Journal of Physical Science and Innovation
ISSN: 2277-0119
Volume 9, No. 2, 2017

NUCLEAR SIZE CORRECTIONS TO THE ISOTOPE SHIFTS OF SINGLE-ELECTRON AND SINGLE-MUON ATOMS

[1]Adamu, A, [2]Langa, E. D., [3]Abbas, B. K., [4]Olawale, A. R. and [5]Balami, B. Y.

[1, 3 & 4] Department of Physics University of Maiduguri, Maiduguri, Nigeria

[2] Department of Basic science and technology, MOLCA, Maiduguri, Nigeria

[5] School of Basic and Applied Science, Federal Polytechnic Mubi, Adamawa State, Nigeria

Email: aliyuadamu703@gmail.com

ABSTRACT

Isotope shift has played vital role in the understanding of atomic spectra. In this work, we determine the isotope shifts in the ground states and first excited states of light and heavy nucleus of single-electron and single-muon atoms, caused by the fact that the nucleus has a finite size distribution of charge. The results showed that the isotope shift due to the finite size of nucleus is increasing with the size of the nucleus for both single-electron and single-muon atoms. The results also showed that the isotope shift is very small for electron atoms as compared to the muon atoms. The difference could be attributed to a difference in the size of the nucleus and effective interaction orbiting particles (electron or muon) with the atomic nucleus. This study may also improve the knowledge of isotope shift for both light and heavy nucleus of single-electron and single-muon atoms caused by the fact that the nuclear has a finite size.

Keywords: Muon, Single-electron Atom, Single-muon Atom, Isotope Shift, Nuclear Size.

INTRODUCTION

The correction to the energy of the atom due to the finite size of the nucleus leads to the isotope shift between the energy levels of two atoms with nuclei that have the same atomic number Z but different mass numbers A [1-3]. A large number of publications exist relating to both nuclear size effects and the isotope shift (see, for example, [4-7]). Recent measurements by Garching [4] and Paris [8] have greatly improved our knowledge of the isotope shift between deuterium and normal hydrogen. Due to their much increased precision [9,10], these measurements now rival the traditional relativistic electron scattering [11] for determining the nuclear sizes of these isotopes and their differences. This new level of precision has led to a reexamination of many contributions to the level shifts [12,13] and to the calculation of higher-order Quantum Electrodynamics processes [14].

A muon is a negatively charged elementary particle from the lepton family with a mass that is about 207 times that of an electron. Muonic atom is an atom in which one of the electrons has been replaced by a negatively charged muon [15-17]. The atomic properties are strongly affected by the orbiting particle mass m [18-20], e.g., the Bohr energy scales linearly with m while the Bohr radius as $1/m$. Because of the very large mass of a muon compared with an electron, the muonic wave function has a large overlap with the nucleus [21] this enhanced the nuclear size effect [22,23] and thus significantly altered the familiar spectra associated with that atom [1,15].

The present paper intends to improve the knowledge of isotope shift by determine its magnitude in the ground states and the first excited states for both light and heavy nucleus of single-electron and single-muon atoms caused by the fact that the nuclear has a finite size. This may lead us to a better understanding of isotope shifts in both light and heavy nucleus of single-electron and single-muon atoms caused by the fact that the nuclear has a finite size.

THEORY

The Point Charge Potential

In an elementary picture, the motion of an orbiting particle in simple hydrogen can be extracted from the well known Schrodinger equation of the hydrogen atom [24]. The energy levels of an electron can be found from quantum mechanics by solving the Schrodinger equation in spherical coordinates, using Coulomb's law for the potential energy [25]:

$$U(r) = \frac{Zke^2}{r} \tag{1}$$

According to Schrödinger equation, the energy levels of an electron in the electrostatic potential (1) of a fixed point charge $+Ze$ are given by:

$$E_n = -\frac{Z^2 m_e e^4}{2\hbar^2 (4\pi\epsilon_0)^2}\frac{1}{n^2} \qquad n = 1, 2, 3, \ldots \tag{2}$$

where m_e and e are the electronic mass and charge, respectively [26] and n is termed the principal quantum number. This solves the problem of determining the energy levels of the discrete spectrum in a coulomb field (1) [27,28].

