GAUSSIAN WHITE NOISE PROCESS IN RADIOLOGY AND MEDICAL IMAGING

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ABSTRACT

This paper highlights the impact of noise on diagnostic image quality and the methods used to de-noise the images for improved quality and better clinical diagnosis. The various types of noise sequences interfering with the diagnostic image especially, the Gaussian white noise was discussed. Since the first two moments of a normally distributed process is sufficient to characterize it completely, the mean and variance of higher powers of the linear Gaussian white noise process $Y_t = X_t^b,\ b = 1, 2, 3, ...$ were determined. The results show that for fixed σ, increase in power of the process leads to increase in variance. As noise production is inherent in the modality for the image acquisition, and against the backdrop that repeat studies have some unpalatable economic, health and social consequences, we recommend that health managers should rather, go for equipments with high signal to noise ratio when procuring medical imaging equipments and should at all times hire the services of quality assurance personnel.

Keywords: Gaussian white noise, time series, image quality, medical diagnosis, computed tomography.

INTRODUCTION

Noise processes are known to constitute a nuisance in medical image processing. They degrade the final image if nothing is done to eliminate or reduce the noise process. Simply put, image noise is the random variation in the image brightness arising from photon fluctuations in the image recording medium. Image quality is an important topic of discuss in the field of radiography and medical imaging. The main purpose of medical imaging is to produce images with clear and detail outlines of the anatomical structures of the body part of interest to enhance medical diagnosis. The presence of noise in the image suppresses structural details thus making it difficult to achieve accurate medical diagnosis. But, the methods of acquisition of the images make them vulnerable to noise interferences. Images acquired by digital processing as is the case in ultrasound imaging, digital radiography, Computed Tomography (CT), Magnetic Resonance Imaging (MRI), etc. are more vulnerable to noise interference. In fact, noise formation is part and parcel of medical images production because of the various modes of acquisition of these images. It is very important that the noise recorded in the image is very low or eliminated completely in order to enhance image quality and medical diagnosis. Modes of acquisition of medical images in radiography and imaging include conventional x-ray imaging (radiography) in which an x-ray film, sandwiched in between two intensifying screens, is exposed to emergent x-ray photons from the patient's body part, and fluoroscopy in which signals from the patient's body part are made to pass through a system of cameras, photomultipliers, closed circuit television (CCTV) before being displayed on the visual display unit (VDU) and followed by radiography (x-ray imaging). Other methods of image acquisition are Computed Tomography (CT), Magnetic Resonance Imaging (MRI), digital radiography, digital mammography, diagnostic ultrasound imaging, nuclear medical imaging with Single Photon Emission Computed Tomography (SPECT), and

Positron Emission Tomography (PET) (Prudhvi and Venkateswarlu, 2012). These different methods of image acquisition are associated with production of different types of noise components of different intensities and frequencies. The types of noise commonly seen in medical imaging include the quantum noise that is common to conventional radiography, the speckle and Poisson noises which are generally referred to as multiplicative noise because their variance is not constant but depends on the parameters of the noise model to be estimated (Sanches et al., 2008). These types of noises occur, in varying degrees, in imaging modalities such as ultrasound scanning (Burckhardt, 1978), PET/SPECT (Ollinger and Fessler, 1997), functional MRI (Hagberg et al., 2001) and fluoroscopy. The Gaussian white noise is commonly associated with computed tomography (Gravel et al., 2004) and low intensity MRI (Bao and Zhang, 2003). Image processing in conventional radiography and fluoroscopy produces very minimal noise interference that compromises in the image quality is more as a result of chemical processing fault, positioning fault, processing equipment fault and film handling faults (Arimie, 2012). On the other hand, digital image processing as is the case with Computed Tomography (CT), Magnetic Resonance Imaging (MRI), digital radiography, digital mammography, diagnostic ultrasound imaging, nuclear medical imaging with Single Photon Emission Computed Tomography (SPECT), and Positron Emission Tomography (PET) produces high degrees of noises that interfere with the image quality. Our objective in this paper is to examine the Gaussian white noise process which is the main noise process associated with the modes of image acquisition and processing commonly found in health care facilities in Nigeria. The distribution of the noise process and its moments and higher moments would be examined. Also to be examined are methods used to eliminate or reduce noises in the diagnostic images for improved image quality.

