# APPLICATION OF LEAST ABSOLUTE SUM (LAS) DEFORMATION DETECTION METHOD USING COORDINATE DIFFERENCES FROM DIFFERENT OBSERVATIONAL CAMPAIGNS 

Omogunloye O.G., Shittu O.G., Ipadeola A.O. and Ojegbile B.M<br>Department of Surveying and Geoinformatics Engineering University of Lagos, Akoka-Lagos, Nigeria E-mail:- gabolushohan@yahoo.com,


#### Abstract

Deformation study is one of the main research fields in geodesy. Deformation study comprises measurement, processing and analysis phases, Measurement techniques can be divided into geotechnical, structural and geodetic methods. Geotechnical and structural methods uses equipment such as tiltmeters, Pseodolites, Laser scanners e.t.c to measure changes in length, inclination, relative height, strains e.t.c. The geodetic methods are of two basic types, the reference and relative methods. This study focuses on the deformation analysis using the geodetic method known as the Least Absolute Sum Method. The method consists mainly of the independent adjustment of each of two epoch data, compatibility test on their a posteriori variances, followed by determination of Trend of movements for all the common points in the monitoring network. A triangulation network was designed (carefully selected) consisting of 45 YTT series second order control points within the study area (Lagos State) resulting in a total of 63 triangles, 189 observations and 90 unknown parameters with 99 degrees of freedom. The network adjustment was done using the method of least squares observation equations. The estimated variance factors for the 2D (horizontal) network were $7.82989325645394 \mathrm{e}-08$ and $7.7207636996395 \mathrm{e}-08$ while 0.03944 and 0.052339 represent


the estimated variance factors for the 1D (height) for the first and second epochs networks respectively. The compatibility of the two epoch data was tested with the variance ratio and compatibility test criteria. Actual displacement vectors were computed and transformed into the same computational base using S-transformation by Least Absolute Sum (LAS), stable and unstable points within the monitoring network were determined using Single Point displacement test, the displacement vector magnitude was computed for the two methods, represented graphically to indicate possible trend of movements that might have occurred. This study finds applications in studying the deformation of large engineering structures such as high rise buildings, bridges, dams, oil exploration zones, mining sites and land slide monitoring.

Keywords: Deformation, Analysis, Least Absolute Sum (LAS).

## INTRODUCTION

One important application of survey control networks is the detection of expected deformations at a specified area. This is done by measurements made at successive epochs and the most probable values of the coordinates are obtained using the well-known method of least squares (James, 1985). Any object, when acted upon by external forces, deforms, or exhibits changes in its size or shape. These observable changes are manifestations of internal stresses or pressures produced by the physical interaction of the external forces and the material itself. Materials either fail or tear when stresses exceed certain critical values. (Chrzanowski et al., 1986). It is this risk of failure which practically necessitates deformation monitoring surveys, which allow the implementation of mitigating constructive procedures or evacuations to take place early enough, to prevent loss of life and material.

Generally, the deformation measurement techniques can be divided into geotechnical, structural and geodetic methods. Geotechnical and structural methods are direct measurement methods, which use special equipment to measure changes in length, inclination, relative height, strain, etc. (Teskey and Porter,1988; Chrzanowski, 1986). On the other hand, in the geodetic method there are two basic types of geodetic monitoring networks; namely the reference and relative networks (Chrzanowski et al., 1986). In a reference network, some of the points or stations are assumed to be located outside of the deformable body or object, thus serving as reference points for the determination of the absolute displacements of the object points. However, in a relative network, all surveyed points are assumed to be located on the deformable body.

This study will focus only on the geodetic method using a relative network. In a geodetic monitoring network, the object or area under investigation is usually represented by a number of points which are permanently monumented or marked. All the points are then observed in two or more epochs of time. The geodetic monitoring network can be either a conventional (terrestrial) network, a photogrammetry (i.e., aerial or close-range) network, Global Positioning System (GPS) network or a combination of these network types.

Deformation analysis using the geodetic method mainly consists of a two-step analysis via independent adjustment of the network of each epoch which involves testing coordinate differences for significance, by comparison to the accuracy of their determination, followed by deformation detection between the two epochs. During deformation
analysis it is important to determine the trend of movements (displacements) for all the common points in a monitoring network. The trend of movements, then form a basis for preliminary identification of the actual deformation models. Although deformation analysis is applicable to one-dimensional (1-D), two dimensional (2D) and three-dimensional (3-D) monitoring networks, for this study a 2D (horizontal) and 1D (vertical) networks of secondary controls located around Lagos State were investigated, a robust method, Least Absolute Sum (LAS) was used for deformation detection and analysis.

## STUDY AREA

The study area is Lagos state and it is the commercial nerve centre and the most populous city in Nigeria. Lagos State is Nigeria's largest commercial, financial and industrial hub. It has industrial zones around the state with over 2000 small, medium and large scale industries. It is regarded as the smallest state in the country; however, it has the highest population density in the nation. Lagos is geographically located on latitudes and longitudes $6^{\circ} 35^{\prime} \mathrm{N} 3^{\circ} 45^{\prime} \mathrm{E}$ and $6.583^{\circ} \mathrm{N} 3.750^{\circ} \mathrm{E}$ Coordinates. Lagos State has a land mass of about 3,577 square kilometres with about 787 constituting lagoons, swamps, marches and creeks. Lagos harbours most of the high rise buildings, bridges and engineering structures prone to deformation or subsidence. Lagos has several networks of control points spread across different parts of the state to which surveys are tied. For this study, Secondary controls located in Lagos state were used.