Finite Size Nucleus

It is well known that the discrete eigenvalues of the Coulomb potential closely approximate the bound state energies of a single-electron atom. On the other hand, the unphysical infinity in the $1/r$ potential at the origin makes it necessary that this potential be modified for values of r inside a region about the origin that can be identified with the nucleus of the atom. The resulting energy shifts produced by the nuclear size effect are dependent only on the assumed form for the potential energy inside the nucleus [23]. The potential for a finite-size nucleus is given as [15,29,30]:

$$U(R) = -\frac{Zke^2}{R}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right) \tag{3}$$

where $R = r_0 A_i^{1/3}$, with $r_0 = 1.2 \times 10^{-15}\ m$.

In the case of electronic or light muonic atoms, the magnitude of the nuclear size effect allows the nuclear size correction to the energy levels to be accurately calculated by the use of perturbation theory [2,31]. Since the electron wave function varies slowly over the nuclear volume, the energy of an electron in a state ψ_{nlm} will depend partly on the expectation of the potential [30,32]:

$$\Delta E_{nlm} = \int \psi^*_{nlm}\{U(R) - U(r)\}\psi_{nlm} d\tau$$

$$= \frac{Zke^2}{R} 4\pi \int \psi^*_{nlm}\left\{\frac{r^2}{2R^2} - \frac{R}{r} - \frac{3}{2}\right\}\psi_{nlm} r^2 dr \tag{4}$$

For the $1s$ and $2s$ lowest state, we have [26]:

$$\left.\begin{aligned} \psi_{1s} &= \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{\sqrt{\pi}} e^{-Zr/a_0} \\ \psi_{2s} &= \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} \frac{1}{\sqrt{8\pi}} \left(1 - \frac{r}{a_0}\right) e^{-Zr/a_0} \end{aligned}\right\} \quad (5)$$

The energy level shifts ΔE_{nlm} due to the finite size of nucleus can be calculated using (5). Taking the approximation $r << a_0$, i.e., $e^{-Zr/a0} \approx 1$ and assuming the wave function to remain constant over the region of integration, we obtain:

$$\Delta E_{1s} = \frac{Zke^2}{R} 4\pi \int \psi_{1s}^* \left\{\frac{r^2}{2R^2} - \frac{R}{r} - \frac{3}{2}\right\} \psi_{1s} r^2 dr$$

$$= 0.8|E_e|Z^4 \left(\frac{R}{a_0}\right)^2 \quad (6a)$$

$$\Delta E_{2s} = \frac{Zke^2}{R} 4\pi \int \psi_{2s}^* \left\{\frac{r^2}{2R^2} - \frac{R}{r} - \frac{3}{2}\right\} \psi_{2s} r^2 dr$$

$$= 0.2 \frac{|E_e|}{4} Z^4 \left(\frac{R}{a_0}\right)^2 \quad (6b)$$

We can write the energy level shift for muonic atoms as:

$$\Delta E_{1s}^{\mu} = 0.8|E_{\mu}|Z^4 \left(\frac{R}{a_{\mu}}\right)^2 \quad (7a)$$

$$\Delta E_{2s}^{\mu} = 0.2 \frac{|E_{\mu}|}{4} Z^4 \left(\frac{R}{a_{\mu}}\right)^2 \quad (7b)$$

where the ground state energy for electron and muon atom are respectively:

$$E_e = \frac{Z^2 e^2}{2a_0} = 13.6\ eV \quad (8)$$

$$E_{\mu} = \frac{Ze^2}{2a_{\mu}} = 2.8\ keV \quad (9)$$

where $a_0 = 5.29 \times 10^{-11}\ m$ and $a_{\mu} = 2.56 \times 10^{-13}\ m$ is the Bohr radius for electronic and muonic atom respectively. Equations ($6a$), ($6b$), ($7a$) and ($7b$) solve the problem of determining the $1s$ and $2s$ energy levels of the discrete spectrum in a finite size potential (3) of both electron – atom and muon – atom.

