equation (1) via the log-likelihood constructed using the normal scale mixture model. Assuming that the observations $Y=(y_1,y_2,\ldots,y_n)$ generated from the model in equation (1). Let $Z=(z_1,\ldots,z_n)$ be a $g\,x\,n$ unobservable weight $\alpha=(\alpha_1,\ldots,\alpha_{g-1})^T$, the degree of freedom $v=(v_1,\ldots,v_g)^T$, $\theta=(\theta_1^T,\ldots,\theta_g^T)^T$ with $\theta_k=(\phi_{k0,},\ldots,\phi_{kp_k})^T$ random matrix, where $Z=Z_{kt}(t=1,\cdots,n)$, is a g-dimensional column indicator vector showing the origin of the K^{th} observation, that is, $Z_{kt}=1$, if y_t is generated from the K^{th} component of the model and $Z_{kt}=0$ otherwise.

Considering another missing random matrix $W=(W_1,\ldots,W_n)$, where $W=(W_{kt})(t=1,\cdots,n)$ is also a g-dimensional vector. Given $Z_{tk}=1$, the conditional distribution of W_{kt} is $\{W_{kt}/Z_{kt}=1\}\sim gamma\left\{\frac{v_k}{2},\frac{(v_k-2)}{2}\right\}$ and $W_1,\ldots,W_n$ are distributed independently. Let the mixing

such that the vector parameters is defined as $\Psi=(\alpha^T,\theta^T,v^T)^T$

So, the conditional log-likelihood of (1) I additive such that

$$\ell=\ell_1(\alpha)+\ell_2(v)+\ell_3(\theta) \quad (3)$$

Where,

$$\ell_1(\alpha) = \sum_{k=1}^{g} \sum_{t=p+1}^{n} Z_{kt} \log(\alpha_k)\text{ ,}$$

$$\ell_2(\alpha) = \sum_{k=1}^{g} \sum_{t=p+1}^{n} Z_{kt} \left[-\log\left\{ \Gamma\left(\frac{1}{2}v_k\right)\right\} + \left(\frac{1}{2}v_k\right)\log\left(\frac{v_k-2}{2}\right) + \left(\frac{1}{2}v_k\right)\log((W_{kt}) - W_{kt}) - \log(W_{kt}) + W_{kt} \right]$$

,

$$\ell_3(\theta) = \sum_{k=1}^{g} \sum_{t=p+1}^{n} \left[-\frac{1}{2}\left\{\log(2\pi) + \log\sigma_k^2 - \log(W_{kt})\right\} - \frac{e_{kt}^2 W_{kt}}{2\sigma_k^2} \right]$$

Such that,

$$e_{kt} = y_t - \phi_{k0} - \phi_{k1} y_{t-1} - \cdots - \phi_{k\,pk} y_{t-pk}$$

$$\frac{\partial \ell_1}{\partial \alpha_k} = \sum_{t=p+1}^{n} \left(\frac{Z_{kt}}{\alpha_k} - \frac{Z_{gt}}{\alpha_g} \right) \quad (k = 1, \cdots, g-1)\text{ ,}$$

$$\frac{\partial \ell_2}{\partial v_k} = \sum_{t=p+1}^{n} Z_{kt} \left[-\frac{1}{2}\psi\left(\frac{1}{2}v_k\right) + \frac{1}{2}\log\left(\frac{v_k-2}{2}\right) + \frac{1}{2}\left(\frac{v_k-2}{2}\right) + \frac{1}{2}\left\{\log(W_{kt}) - W_{kt}\right\} \right] \quad (k = 1, \cdots, g)\text{ ,}$$

$$\frac{\partial \ell_3}{\partial \phi_{ki}} = \sum_{t=p+1}^{n} \frac{Z_{kt} W_{kt} u(y_t, i) e_{kt}}{\sigma_k^2} \quad (k = 1, \cdots, g;\ i = 0, \cdots, p_k)$$

The parameters in (3) will be estimated by maximizing the log-likelihood via the iterative EM procedure.