## DATA ACQUISITION

The study has been executed with an existing geodetic data acquired using the conventional surveying technique. An existing data of a set of control points was used to design a reference network. The data used were second order two dimensional control point coordinates obtained from the office of the SurveyorGeneral of Lagos State while the Orthometric heights for these selected stations in the network are derived from EGM 2008. A total of 45 common stations coordinates were used for the two epochs. Note that second epoch data in this case was simulated from the adjustment of the first epoch data for the purpose of this study. Table 3.0, below shows the coordinates of the first and second epoch data.

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Table 3.0: The Coordinates of the First and Second Epoch Data

|  |  | FIRST EPOCH |  |  | SECOND EPOCH |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{S} / \mathbf{N}$ | $\begin{aligned} & \text { CONTROL } \\ & \text { POINT } \\ & \text { NAME } \\ & \hline \end{aligned}$ | EASTINGS(m) | NORTHINGS(m) | HEIGHT(m) | EASTINGS(m) | NORTHINGS(m) | HEIGHT(m) |
| 1 | YTT1 | 512770.871400334; | 718266.132200109; | 22.69139892 | 512770.871403101; | 718266.132201002; | 22.6714836 |
| 2 | YTT2 | 514506.700499577; | 718531.839799538; | 22.69178864 | 514506.700502712; | 718531.839892683; | 22.6177766 |
| 3 | YTT3 | 512893.348699673; | 714574.324598699; | 22.32421379 | 512893.348696706; | 714574.324682748; | 22.0283972 |
| 4 | YTT4 | 515558.463298852; | 713569.142998863; | 22.31915793 | 515558.463313656; | 713569.143971483; | 22.1792805 |
| 5 | YTT5 | 516586.611797575; | 714276.855800185; | 22.44819168 | 516586.611803276; | 714276.855808693; | 22.4573541 |
| 6 | YTT6 | 518643.696295812; | 713094.787300631; | 22.27584535 | 518643.696301644; | 713094.787305278; | 22.1277046 |
| 7 | YTT7 | 514352.907099268; | 714685.214899466; | 22.40743677 | 514352.90709294; | 714685.215000243; | 22.3007829 |
| 8 | YTT8 | 517061.729398256; | 715437.606801309; | 22.54378808 | 517061.729400602; | 715437.606814866; | 22.6728747 |
| 9 | YTT9 | 518422.044396225; | 714609.031901365; | 22.43557176 | 518422.044396833; | 714609.031912447; | 22.4370771 |
| 10 | YTT10 | 520125.232796594; | 713647.970001077; | 22.37953996 | 520125.232796485; | 713647.97000797; | 22.3638133 |
| 11 | YTT11 | 521363.15129068; | 715052.213702974; | 22.48641337 | 521363.151281339; | 715052.213730621; | 22.569482 |
| 12 | YTT12 | 518498.663596491; | 716974.489604158; | 22.69806547 | 518498.663595936; | 716974.489641336; | 22.9511293 |
| 13 | YTT13 | 514108.928199661; | 717481.663299636; | 22.61380567 | 514108.928201137; | 717481.663397287; | 22.5929253 |
| 14 | YTT14 | 515601.588799851; | 717526.274999398; | 22.70542529 | 515601.588808979; | 717526.275961109; | 22.868333 |
| 15 | YTT15 | 516950.750999607; | 716775.036400767; | 22.6656666 | 516950.7510017; | 716775.036407083; | 22.851861 |
| 16 | YTT16 | 517138.43110192; | 717714.634600756; | 22.76961739 | 517138.431104485; | 717714.634610646; | 23.058185 |
| 17 | YTT17 | 520079.581892186; | 717605.081806163; | 22.75625371 | 520079.58188775; | 717605.081862575; | 23.0669356 |
| 18 | YTT18 | 521384.589782752; | $716820.772199095 ;$ | 22.65590067 | 521384.589767459; | 716820.772291492 ; | 22.8672022 |
| 19 | YTT19 | 521584.838793279 ; | 713648.512600229 ; | 22.32802007 | 521584.838781533; | 713648.512604235; | 22.2474788 |
| 20 | YTT20 | 523697.284691038; | 712610.341101032; | 22.17827755 | 523697.284674975; | 712610.341115527; | 21.9320408 |
| 21 | YTT21 | 525256.684295581; | 712069.400902666; | 22.09307778 | 525256.684279801; | 712069.400939104; | 21.7478426 |
| 22 | YTT22 | 523497.609891544; | 714124.578899686; | 22.3578494 | 523497.609877763; | 714124.578999448; | 22.304579 |