The Isotope Shift

If we have ΔE_A and $\Delta E_{\acute{A}}$ as the energy shifts in the $1s$ and $2s$ states in different isotopes A and $\acute{A}$ with finite size nucleus, then their isotope shift δE can be measured using

$$\delta E = \Delta E_{\acute{A}} - \Delta E_A \tag{10}$$

Hence:

$$\delta E_{1s} = 5.6 \times 10^{-9} Z^4 \left(A_{\acute{A}}^{2/3} - A_A^{2/3} \right) \tag{11}$$

$$\delta E_{2s} = 3.5 \times 10^{-10} Z^4 \left(A_{\acute{A}}^{2/3} - A_A^{2/3} \right) \tag{12}$$

We can write the isotope shift for muonic atoms as:

$$\delta E_{1s}^{\mu} = 4.92 \times 10^{-2} Z^4 \left(A_{\acute{A}}^{2/3} - A_A^{2/3} \right) \tag{13}$$

$$\delta E_{2s}^{\mu} = 3.076 \times 10^{-3} Z^4 \left(A_{\acute{A}}^{2/3} - A_A^{2/3} \right) \tag{14}$$

Equations (6*a*), (6*b*), (7*a*) and (7*b*) can be used to calculate the energy shifts in the ground states (1*s*) and first excitation states (2*s*) due to finite-sized nucleus of electronic and muonic hydrogen-like atoms respectively.

DISCUSSION

The energy shifts in the ground states (1*s*) and first excitation states (2*s*) of electronic and muonic hydrogen-like atoms are calculated using (6*a*), (6*b*), (7*a*) and (7*b*) and their values are represented in Table 1:

Table 1: The Energy Shifts in the Ground States (1*s*) and First Excitation States (2*s*) of Electronic and Muonic Hydrogen-Like Atoms with their Isotopes

Nuclei $^{A}X_{Z}$	*R (fm)*	*Electronic Atoms*		*Muonic Atoms*	
		ΔE_{1s} (eV)	ΔE_{2s} (eV)	ΔE^{μ}_{1s} (eV)	ΔE^{μ}_{2s} (eV)
$^{1}H_{1}$	1.2000	0.5598×10^{-8}	3.4987×10^{-10}	0.4922×10^{-1}	3.0737×10^{-3}
$^{3}H_{1}$	1.7307	1.1645×10^{-8}	7.2784×10^{-10}	1.0238×10^{-1}	6.3986×10^{-3}
$^{6}Li_{3}$	2.1805	1.4973×10^{-6}	0.9358×10^{-7}	$1.3158\times10^{+1}$	8.2270×10^{-1}
$^{9}Li_{3}$	2.4961	1.9618×10^{-6}	1.2261×10^{-7}	$1.5941\times10^{+1}$	9.9633×10^{-1}
$^{21}Na_{11}$	3.3107	6.2391×10^{-4}	3.8994×10^{-5}	$5.4829\times10^{+3}$	$3.4260\times10^{+2}$
$^{23}Na_{11}$	3.4126	6.6291×10^{-4}	4.1432×10^{-5}	$5.8258\times10^{+3}$	$3.6456\times10^{+5}$
$^{107}Ag_{47}$	5.6969	6.1572×10^{-1}	3.8482×10^{-2}	$5.4110\times10^{+5}$	$3.3816\times10^{+5}$
$^{109}Ag_{47}$	5.7322	6.2333×10^{-1}	3.8961×10^{-2}	$5.4782\times10^{+5}$	$3.4255\times10^{+5}$
$^{151}Eu_{63}$	5.6969	2.5015×10^{0}	1.5633×10^{-1}	$2.1975\times10^{+7}$	$1.3736\times10^{+6}$
$^{153}Eu_{63}$	5.7322	2.5235×10^{0}	1.5773×10^{-1}	$2.2171\times10^{+7}$	$1.3863\times10^{+6}$
$^{203}Pb_{81}$	5.7322	8.3262×10^{0}	5.2042×10^{-1}	$7.3537\times10^{+7}$	$4.5716\times10^{+6}$
$^{205}Pb_{81}$	5.7322	8.3810×10^{0}	5.2381×10^{-1}	$7.3631\times10^{+7}$	$4.6017\times10^{+6}$
$^{235}U_{92}$	5.7322	$1.5278\times10^{+1}$	9.5482×10^{-1}	$1.3422\times10^{+8}$	$8.3889\times10^{+6}$
$^{238}U_{92}$	5.7322	$1.5407\times10^{+1}$	9.6292×10^{-1}	$1.3536\times10^{+8}$	$8.4606\times10^{+6}$