E-STEP: Assuming that Ψ is known. The missing data Z, W and log W in the log-likelihood are replaced by their expectations, conditional over the parameters and observed data Y.

$$\tau_{kt} = E_\Psi(Z_{kt} / y_t) = \frac{\alpha_k \sigma_k^{-1} f v_k\left(\frac{e_{kt}^2}{\sigma_k^2}\right)}{\sum_{j=i}^{g} \alpha_j \sigma_j^{-1} f v_j\left(\frac{e_{kt}^2}{\sigma_k^2}\right)} \quad (k = 1, \ldots, g)$$

$$\eta_{kt} = E_\Psi(W_{kt} / y_t, Z_{kt} = 1) = \frac{v_k + 1}{\frac{e_{kt}^2}{\sigma_k^2} + (v_k - 2)} \quad (k = 1, \ldots, g)$$

$$E_\Psi(\log W_{kt} / y_t, Z_{kt} = 1) = \log \eta_{kt} + \left\{ \psi\left(\frac{v_k+1}{2}\right) - \log\left(\frac{v_k+1}{2}\right) \right\} \quad (k = 1, \ldots, g)$$

where

$\psi(s) = \frac{\{d\Gamma(s)/ds\}}{\Gamma(s)}$ is the Digamma distribution.

M-STEP: Assuming that the missing data are known. By maximizing the log-likelihood function in (1) . The estimates of the mixing proportions and component standard deviations are:

$$\alpha_k = \frac{\sum_{t=p+1}^{n} \tau_{kt}}{n-p} \qquad (k = 1,\dots,g)$$

$$\sigma_k = \left(\frac{\sum_{t=p+1}^{n} \tau_{kt}\, \eta_{kt}\, \hat{e}_{kt}^{2}}{\sum_{t=p+1}^{n} \tau_{kt}} \right)^{\frac{1}{2}} \qquad (k = 1,\dots,g)$$

Where, $\phi_{k0}, \phi_{k1}, \dots, \phi_{k\,pk}$, $(k = 1,\dots,g)$ estimates are to be obtained by solving the following system of equations.

$$\sum_{t=p+1}^{n} \tau_{kt}\, \eta_{kt}\, y_t\, u(y_t, i) = \sum_{j=0}^{p_k} \phi_{kj} \sum_{t=p+1}^{n} \tau_{kt}\, \eta_{kt}\, y_t\, u(y_t, i) \quad (k = 1,\dots,g; i = 0,\dots p_k)$$

where $u(y_t, i) = 1$ for $i = 0$ and $u(y_t, i) = y_{t-i}$ for $i > 0$.

The estimates of the degree of freedom must satisfy the following equations,

$$\left(\frac{v_k}{v_k - 2} \right) + \log\left(\frac{v_k - 2}{2} \right) + \psi\left(\frac{v_k^{(m)} + 1}{2} \right) + -\log\left(\frac{v_k^{(m)} + 1}{2} \right) + \frac{1}{\sum_{t=p+1}^{n} \tau_{kt}^{(m)}} \sum_{t=P+1}^{n} \tau_{kt}^{(m)} (\log \eta_{kt}^{(m)} - \eta_{kt}^{(m)}) = 0 \quad (k = 1,\dots,g)$$

$v_k^{(m)}$ is the estimated v_k in the m^{th} iteration of the EM algorithm. The estimates of the parameters are then obtained by iterating the two steps until convergence is achieved.

Model Selection Criteria for TMAR Model

The AIC and BIC selection criteria for MAR depends on the maximum (observed) log-likelihood automatically generated by an EM estimation.

$$l = \sum_{t=p+1}^{n} \log\{f(y_t / F_{t-1})\} = \sum_{t=p+1}^{n} \log\left\{\frac{d}{dy_t} F(y_t / F_{t-1})\right\}$$

Wong and Li (2000), proposed the selection criteria for the two criteria to be

$$BIC = -2l + \log(n-p) + \left(4g - 1 + \sum_{k=1}^{g} p_k\right)$$

$$AIC = -2l + 2\left(4g - 1 + \sum_{k=1}^{K} p_k\right)$$

Experimental Work

The data used in this research was the all Nigeria banks volume of shares from 4th January 2007 to 20th April 2015.The data set was extracted from the Nigeria Stock Exchange (NSE), such that two thousand and forty-four (2044) data points were extracted from the website.