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| 23 | YTT23 | $525443.708593041 ;$ | $714191.748497196 ;$ | 22.32616293 | $525443.708582496 ;$ | $714191.748578731 ;$ | 22.212263 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 24 | YTT24 | $527124.733799406 ;$ | $713617.755492013 ;$ | 22.25907939 | $527124.733796371 ;$ | $713617.755536764 ;$ | 22.0740095 |
| 25 | YTT25 | $522501.845287421 ;$ | $715583.224899734 ;$ | 22.49919981 | $522501.845274277 ;$ | $715583.225000239 ;$ | 22.5282929 |
| 26 | YTT26 | $526736.830187412 ;$ | $715474.552392427 ;$ | 22.45949366 | $526736.830178688 ;$ | $715474.552433881 ;$ | 22.4668134 |
| 27 | YTT27 | $527887.037415264 ;$ | $714977.706471458 ;$ | 22.40016023 | $527887.037436763 ;$ | $714977.706532537 ;$ | 22.3521075 |
| 28 | YTT28 | $518840.786597038 ;$ | $718875.794609559 ;$ | 22.89331219 | $518840.786598495 ;$ | $718875.794694225 ;$ | 23.3294166 |
| 29 | YTT29 | $520145.435490858 ;$ | $718953.625408137 ;$ | 22.9183092 | $520145.435486309 ;$ | $718953.625481671 ;$ | 23.4036233 |
| 30 | YTT30 | $522444.869566192 ;$ | $719783.514112478 ;$ | 22.97815103 | $522444.869536483 ;$ | $719783.514127727 ;$ | 23.494706 |
| 31 | YTT31 | $522025.385573317 ;$ | $718114.274704924 ;$ | 22.79568104 | $522025.385549735 ;$ | $718114.274751322 ;$ | 23.1433067 |
| 32 | YTT32 | $523186.583369713 ;$ | $717539.965614425 ;$ | 22.71579058 | $523186.583341969 ;$ | $717539.965648825 ;$ | 22.9760665 |
| 33 | YTT33 | $528705.879517029 ;$ | $713817.503986232 ;$ | 22.26302379 | $528705.879534535 ;$ | $713817.504864103 ;$ | 22.0818816 |
| 34 | YTT34 | $528043.110511926 ;$ | $712435.484798928 ;$ | 22.13055794 | $528043.110515779 ;$ | $712435.484909801 ;$ | 21.8191489 |
| 35 | YTT35 | $528419.988315911 ;$ | $710633.958211361 ;$ | 21.92731111 | $528419.98831332 ;$ | $710633.958237693 ;$ | 21.4137506 |
| 36 | YTT36 | $529967.93452679 ;$ | $711032.684607905 ;$ | 21.95829762 | $529967.934544663 ;$ | $711032.684711616 ;$ | 21.4742604 |
| 37 | YTT37 | $528261.861876 ;$ | $717210.698619623 ;$ | 22.63104409 | $528261.861862215 ;$ | $717210.698704528 ;$ | 22.8097254 |
| 38 | YTT38 | $526425.689061496 ;$ | $718724.127100844 ;$ | 22.81301857 | $526425.689028949 ;$ | $718724.127113802 ;$ | 23.1712292 |
| 39 | YTT39 | $525076.468986405 ;$ | $719408.819474891 ;$ | 22.91308931 | $525076.468977532 ;$ | $719408.819551119 ;$ | 23.3689151 |
| 40 | YTT40 | $526225.935350995 ;$ | $720282.574474673 ;$ | 22.98975953 | $526225.935307325 ;$ | $720282.574549655 ;$ | 23.5230335 |
| 41 | YTT41 | $528493.426463876 ;$ | $718448.80777251 ;$ | 22.75950388 | $528493.426333629 ;$ | $718448.807829748 ;$ | 23.0647514 |
| 42 | YTT42 | $527884.34385114 ;$ | 720371.80872944 | 22.9684489 | 527884.34360357 | $720371.80879708 ;$ | 23.481299 |
| 43 | YTT43 | $523273.527400817 ;$ | $721154.484610349 ;$ | 23.12072684 | 523273.527204784 | 721154.484704536 | 23.78354 |
| 44 | YTT44 | $524356.490404865 ;$ | $722381.886575353 ;$ | 23.23512038 | $524356.488142223 ;$ | $722381.886658528 ;$ | 24.0128619 |
| 45 | YTT45 | $525882.380108575 ;$ | $722017.811261183 ;$ | 23.17551393 | $525882.380020785 ;$ | $722017.811315488 ;$ | 23.8941837 |

## Initial Checking of Data and Test on Variance Ratio

Before deformation analysis can be carried out, it is important to perform initial checking on the input data and test on the a-posterior variance factors of both epochs (Omogunloye 1988; 1990; 2006 and 2010). This is to ensure that common points, same approximate coordinates and same point's names were used in the two campaigns. The a posteriori variance factors of both epochs were then tested for their compatibility. The null and alternative hypotheses used are as proposed by (Setan 1995; Caspary 1987; Chen et al. 1990; Cooper 1987; Singh 1999)

$$
H_{o}: \sigma_{o 1}^{2}=\sigma_{o 2}^{2}
$$

and

$$
H_{a}: \sigma_{o 1}^{2}>\sigma_{o 2}^{2} \text { or } \sigma_{o 2}^{2}>\sigma_{o 1}^{2}
$$

With $\sigma_{o 1}^{2}$ and $\sigma_{o 2}^{2}$ being the a-posteriori variance factors for the first and second campaigns respectively.