TABLE 2: The Isotope Shifts for Single-Electron and Single-Muon Atoms

Nuclei	δE_{1s}(eV)	δE_{2s}(eV)	δE^{μ}_{1s} (eV)	δE_{2s}(eV)
$^{3}H - {}^{1}H$	6.0486×10^{-9}	3.7803×10^{-10}	5.3162×10^{-2}	3.3249×10^{-3}
$^{9}Li - {}^{6}Li$	3.1630×10^{-7}	1.9769×10^{-8}	2.7835×10^{0}	1.7397×10^{-1}
$^{23}Na - {}^{21}Na$	3.9000×10^{-5}	2.4382×10^{-6}	$3.4289\times10^{+2}$	$2.1431\times10^{+1}$
$^{109}Ag - {}^{109}Ag$	$7.6\ 121\times10^{-3}$	4.7941×10^{-4}	$6.7291\times10^{+4}$	$4.2056\times10^{+3}$
$^{153}Eu - {}^{151}Eu$	2.2000×10^{-2}	1.4000×10^{-3}	$1.9600\times10^{+5}$	$1.2700\times10^{+4}$
$^{205}Pb - {}^{203}Pb$	5.4800×10^{-2}	3.3900×10^{-3}	$9.4000\times10^{+4}$	$3.0100\times10^{+4}$
$^{238}U - {}^{235}U$	1.2900×10^{-1}	8.1000×10^{-3}	$1.1400\times10^{+6}$	$7.1700\times10^{+4}$

TABLE 3: The Isotope Shifts for Single-Electron and Single-Muon Atoms

Nuclei	Log δE_{1s}	Log δE_{2s}	Log δE^{μ}_{1s}	Log δE^{μ}_{2s}
H	-8.2183	-9.4225	-1.2744	-2.4782
Li	-6.4999	-7.7040	0.4446	-0.7595
Na	-4.4089	-5.6129	2.5351	1.3310
Ag	-2.1185	-3.3193	4.8279	3.6238
Eu	-1.6576	-2.8538	5.2923	4.1038
Pb	-1.2612	-2.4698	5.9731	4.4786
U	-0.8894	-2.0915	6.0569	4.8555

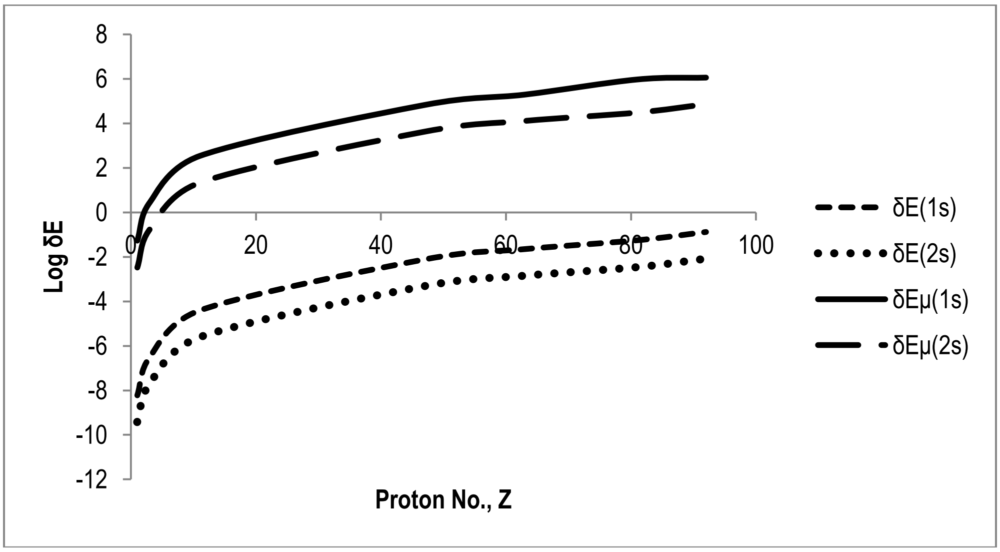

Figure 1: A graph of isotope shifts against the proton number Z for single-electron and single-muon atoms

Figure 1 showed the relationship between the isotope shifts in the 1s and 2s states muonic and electronic atoms. It can be observed from the Figure 1 that the isotope shift in $2s$ states are smaller than in the $1s$ states for both muon and electron hydrogen-like atoms. There is a rapid increase in the isotope shifts in 1s and 2s states with increasing Z. The isotope shifts are very small for light Z atoms. The shifts become more important with increasing Z for both electron and muon atoms. This is due to the nuclear size effect. The order of magnitude of isotope shifts for different orbiting particles in different energy states are as follows:

$$\delta E_{1\mathrm{s}}^{\mu} > \delta E_{2\mathrm{s}}^{\mu} > \delta E_{1s} > \delta E_{2s}$$