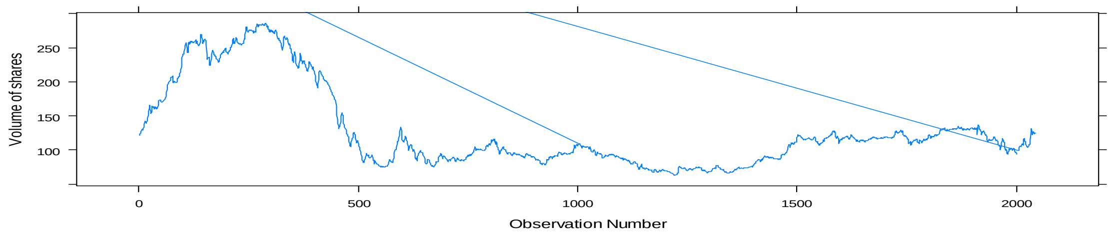

Figure 1: Time Plot of the original series of the Nigeria all banks volume of shares.

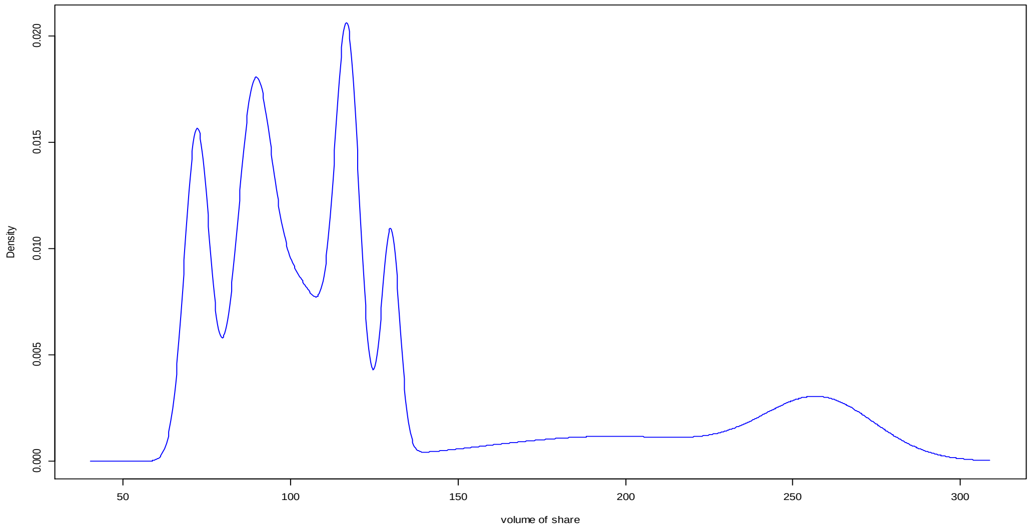

Figure 2: Graph of the Kernel Densities plot of the volume of shares.

From Figure 2, it is obvious that the first four (regimes) curvatures possessed 'thin-tailed' (high peaked) curve, implying **Leptokurtic distributions**, that is kernel densities with higher positive kurtosis that clustered around their means, in other words, the first four curvatures are affected by excess positive kurtosis while the last kernel density possessed a 'fat-tailed' curve (**Platykurtic in nature**), meaning ,the last regime was influenced by a negative excess kurtosis. This suggested that models allied with Normal distribution might not describe the data well.

Table 1. Parameters Estimated for the MAR (5: 2, 1, 6, 2, 2) model

K	Mixing weight α_k	σ_k	$\phi_{k\ 1}$	$\phi_{k\ 2}$	$\phi_{k\ 3}$	$\phi_{k\ 4}$	$\phi_{k\ 5}$	$\phi_{k\ 6}$	AIC	BIC	RSME
1	0.1472 (0.0322)	3.8048	1.4347 (0.0516)	-0.4374 (0.0517)					445.84	459.9	10.6061
2	0.1808 (0.0403)	5.1566	0.9992 (0.0627)								
3	0.2163 (0.0521)	10.4370	0.3425 (0.0575)	-0.5889 (0.0963)	0.2932 (0.1021)	-0.1027 (0.1020)	0.1796 (0.0964)	-0.1451 (0.0575)			
4	0.1573 (0.0344)	3.5715	1.2245 (0.0500)	-0.2290 (0.05)							
5	0.2983 (0.0546)	20.6902	0.1492 (0.0399)	-0.1611 (0.0399)							