The test statistic is $\boldsymbol{T}=\frac{\sigma_{0 f}^{2}}{\sigma_{n i}^{2}} \sim \mathbf{F}\left(\boldsymbol{\alpha}, \mathbf{d} \boldsymbol{f}_{j}, \mathbf{d} \boldsymbol{f}_{i}\right)$

With $j$ and $i$ representing the larger and smaller variance factors, $\mathbf{F}$ is the Fisher's distribution, $\alpha$ is the chosen significance level (typically $\alpha=0.05$ ) and $\mathrm{d} \boldsymbol{f}_{\boldsymbol{i}}$ and $\mathbf{d} \boldsymbol{f}_{\boldsymbol{j}}$ are the degrees of freedom for $i$ and $j$ observation campaigns respectively. The above test is accepted if $\boldsymbol{T}<\boldsymbol{F}\left(\alpha, d \boldsymbol{f}_{\boldsymbol{j}}, \mathrm{d} \boldsymbol{f}_{\boldsymbol{i}}\right)$ at a significance level $\alpha$. The failure of the above test may be caused by incompatible weighting between the two campaign observations or incorrect weighting scheme and any further analysis is stopped at such stage.

## TREND ANALYSIS

After the test on the variance ratio, the test is accepted, the displacement vector (coordinates differences) and its cofactor matrix is then computed as follows

$$
\mathbf{d}=\hat{\mathbf{x}}_{2}-\hat{\mathbf{x}}_{1} \quad(90 \times 1 \text { matrix })
$$

$$
\begin{equation*}
\mathbf{Q}_{\mathbf{d}}=\mathbf{Q}_{\widetilde{\mathrm{x}} 1}+\mathbf{Q}_{\widetilde{\mathrm{x}} \overline{2}} \quad(90 \times 90 \text { matrix }) \tag{3.13}
\end{equation*}
$$

$\mathbf{d}$ is the displacement vector, $\mathbf{Q}_{\mathbf{d}}$ is the cofactor matrix of $\mathbf{d}, \hat{x}_{1}$ and $\hat{\chi}_{2}$ are the estimated coordinates of all the common points in the first and second observation epochs respectively (with same datum definition), $\mathbf{Q}_{\widehat{\mathbf{x}} \overline{1}}$ and $\mathbf{Q}_{\widehat{\mathbf{x}} \overline{2}}$ are the cofactor matrix of the estimated coordinates $\hat{\chi}_{1}$ and $\hat{\chi}_{2}$.

## Least Absolute Sum (LAS)

Chen, (1983) has proposed a robust method known as Least Absolute Sum (LAS). This robust method was developed at the University of New Brunswick, Canada. In the LAS method, some points in a reference network cannot be accepted as stable .In other words not every point has equal importance .Hence in the beginning, the weight matrix $(\mathrm{W})$ is accepted as $\mathrm{W}=\mathrm{I}$. While datum determines, this indicates that all points in the network have the same importance. Therefore, the solution is similar to the Helmert transformation, if some points are given unit weight and the others a zero weight, that is, $\mathrm{W}=\operatorname{diag}(\mathrm{I}, \mathrm{O})$.

The LAS methods are used when there is no previous information about the movement of points within the network.

$$
\begin{equation*}
d^{k+1}=\left[\mathbf{I}-\mathbf{H}\left(\mathbf{H}^{\mathrm{T}} \mathbf{W}^{(k)} \mathbf{H}\right)^{-1} \mathbf{H}^{\mathrm{T}} \mathbf{W}^{(\mathrm{k})}\right] \mathrm{d}^{\mathbf{k}}=\mathbf{S}^{(\mathrm{k})} \mathbf{d}^{(\mathrm{k})} \tag{3.14}
\end{equation*}
$$

$I=$ identity matrix
$k=$ number of iterations
$d=$ displacement vector
$S=$ S-transformations matrix
$W=$ weight matrix
Then displacement values (d) are calculated as:

$$
\begin{align*}
& d_{1}=S_{1} d  \tag{3.15}\\
& Q d_{1}=S Q_{d} S^{T}  \tag{3.16}\\
& S=I-H\left(H^{T} W H\right)^{-1} H^{T} W  \tag{3.17}\\
& d_{2}=S_{2} d_{1}  \tag{3.18}\\
& Q d_{2}=S_{2} Q_{d} S_{2}^{T} \tag{3.19}
\end{align*}
$$

where $d_{1}$ and $Q_{d 1}$ are the displacement vector and its cofactor matrix respectively based on the new datum or computational base, $H$ is the inner constraints matrix constructed depending on the union of the datum defects in the two epochs and on the number of common points, and $W$ is the weight matrix with diagonal value of one for datum points and zero elsewhere. Matrix $S$ is symmetric only for the minimum trace solutions. (i.e., all points in the network were defined as datum).The group of selected datum points is then tested for its stability by using Single Point displacement test.

## Formation of Matrix H for the Final S-Transformation

$H$ is a configuration matrix for the datum defect, called inner constraint matrix. Basically, the matrix $\mathbf{H}$ depends on the type of network: 1D, 2D or 3D. For 1D, 2D and 3D networks, H is having maximum dimensions of ( 1 m by 1 ), ( 2 m by 4 ) and (3m by 7) respectively, where $m$ is the number of stations.