Table 4 and Table 5 compared the corrections, ΔE_{1s}, ΔE_{2s}, $\mathit{\Delta} E_{1s}^{\mu}$ and $\mathit{\Delta} E_{1s}^{\mu}$ calculated from our results with the value of ground state corrections $\square\square_{\square\square}$ obtained from Ref. [2] based on the non-relativistic calculation. As can be seen from Tables the agreement between the present results and the results from Ref. [2] is very good.

TABLE 4: The Value of Energy Shifts ΔE_{1s} and ΔE_{2s}Derived from the Present Work and from Non-Relativistic Calculation, ΔE_{NR} in $1s$ States from [2]

Nuclei $^{A}X_{Z}$	ΔE_{1s} (eV)	ΔE_{NR} (eV)	ΔE_{2s} (eV)
$^{1}H_{1}$	0.5598×10^{-8}	0.5600×10^{-8}	3.4987×10^{-10}
$^{3}H_{1}$	1.1645×10^{-8}	0.8890×10^{-8}	7.2784×10^{-10}
$^{6}Li_{3}$	1.4973×10^{-6}	–	0.9358×10^{-7}
$^{9}Li_{3}$	1.9618×10^{-6}	–	1.2261×10^{-7}
$^{21}Na_{11}$	6.2391×10^{-4}	–	3.8994×10^{-5}
$^{23}Na_{11}$	6.6291×10^{-4}	–	4.1432×10^{-5}
$^{107}Ag_{47}$	0.6157	0.6156	0.0386
$^{109}Ag_{47}$	0.6233	0.6232	0.0389
$^{151}Eu_{63}$	2.5015	2.5003	0.1563
$^{153}Eu_{63}$	2.5235	2.5223	0.1577
$^{203}Pb_{81}$	8.3262	8.3220	0.5204
$^{205}Pb_{81}$	8.3810	8.3770	0.5238
$^{235}U_{92}$	15.2782	15.2700	0.9548
$^{238}U_{92}$	15.4071	15.4000	0.9629

TABLE 5: The Value of Energy Shifts ΔE^{μ}_{1s} and ΔE^{μ}_{2s} Derived from the Present Work and from Non-Relativistic Calculation, ΔE_{NR} in $1s$ from [2]

Nuclei $^{A}X_{Z}$	ΔE^{μ}_{1s} (eV)	ΔE^{μ}_{NR} (eV)	ΔE^{μ}_{2s} (eV)
$^{1}H_{1}$	0.04922	0.0450	0.0031
$^{3}H_{1}$	0.10238	0.0785	0.0064
$^{6}Li_{3}$	13.1580	–	0.8227
$^{9}Li_{3}$	15.9410	–	0.9963
$^{21}Na_{11}$	5482.90	–	342.600
$^{23}Na_{11}$	5825.80	–	364.560
$^{107}Ag_{47}$	$5.4110\times10^{+5}$	$5.4417\times10^{+5}$	$3.3816\times10^{+5}$
$^{109}Ag_{47}$	$5.4782\times10^{+5}$	$5.5093\times10^{+5}$	$3.4255\times10^{+5}$
$^{151}Eu_{63}$	$2.1975\times10^{+7}$	$2.2102\times10^{+7}$	$1.3736\times10^{+6}$
$^{153}Eu_{63}$	$2.2171\times10^{+7}$	$2.2297\times10^{+7}$	$1.3863\times10^{+6}$
$^{203}Pb_{81}$	$7.3537\times10^{+7}$	$7.8029\times10^{+7}$	$4.5716\times10^{+6}$
$^{205}Pb_{81}$	$7.3631\times10^{+7}$	$7.8533\times10^{+7}$	$4.6017\times10^{+6}$
$^{235}U_{92}$	$1.3422\times10^{+8}$	$1.3498\times10^{+8}$	$8.3889\times10^{+6}$
$^{238}U_{92}$	$1.3536\times10^{+8}$	$1.3613\times10^{+8}$	$8.4606\times10^{+6}$