GAUSSIAN WHITE NOISE PROCESSES

Gaussian noise is a statistical noise sequence with probability density function equivalent to that of the normal (Gaussian) distribution. In other words, the values that the noise can take on are Gaussian-distributed (Khera and Malhotra, 2014). A special case of the Gaussian noise is the linear Gaussian white noise process which, in addition to being distributed normally, is identically distributed and statistically independent (and hence uncorrelated). It is assumed to have zero mean and finite variance $\sigma^2 < \infty$. Linear Gaussian white noise processes are important class of stationary processes that form the building blocks of most time series processes. In statistics, the linear Gaussian white noise process, denoted by e_t, $t \in Z$, is defined (Brockwell and Davies, 2002; Greene, 2005; Brooks, 2013) as having

$$E(e_t) = \mu \tag{2.1}$$

$$E(e_t^2) = Var(e_t) = \sigma^2 \tag{2.2}$$

$$R(k) = Cov(e_t, e_{t+k}) = \begin{cases} \sigma^2, & k = 0 \\ 0, & k \neq 0 \end{cases} \tag{2.3}$$

$$\rho_k = \frac{R(K)}{R(0)} = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases} \tag{2.4}$$

Where R (k) is the auto-covariance function and ρ_k is the autocorrelation function at lag k. The partial autocorrelation function is

$$\phi_{kk} = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases} \tag{2.5}$$

As a normally distributed process, the linear Gaussian white noise process is completely characterized by its first two moments – the mean and variance (Kunst, 2004; Iwueze, 2006).

HIGHER MOMENTS OF THE LINEAR GAUSSIAN WHITE NOISE PROCESS

For the linear Gaussian white noise process, $X_t = e_t,\ t \in Z,\ e_t \sim iid\ N(0, \sigma^2)$ we define the nth central moment of the process as

$$E(e^b) = \begin{cases} (2a-1)!!\sigma^{2a}, & b = 2a,\ b,\ even \\ 0, & b,\ odd \end{cases} \quad (2.6)$$

$$where, (2a-1)!! = \prod_{c=1}^{a} (2c-1) \qquad (Ibrahim,\ 2013)$$

Then, the even moments are as shown in Table 2.1 and all the odd moments are zero. The expansion of the product $\prod_{c=1}^{a}(2c-1)$ is shown in Table 2.2.

Table 2.1: The Even Moments of the Linear Gaussian White Noise Process

b	a = b/2	$E(e^b)$
2	1	σ^2
4	2	$3\sigma^4$
6	3	$15\sigma^6$
8	4	$105\sigma^8$
10	5	$945\sigma^{10}$

Table 2.2: Expansion of the Product $\prod_{c=1}^{a}(2c-1)$

b	a	$\prod_{c=1}^{a}(2c-1)$
2	1	(1) = 1
4	2	(1 * 3) = 3
6	3	(1 * 3 * 5) = 15
8	4	(1 * 3 * 5 *7) = 105
10	5	(1 * 3 * 5 * 7 * 9) = 945
12	6	(1 * 3 * 5 * 7 * 9 * 11) = 10,395
14	7	(1 * 3 * 5 * 7 * 9 * 11 * 13) = 135,135
16	8	(1 * 3 * 5 * 7 * 9 * 11 * 13 * 15) = 2,027,025
18	9	(1 * 3 * 5 * 7 * 9 * 11 * 13 * 15 * 17) = 34,459,425
20	10	(1 * 3 * 5 * 7 * 9 * 11 * 13 * 15 * 17 * 19) = 654,729,075

Let $Y_t = X_t^b,\ b = 1,2,3,......$ be the higher moment of the linear Gaussian white noise process where, $X_t = e_t$.

$$E(Y_t) = E(X_t^b) = E(e_t^b) \quad (2.7)$$

$$Var(Y_t) = Var(X_t^b) = Var(e_t^b)$$

$$= E[(e_t^b)^2] - [E(e_t^b)]^2 \quad (2.8)$$

Using Table 2.1 and equations (2.7) and (2.8) respectively, the mean and variance of the higher moments of the linear Gaussian white noise process were calculated. The results are shown in Table 2.3.

Table 2.3: Mean and Variance of $Y_t = X_t^b,\ b = 1,2,3,......$

b	$Y_t = X_t^b$	$E(Y_t) = E(X_t^b)$	$Var(Y_t)$
1	X_t	0	σ^2
2	X_t^2	σ^2	$2\sigma^4$
3	X_t^3	0	$15\sigma^6$
4	X_t^4	$3\sigma^4$	$96\sigma^8$
5	X_t^5	0	$945\sigma^{10}$
6	X_t^6	$15\sigma^6$	$10170\sigma^{12}$
7	X_t^7	0	$135135\sigma^{14}$
8	X_t^8	$105\sigma^8$	$2016000\sigma^{16}$
9	X_t^9	0	$34459425\sigma^{18}$
10	X_t^{10}	$945\sigma^{10}$	$653836050\sigma^{20}$

From Table 2.3 it is clear that for fixed σ, increase in power, *b* leads to increase in variance. Simulation of $Var(Y_t)$ for b = 2, 3 and 4 shows that If $Y_t = X_t^b$ then,

(i) Y_t is a linear white noise

(ii) Var(Y_t) depends on *b*. that

is, $Var(X_t^{b1}) < Var(X_T^{b2})$ provided $b_1 < b_2$ for fixed σ (see figure 2.1).