Equation (3.20) shows the components of the matrix $H$ for a $1 D$ network $\mathrm{H}^{\mathrm{T}}=\left(\begin{array}{llll}1 & 1 & 1 & 1 \\ 1 & 1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots\end{array} \quad(1 \times 45\right.$ matrix $) \quad$ [3.20]

For 2D surveying networks, the first two rows of the matrix $H$ represent the translations in the
x and y directions ( $\boldsymbol{t x}$ and $\boldsymbol{t} \boldsymbol{y}$ ), the third row defines the rotation about the z axis $(r z)$ and the last row is the scale of the network. Equation (3.2.4.1) shows the components of the matrix $H$ for a

2D network $\boldsymbol{H}^{T}\left[\begin{array}{ccccccc} & & & & & & \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{0} & \mathbf{1} & \cdots & \mathbf{1} & \mathbf{0} \\ \boldsymbol{y}_{1}^{o} & -\boldsymbol{x}_{1}^{o} & y_{2}^{o} & -\boldsymbol{x}_{2}^{o} & \cdots & -y_{m}^{o} & -\boldsymbol{x}_{m}^{o} \\ \boldsymbol{x}_{1}^{o} & \boldsymbol{y}_{1}^{o} & x_{2}^{o} & y_{2}^{o} & \cdots & \boldsymbol{x}_{m}^{o} \boldsymbol{c} & \boldsymbol{y}_{m}^{o}\end{array}\right] \quad$ (4 $\times 90$ matrix)

Where $\boldsymbol{x}_{\boldsymbol{i}}^{o}$ and $\boldsymbol{y}_{i}^{o}, \boldsymbol{z}_{\boldsymbol{i}}^{o}$ are the coordinates of point $\boldsymbol{p}_{\boldsymbol{i}}$ which are reduced to the centroid or centre of gravity of the network, i.e.,

$$
\begin{array}{ll}
x_{i}^{o}=x_{i}-\frac{\left(\sum_{i=1}^{m} x_{i}\right)}{m} & (90 \times 1 \text { matrix })  \tag{3.22}\\
y_{i}^{o}=y_{i}-\frac{\left(\sum_{i=1}^{m} y_{i}\right)}{m} & (90 \times 1 \text { matrix })
\end{array}
$$

With $x_{i}, y_{i}$, the approximate coordinates of point $p_{i}$ and m is the number of common points in the network. (Kuang, 1996; Ozturk and Serbetci, 1992; Singh and Setan , 2001).The first two rows of the inner constraint matrix $\left(\boldsymbol{H}^{\boldsymbol{T}}\right)$ take care of the translations in the $x$ and $y$ directions, while the third row defines the rotation about the vertical $(Z)$ axis and the last row defines the scale of the network. For a trilateration network, the last row of $H^{T}$ is omitted (Caspary 1987; Cooper and Cross 1991; Setan 1997; Chen et al. 1990; Singh 1999).In the first transformation $(k=1)$ the weight matrix is taken as identity $\left(W^{(K)}=l\right)$ for all the common points, this indicates that all the points in the network have the same importance. The weight matrix for LAS
$\mathrm{W}^{\mathrm{K}}=\operatorname{diag}$
The iterative procedure continues until the absolute differences between the successive transformed displacements of all the comation pqints i.e

## [3.25]

are smaller than a tolerance value $\delta$ (say 0.001 m ). It is possible that during the iterations some dxi, dyi, dzi may approach zero causing numerical instability because $\mathrm{W}^{\mathrm{K}}$ becomes very large. There are two ways to solve this problem, either
$\checkmark$ Setting a lower bound value e.g 0.0001 m . If $\mathrm{d}_{\mathrm{j}}{ }^{(\mathrm{k})}$ is smaller than the lower bound value, its weight is set to zero, or replacing equation [3.26] as


Where dicks is the $\delta$ component of the vector dk after kth iteration.
In this study the Least Absolute Sum minimizes the sum of the lengths of the displacements i.e

$$
\begin{equation*}
\Sigma \sqrt{(d x i)^{2}+(d y i)^{2}} \quad \longrightarrow \quad \text { minimum } \tag{3.27}
\end{equation*}
$$

In the final iteration, the cofactor matrix of the displacement vector is computed as

$$
\begin{equation*}
\boldsymbol{Q}_{d}{ }^{k+1}=\boldsymbol{S}^{(k)} \boldsymbol{Q}_{d}\left(\boldsymbol{S}^{(k)}\right)^{T} \quad(90 x 90 \text { matrix }) \tag{3.28}
\end{equation*}
$$

For 1D networks, there are some differences for the calculation of $d^{\prime}$ and $Q_{d}$ '. First, the displacements $d$ are arranged in increasing order. The median is assigned unit weight 1 and zero weight is assigned to the other displacements $d$. If the total number of $d$ is an even number, the two middle (median) displacements $d$ are assigned unit weight 1 and zero weight is assigned to the other displacements $d$, Then, the new vector of displacements $d$ ' and its cofactor matrix
$d^{\prime}=\min \sum\left|d_{i}-t z\right| \Rightarrow$
$Q_{d}{ }^{2}=S Q_{d}(S)^{T}$
where tz is the mean value of the middle displacements and di is the displacement of point i.
$\mathrm{S}=\mathrm{I}-\mathrm{H}\left(\mathrm{H}^{\mathrm{T}} \mathrm{WH}\right)^{-1} \mathrm{H}^{\mathrm{T}} \mathrm{W}$

The stability information of each common point j is then determined through a single point test as below (Setan 1995; Setan and Singh 1998)

$$
\begin{equation*}
T_{j}=\frac{\left.\left(d_{j}^{(k+1)}\right)^{T}\left(Q_{d} j^{(k+1)}\right)^{n-1} d_{j}^{(k+1)}\right)}{2 \sigma_{o}^{2}} \sim F(\alpha, 2, \mathrm{df}) \tag{3.30}
\end{equation*}
$$