The finite size Nuclear size corrections to the states *p, d, f, …* are very small [1] and can be ignored for both single-electron and single-muon atoms. Here, we calculated the values of the energy shift in the $1s$ and $2s$ levels of single-electron and single-muon atoms due to the finite size of nucleus. We compared our results with the correction to the ground state energy of the hydrogenic atom obtained by Ref. [2] from first-order non-relativistic perturbation theory:

$$(\Delta E)_{\mathrm{NR}} = \frac{2}{5}(Z\alpha)^4 R^2 mc^2 \qquad (15)$$

This is approximately equal to the standard expression for the nuclear size correction to the n^{th} electronic level of a one-electron atom [4,5,7]:

$$E_{\text{Finite Size}} = \frac{2(Z\alpha)^4\mu^3}{3n^2}\langle R^2\rangle\delta_{l0} \qquad (16)$$

where, μ and l are the reduced mass and orbital angular momentum quantum number respectively, α is the fine structure constant and R is the radius of the nucleus.

Table 4 and Table 5 showed that our results are in good agreement with the calculated values of the energy shift from the standard expressions (15) and (16).

CONCLUSION

The muonic atoms are much more sensitive to the charge distribution of the nucleus than electron atoms. Hence the relative contribution of the finite nuclear size to the isotope shift in muonic atoms is much more than the corresponding values for electron atoms, the difference is of about order of 10^{+8}. This help identifying how significance isotope shift could be. Since the nuclear finite size effect is proportional to the square of the nuclear radius, R and to the mass of the orbiting particles (electron or muon), any difference in isotope shift obtained from $\Delta\square_{1\square}$ and $\Delta\square_{2\square}$ could be attributed to a difference in the size of the nucleus and in the effective interaction of orbiting particles with the atomic nucleus. A better theoretical calculation for the different in nuclear size implies also an increase of the accuracy of future muonic atom experiments.

RECOMMENDATIONS

This study is based on the isotope shifts in the $1s$ and $2s$ states of both single-electron and single-muon atoms. It is recommended that for further study one should expand this to isotope shifts for optical transition were isotope shifts can be detected.

REFERENCES

[1]. A. Adamu, P. Tartius and S. A. Amshi, The Spectroscopy of Single Electron and Muonic Atoms. *Journal of Physical Science and Innovations, Vol. 8, No. 2,* (2016) 1 – 13.

[2]. R. T. Deck, J. G. Amar and G. Fralick, Nuclear size corrections to the energy levels of single-electron and -muon atoms. *J. Phys. B: At. Mol. Opt. Phys. 38* (2005) 2173–2186.

[3]. F. A. Babushkin. Isotope Shift of Spectral Lines. *J. Exptl. Theoret. Phys. (U.S.S.R.) 44,* (1963) 1661-1667.

[4]. K. Pachucki, M. Weitz and T.W. Hnsch, *Phys. Rev. A492255* (1996).

[5]. J. L. Friar, J. Martorell and D. W. L. Sprung, *Phys. Rev. A564579* (1997).

[6]. A. Huber, T. Udem, B. Gross, J. Reichert, M. Kourogi, K. Pachucki, M. Weitz and T W. Hnsch, *Phys. Rev. Lett. 80468* (1998).

[7]. K. Pachucki, D. Leibfried, M. Weitz, A. Huber, W. Konig, and T. W. Hansch, *J. Phys. B29, 177* (1996).

[8]. B. de Beauvoir, F. Nez, L. Julien, B. Cagnac, F. Biraben, D. Touahri, L. Hilico, O. Acef, A. Clairon, and J. J. Zondy, *Phys. Rev. Lett. 78, 440* (1997).

[9]. A. Antognini, F. Nez, K. Schuhmann et al., Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen, *Science* 339, 417 (2013), 4.

[10]. T. W. Hansch, Invited talk at APS Spring Meeting, Washington DC, April 20, 1997; T. W. Hansch (private communication).

[11]. J. L. Friar and J. W. Negele, *Adv. in Nucl. Phys.* ***8***, (1975) *219*.

[12]. K. Pachucki and S. G. Karshenboim, J. Phys. B28, L221 (1995).

[13]. I. B. Khriplovich, A. I. Milstein, and R. A. Senkov, Phys. Lett. A221, 370 (1996).