Table 2.4: Simulation of $Var(Y_t)$ for b = 2, 3 and 4 given that $Y_t = X_t^b$

σ	b = 2 $Var(Y_t) = 2\sigma^4$	b = 3 $Var(Y_t) = 15\sigma^6$	b = 4 $Var(Y_t) = 96\sigma^8$
0.1	0.0002	0.0000	0.0000
0.2	0.0032	0.0010	0.0002
0.3	0.0162	0.0109	0.0063
0.4	0.0512	0.0614	0.0629
0.5	0.1250	0.2344	0.3750
0.6	0.2592	0.6998	1.6124
0.7	0.4802	1.7647	5.5342
0.8	0.8192	3.9322	16.1061
0.9	1.3122	7.9716	41.3249
1.0	2.0000	15.0000	96.0000

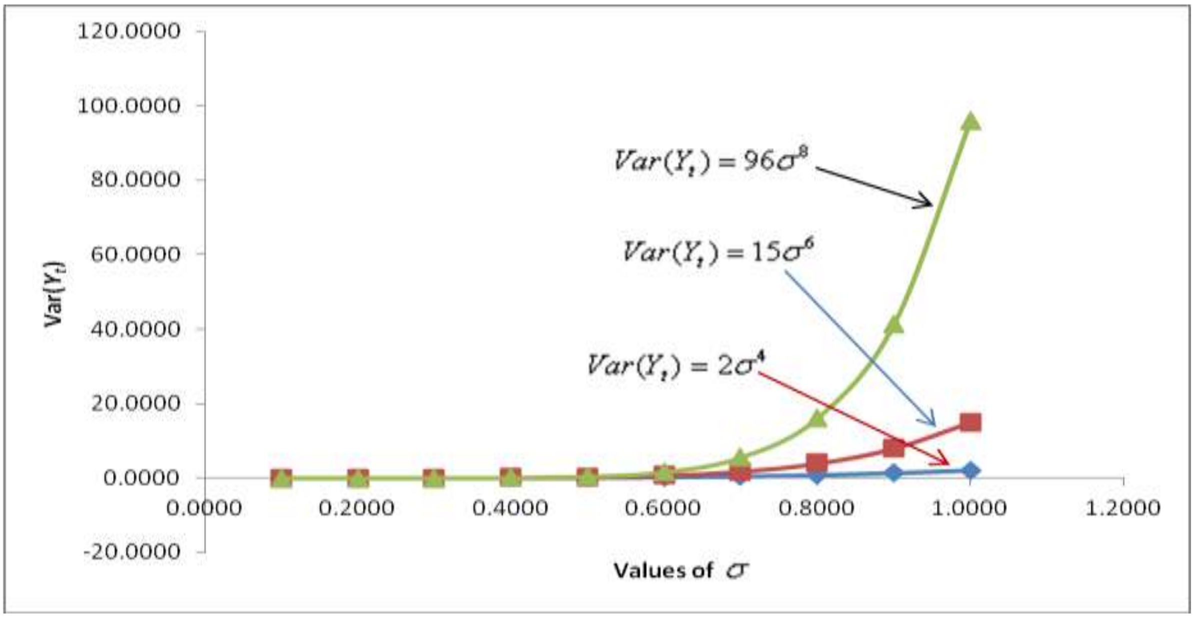

Figure 2.1: The graph of Var (Y_t) against the values of σ for b =2, 3 and 4

METHODS OF DENOISING THE IMAGE

Image de-noising is a subject matter that has taken the attention of many researchers in image processing world for a long time now. Much progress has been made in this area but still, there is so much to be done to guarantee noise free medical images, as accurate diagnosis of the patient's ill condition of health depends on it. Some techniques for de-noising medical images have been proposed. They include the use of diffusion filters and discrete wavelet transforms. Diffusion filters are efficient if the noise level is low but if the noise level is high, the use of a combination of filters is advised in order to achieve a better image enhancement. Another method used to de-noise the image is called threshold technique. According to Khera and Malhotra (2014), threshold technique can be used to create binary image from gray scale image. In this technique, an image is segmented by setting all pixels which have intensity values above a threshold value determined, abinitio, to be a forehand value and all the remaining pixels to a background value. Further discussions on this can be found in Sanches et al., (2008) and Khera and Malhotra (2014). Although it is not the responsibility of practitioners in radiology and imaging to measure noise levels in the image or even to de-noise the image, it is important that they are aware that noise production in imaging is inherent in the imaging systems so that they can properly advise managers in the field on the most appropriate equipment to buy and also, to advise on the need for quality assurance personnel whose responsibility it is to assess, on regular basis, the suitability of an imaging equipment for continued usage. Before procurement of such expensive equipment it is important to check that the various parameters and specifications given by the manufacturers would meet the need of the healthcare facility. For example, equipment which can acquire images and process them with high signal to noise ratio (SNR) would definitely be more efficient than one with low SNR. In other words a diagnostic image with low noise sequence is better than one with high noise sequence. The cost implication of a noisy image cannot be overemphasized. Aside the economic waste arising from repeated studies, the radiation hazard associated with repeated studies in CT and digital radiography is enormous. For these reasons, measurement of image quality is very crucial.

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