Where;
$\boldsymbol{d}_{j}, \boldsymbol{Q}_{\boldsymbol{d} j}=$ displacement vector and its cofactor matrix respectively for each common point j or pooled variance factor.
$\boldsymbol{\sigma}_{o}^{2}=\frac{\left[d f_{1}\left(\sigma_{0}^{2}\right)+d f_{2}\left(\sigma_{o v 2}^{2}\right)\right]}{d f}$, common or pool variance factor
$\left(\boldsymbol{\sigma}_{o 1}^{2}\right),\left(\sigma_{o z}^{2}\right)=$ a posteriori variance factors of first and second epochs respectively
$\boldsymbol{d} \boldsymbol{f}_{1}, \boldsymbol{d} \boldsymbol{f}_{\mathbf{2}}=$ degrees of freedom of first and second epochs
$\boldsymbol{d} \boldsymbol{f}=\boldsymbol{d} \boldsymbol{f}_{1}+\boldsymbol{d} \boldsymbol{f}_{\mathbf{2}}$, sum of degrees of freedom of first and second epochs
$=$ significance level (usually chosen as 0.05)

If the above test passes (i.e., $\boldsymbol{T}_{j}<F(\alpha, 2, d f)$ ) then the point is assumed to be stable at a significance level $\alpha$. Otherwise, if the test fails (i.e., $\boldsymbol{T}_{\boldsymbol{j}} \geq \mathbf{F}(\boldsymbol{\alpha}, \mathbf{2}, \mathbf{d f})$ ) then the point is assumed to be deformed (moved).

## RESULTS AND DATA ANALYSIS

Table 4.1a: 2D (X, Y) Network Adjustment Summary

| PARAMETER | FIRST EPOCH | SECOND EPOCH |
| :--- | :--- | :--- |
| Datum Definitions | 2 | 2 |
| No of Station | 45 | 45 |
| No of Observation $(\boldsymbol{n})$ | 189 | 189 |
| No of Parameters $(\boldsymbol{m})$ | 90 | 90 |
| Degree of Freedom <br> (df- $\boldsymbol{n}-\boldsymbol{m}$ ) | 99 | 99 |


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| :---: | :---: | :---: | :---: |
| Convergence Limit | 0.00001 | 0.00001 |  |
| A-posteriori Variance $(\sigma)$ | 7.82989325645394e08 | $7.96836000130844 \mathrm{e}-$ 08 |  |
| Trace of the Covariance Matrix of the Adjusted parameter | $\begin{aligned} & \text { 5.183975843652210e- } \\ & 06 \end{aligned}$ | $5.27565084002794 \mathrm{e}-$ 06 |  |
| Trace of the Adjusted Observation Matrix | $\begin{aligned} & 7.04690393080854 \mathrm{e}- \\ & 06 \end{aligned}$ | $7.17152400117759 \mathrm{e}-$06 |  |

Table 4.1b: 1D (Height) Network Adjustment Summary

| PARAMETER | FIRST EPOCH | SECOND EPOCH |
| :--- | :--- | :--- |
| Datum Definitions | 2 | 2 |
| No of Station | 45 | 45 |
| No of Observation ( $n$ ) | 107 | 107 |
| No of Parameters (m) | 45 | 45 |
| Degree of Freedom <br> (df- $n-m$ ) | 62 | 62 |
| A-posteriori Variance <br> ( $\sigma$ ) | 0.0394472461577893 |  |
| Trace of the Covariance <br> Matrix of the Adjusted <br> parameter | 1.040613555969225 | 4.018695177022139 |
| Trace of the Adjusted <br> Observation Matrix | 1.77512607710052 | 6.85527356791522 |

## Deformation Analysis Result

After the network adjustment, the obtained results, especially the adjusted coordinates and the cofactor matrices were used for the computation of the
displacement vector and the cofactor matrix of the displacement vector. The trend analysis and deformation detection were carried out using the LAS method. At the degrees of freedom of the epoch, the Fisher's critical value obtained at 0.05 ( $95 \%$ ) significant level is 1.39 . The result of the variance ratio test of the two epochs shows the test statistic (T) value is 1.020884677924254 . The displacement vector (d), cofactor matrix of the displacement vector ( Qd ), the inner constraint matrix (H), weight matrix (W), S-transformation matrix (S) and other parameters of the LAS were all computed. The results of the displacement vector (d) after adjustment of the network, the first iteration displacement vector (d1) and the second iteration displacement vector (d2) after transformation by Least Absolute Sum method the final single point displacement ( dp ) are as shown in Table 4.2 , Table 4.3, and Table 4.4.