Table 2: Parameters Estimated for the TMAR (5: 2, 1, 6, 2, 2) model

K	v_k (d.f)	Mixing weight α_k	σ_k	$\phi_{k\ 1}$	$\phi_{k\ 2}$	$\phi_{k\ 3}$	$\phi_{k\ 4}$	$\phi_{k\ 5}$	$\phi_{k\ 6}$	AIC	BIC	RSME
1	4.4560 (1.2923)	0.1500 (0.0300)	3.8048	1.4100 (0.0490)	-0.5013 (0.0501)					438.98	454.54	8.2960
2	4.2310 (1.5672)	0.1859 (0.0456)	5.1566	0.8022 (0.0602)								
3	4.4190 (1.4563)	0.2121 (0.0642)	10.4370	0.2987 (0.0515)	-0.6431 (0.0901)	0.2846 (0.1000)	-0.1027 (0.1020)	0.1738 (0.0941)	-0.1891 (0.0501)			
4	4.0724 (1.8951)	0.1564 (0.0471)	3.5715	1.2121 (0.0429)	-0.3287 (0.0481)							
5	4.2349 (1.9024)	0.2955 (0.0548)	20.6902	0.1490 (0.0289)	-0.2876 (0.0165)							

****In parentheses are the standard errors of estimates

$$TMAR(5:2,\,1,\,6,\,2,\,2)=F(y_t\,|\,f_{t-1})=0.1500\,F_{4.4560}\left(\frac{y_t-1.410y_{t-1}+0.5013y_{t-2}}{3.8048}\right)+0.1859F_{4.2310}\left(\frac{y_t-0.8022y_{t-1}}{5.1566}\right)$$
$$+0.2121\mathrm{F}_{4.4190}\left(\frac{y_t-0.2987y_{t-1}+0.6431y_{t-2}-0.2846y_{t-3}+0.1027y_{t-4}-0.1738y_{t-5}+0.1891y_{t-6}}{10.4370}\right)$$
$$+0.1564\mathrm{F}_{4.0724}\left(\frac{y_t-1.2121y_{t-1}+0.3287y_{t-2}}{3.5715}\right)+0.2955F_{4.2349}\left(\frac{y_t-0.1490y_{t-1}+0.2876y_{t-2}}{20.6902}\right)$$

It is stubbed and obvious from figure 1 that the original series of the volume of shares is non-stationary and that regime one and regime four possessed a non-stationary process but Wong & Li (2000) ascertained that non-stationary of the original series would not pose a distortion to parameter estimation since combining stationary regime(s) with non-stationary regime(s) still makes the overall process stationary. Since the degree of freedom for each regime is greater than four, that is $v_k > 4,\ for\ k = 1, 2, 3, 4, 5$ kurtosis exist in this TMAR model. Having fitted the data to both TMAR and MAR with both models characterized with shape changing mean, shape changing variance (change point detection) and shape changing conditional distributions it was noted that apart from the smaller standard errors of estimates in TMAR model to that of estimates in

MAR model due to distortion in parameters, TMAR model was able to capture and accommodate excess kurtosis the white noise (Gaussian) MAR cannot detected. In term of the model performance, AIC= 438.98 and BIC=454.54 for TMAR model was smaller than that of the AIC= 445.84 and BIC=459.9 for MAR model, which makes the TMAR model better. Also, in terms of the error or fluctuation that might arises from their forecasting techniques, the Residual Mean Square Error (RMSE) of 8.2960 and 10.6061 for TMAR model and MAR model respectively were recorded, this makes TMAR model to have lesser forecasting error compare to MAR.

CONCLUSION

Having considered the model performance for the two mixture models, it was noted that the TMAR model out-performed the MAR model in the presence of excess kurtosis because subjecting a normally (mesokurtic) distributed white noise to MAR model parameters' estimation and volatilities might overestimate at high levels of significance if the return distribution is leptokurtic . Having said this, it was also noted that the volatility in regime five and regime three (20.6902 and 10.4370), respectively, were higher compare to other regime one, regime two and regime four (3.8048, 5.1566 and 3.5715), respectively. By implication, banks' shares changed rapidly and unpredictably (worsened) in the fifth regime (around 2015) and in the third regime (2013).

Lastly, the predictive power of the TMAR model has a lesser forecasting error compared to the predictive power of MAR model.

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Reference to this paper should be made as follows: J. F. Ojo and R.O. Olanrewaju (2017), Performance of All Nigeria Banks' Shares Using Student- *T* mixture Autoregressive Model. *J. of Engineering and Applied Scientific Research*, Vol. 9, No. 1, Pp. 69 - 82