[14]. J. L. Friar, J. Martorell and D. W. L. Sprung, Nuclear Sizes and the Isotope Shift, *Phys. Rev. A56:* (1997) 4579 – 4586

[15]. A. Palffy, Nuclear effects in atomic transitions. *arXiv:1106.3218v1* [physics.atom-ph] 16 Jun 2011.

[16]. E. Borie, Lamb shift in light muonic atoms — Revisited. *Annals of Physics 327 (2012) 733–763.*

[17]. E. Borie, G. A. Rinker, Rev. *Modern Phys. 54 (1982) 67.*

[18]. R. Pohl, A. Antognini, F. Nez et al., The size of the proton, *Nature 466, 213* (2010).

[19]. A. Antognini, F. Nez, K. Schuhmann et al., Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen, *Science* 339, 417 (2013), 4.

[20]. A. Antognini, Muonic atoms and the nuclear structure for the CREMA collaboration. *arXiv:1512.01765v1 [physics.atom-ph]* 6 Dec 2015].

[21]. J. M. Dong, W. Zuo, H. F. Zhang, W. Scheid, and J. Z. Gu, and Y. Z. Wang, Correlation between muonic levels and nuclear structure in muonic atoms. *arXiv:1111.0494v1* [nucl-th] 2 Nov 2011.

[22]. A. Antognini, Muonic atoms and the nuclear structure for the CREMA collaboration. *arXiv:1512.01765v1 [physics.atom-ph]* 6 Dec 2015].

[23]. T. Nebel, The Lamb Shift in Muonic Hydrogen, *Max-Planck-Institut fur Quantenoptik Hans-Kopfermann Str. 185748 Garching, Deutschland*, (2010).

[24]. B. Cagnac, J.C. Pebay-Peyroula. *Modern atomic Physics, Part 1 Fundamental Principles*. Macmillan, 1975.

[25]. W. Greiner, *Quantum Mechanics, An Introduction,* Fourth Edition, Springer, Berlin, Germany. (2011) 220 – 222.

[26]. G. W. Erickson, Energy Levels of One-Electron Atoms, *J. Phys. Chem. Ref. Data, Vol. 6, No. 3.* (1977) 831 – 869.

[27]. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics, Non-relativistic Theory, Volume 3 of Course of Theoretical Physics*, Third edition, Pergamon Press, Oxford, England. (1991) pp. 119.

[28]. A. Adamu and Y. H. Ngadda, Determination of Nuclear Potential Radii and Its Parameter from Finite – Size Nuclear Model. *International Journal of Theoretical and Mathematical Physics. Vol. 7, No. 1,* (2017) 9 – 13.

[29]. L. Yung-Kuo, *Problems and Solutions on Atomic, Nuclear and Particle Physics*, World Scientific Publishing Co. Pte. Ltd. Singapore, (2000), pp. 61- 68.

[30]. P. Jonsson and C. F. Fischer, SMS92: a program for relative isotope shift calculations, *Computer Physics Communications. Vol. 100, Issue 1 – 2* (1997) 81 – 92.

[31]. D. J. Griffiths, *Introduction to Quantum Mechanics*, Prentice Hall, Inc., New Jersey, USA. (1995) pp. 248 – 251.

BIOGRAPHY

Aliyu Adamu had obtained his B. Sc. Degree in Physics in 2009 and M.Sc. Degree in Theoretical Physics in 2015 both from University of Maiduguri, Borno State. However, the author has experiences in the field of teaching, publications and trainings. Presently he is working as Lecturer II in University of Maiduguri, Borno state, Nigeria. His area of interest is theoretical Physics, with emphasis in Nuclear Physics.

Emmanuel Dauda, Langa's first degree was in physics from Ahmadu Bello University, Zaria in 1990. In addition, he had a National Diploma in Computer Science at Federal Polytechnic, Mubi in 1998. He holds Postgraduate Diplomas in Education and Physics from National Teachers' Institute, Kaduna and Federal University of Technology, Yola in 2007 and 2009 respectively. Currently, a lecturer with the College of Agriculture, Maiduguri, Borno State.

Reference to this paper should be made as follows: Adamu, A, Langa, E. D., Abbas, B. K., Olawale, A. R. and Balami, B. Y. (2017), Nuclear Size Corrections to the Isotope Shifts of Single-Electron and Single-Muon Atoms. J. of Physical Science and Innovation, Vol. 9, No. 2, Pp. 1-14.