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## Table 4.2: Displacement Vector of the 1D Network and Stable and Unstable Point

 Displacement| S/N | CONTROL <br> POINT <br> NAME | Displacement Vector | Displacement Vector on a <br> New Computational Base <br> After S- Transformation |  | Single Point <br> Displacement | $\begin{aligned} & \mathrm{PT}<\mathrm{Fi} \\ & (0.05,2, \mathrm{df}) \\ & \mathrm{PT}<1.550 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{d}_{1}=\mathrm{S}_{1} \mathrm{~d}$ | $\mathrm{d}_{1}=\mathrm{S}_{2} \mathrm{~d}_{1}$ |  | $\mathrm{PT}<1.550$ |
|  |  | dZ(m) | dz1 | dz2 | PTPz |  |
| 1 | YTT1 | -0.01992 | -0.62106 | -0.54265 | 0.811994 | Stable |
| 2 | YTT2 | -0.07401 | -0.59154 | -0.51313 | 0.988851 | Stable |
| 3 | YTT3 | -0.29582 | -0.45274 | -0.37433 | 0.592596 | Stable |
| 4 | YTT4 | -0.13988 | -0.41891 | -0.3405 | 0.648932 | Stable |
| 5 | YTT5 | 0.009162 | -0.40332 | -0.32491 | 0.600531 | Stable |
| 6 | YTT6 | -0.14814 | -0.35374 | -0.27533 | 0.409578 | Stable |
| 7 | YTT7 | -0.10665 | -0.29257 | -0.21416 | 0.261906 | Stable |
| 8 | YTT8 | 0.129087 | -0.28865 | -0.21024 | 0.306232 | Stable |
| 9 | YTT9 | 0.001505 | -0.25564 | -0.17723 | 0.191986 | Stable |
| 10 | YTT10 | -0.01573 | -0.24738 | -0.16897 | 0.189104 | Stable |
| 11 | YTT11 | 0.083069 | -0.2214 | -0.14299 | 0.147021 | Stable |
| 12 | YTT12 | 0.253064 | -0.21416 | -0.13575 | 0.13268 | Stable |
| 13 | YTT13 | -0.02088 | -0.18804 | -0.10963 | 0.070051 | Stable |
| 14 | YTT14 | 0.162908 | -0.18152 | -0.10311 | 0.068049 | Stable |
| 15 | YTT15 | 0.186194 | -0.16077 | -0.08236 | 0.044635 | Stable |
| 16 | YTT16 | 0.288568 | -0.15556 | -0.07715 | 0.040944 | Stable |
| 17 | YTT17 | 0.310682 | -0.12838 | -0.04997 | 0.021027 | Stable |
| 18 | YTT18 | 0.211302 | -0.12742 | -0.04901 | 0.021141 | Stable |
| 19 | YTT19 | -0.08054 | -0.12323 | -0.04482 | 0.017767 | Stable |
| 20 | YTT20 | -0.24624 | -0.106 | -0.02759 | 0.012284 | Stable |
| 21 | YTT21 | -0.34524 | -0.10018 | -0.02177 | 0.013732 | Stable |
| 22 | YTT22 | -0.05327 | -0.09834 | -0.01993 | 0.003962 | Stable |
| 23 | YTT23 | -0.1139 | -0.07841 | 0 | 0 | Stable |
| 24 | YTT24 | -0.18507 | -0.02444 | 0.053976 | 0.079391 | Stable |
| 25 | YTT25 | 0.029093 | 0.021583 | 0.099994 | 0.108622 | Stable |
| 26 | YTT26 | 0.00732 | 0.055404 | 0.133815 | 0.308231 | Stable |
| 27 | YTT27 | -0.04805 | 0.071178 | 0.149588 | 0.449503 | Stable |
| 28 | YTT28 | 0.436104 | 0.078691 | 0.157101 | 0.183723 | Stable |
| 29 | YTT29 | 0.485314 | 0.103798 | 0.182208 | 0.321928 | Stable |
| 30 | YTT30 | 0.516555 | 0.14556 | 0.223971 | 0.528426 | Stable |

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| 31 | YTT31 | 0.347626 | 0.152772 | 0.231183 | 0.514461 | Stable |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 32 | YTT32 | 0.260276 | 0.181064 | 0.259474 | 0.756523 | Stable |
| 33 | YTT33 | -0.18114 | 0.197744 | 0.276154 | 1.883885 | Moved |
| 34 | YTT34 | -0.31141 | 0.203178 | 0.281589 | 2.125777 | Moved |
| 35 | YTT35 | -0.51356 | 0.240122 | 0.318533 | 2.76796 | Moved |
| 36 | YTT36 | -0.48404 | 0.250707 | 0.329118 | 2.839587 | Moved |
| 37 | YTT37 | 0.178681 | 0.328601 | 0.407011 | 2.594411 | Moved |
| 38 | YTT38 | 0.358211 | 0.348322 | 0.426733 | 2.418916 | Moved |
| 39 | YTT39 | 0.455826 | 0.37781 | 0.456221 | 2.445382 | Moved |
| 40 | YTT40 | 0.533274 | 0.405347 | 0.483757 | 2.887476 | Moved |
| 41 | YTT41 | 0.305247 | 0.409051 | 0.487462 | 3.288789 | Moved |
| 42 | YTT42 | 0.51285 | 0.42577 | 0.504181 | 3.274917 | Moved |
| 43 | YTT43 | 0.662813 | 0.55531 | 0.63372 | 4.707015 | Moved |
| 44 | YTT44 | 0.777741 | 0.611166 | 0.689577 | 5.687355 | Moved |
| 45 | YTT45 | 0.71867 | 0.670238 | 0.748648 | 6.843486 | Moved |

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Table 4.3: The Displacement Vector Pattern of the Epoch Data using LAS

|  |  | Displacement Vector <br> (d) |  | Displacement Vector on Transformation By LAS |  | a New | mputation | $\text { Base After } \mathrm{S} \text { - }$ | Single Point <br> Displacement  <br> (PTp)  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Displacement Vector$\left(\mathbf{d}_{1}=\mathbf{S}_{1} \mathbf{d}\right)$ |  | Displacement Vector $\left(\mathrm{d}_{2}=\mathrm{S}_{2} \mathrm{~d}_{1}\right)$ |  |  |  |  |
| S/N | $\begin{aligned} & \text { CONTROL } \\ & \text { POINT } \\ & \text { NAME } \\ & \hline \end{aligned}$ | dX(m) | dY(m) | d1(X) | d1(Y) | d2(X) | d2(Y) | $\begin{array}{\|l} \hline \text { MAGNITUDE } \\ \sqrt{\left(\mathbf{d} 2(X)^{2}\right.} \\ \left.\mathbf{d} 2(\mathbf{Y})^{2}\right) \\ \hline \end{array}$ | PTp (X) | PTP(Y) |
| 1 | YTT1 | $2.77 \mathrm{E}-06$ | $2.59 \mathrm{E}-05$ | $7.20 \mathrm{E}-06$ | -3.17E-05 | $4.25 \mathrm{E}-08$ | -3.88E-05 | $3.880 \mathrm{E}-05$ | 0.000164 | 0.794432 |
| 2 | YTT2 | $3.14 \mathrm{E}-06$ | -0.00011 | $2.26 \mathrm{E}-05$ | $5.17 \mathrm{E}-05$ | $1.54 \mathrm{E}-05$ | $4.46 \mathrm{E}-05$ | $4.718 \mathrm{E}-5$ | 0.186669 | 1.069351 |
| 3 | YTT3 | -2.97E-06 | 0.000105 | $-2.08 \mathrm{E}-05$ | $2.24 \mathrm{E}-05$ | -2.79E-05 | $1.52 \mathrm{E}-05$ | $3.177 \mathrm{E}-05$ | 0.456962 | 0.080737 |
| 4 | YTT4 | $1.48 \mathrm{E}-05$ | -0.00031 | $1.11 \mathrm{E}-05$ | 0.000886 | $3.96 \mathrm{E}-06$ | 0.000879 | $8.79 \mathrm{E}-04$ | 0.007146 | 4.246152 |
| 5 | YTT5 | $5.70 \mathrm{E}-06$ | 0.000124 | $1.44 \mathrm{E}-05$ | -7.86E-05 | $7.20 \mathrm{E}-06$ | -8.58E-05 | 8.61 E-05 | 0.024044 | 4.110841 |
| 6 | YTT6 | $5.83 \mathrm{E}-06$ | $4.10 \mathrm{E}-05$ | $2.28 \mathrm{E}-05$ | -0.0001 | $1.57 \mathrm{E}-05$ | -0.00011 | $1.570 \mathrm{E}-05$ | 0.111269 | 7.19099 |
| 7 | YTT7 | -6.33E-06 | 0.000162 | $-1.23 \mathrm{E}-05$ | $3.08 \mathrm{E}-05$ | -1.94E-05 | $2.36 \mathrm{E}-05$ | 3.053 E-05 | 0.16821 | 0.31783 |
| 8 | YTT8 | $2.35 \mathrm{E}-06$ | 0.000133 | $2.19 \mathrm{E}-05$ | -6.77E-05 | $1.48 \mathrm{E}-05$ | -7.48E-05 | 7.625 E-05 | 0.103594 | 3.158588 |
| 9 | YTT9 | $6.08 \mathrm{E}-07$ | $3.72 \mathrm{E}-05$ | $2.54 \mathrm{E}-05$ | -8.51E-05 | $1.83 \mathrm{E}-05$ | -9.22E-05 | $9.399 \mathrm{E}-05$ | 0.157434 | 4.870269 |
| 10 | YTT10 | -1.09E-07 | $2.52 \mathrm{E}-05$ | $3.17 \mathrm{E}-05$ | -0.00011 | $2.46 \mathrm{E}-05$ | -0.00011 | $1.127 \mathrm{E}-05$ | 0.293329 | 7.62088 |
| 11 | YTT11 | -9.34E-06 | $8.80 \mathrm{E}-05$ | $4.08 \mathrm{E}-05$ | -8.36E-05 | $3.37 \mathrm{E}-05$ | -9.07E-05 | 9.675 E-05 | 0.565388 | 4.660503 |
| 12 | YTT12 | -5.55E-07 | $8.99 \mathrm{E}-05$ | $3.97 \mathrm{E}-05$ | -4.13E-05 | $3.26 \mathrm{E}-05$ | -4.84E-05 | 5.835 E-05 | 0.521227 | 1.323015 |
| 13 | YTT13 | $1.48 \mathrm{E}-06$ | $1.26 \mathrm{E}-05$ | $1.12 \mathrm{E}-05$ | $5.07 \mathrm{E}-05$ | $4.09 \mathrm{E}-06$ | $4.35 \mathrm{E}-05$ | 5.970 E-05 | 0.010728 | 0.872408 |
| 14 | YTT14 | $9.13 \mathrm{E}-06$ | -0.00053 | $3.06 \mathrm{E}-05$ | 0.000906 | $2.35 \mathrm{E}-05$ | 0.000899 | 8.993 E-05 | 0.300271 | 4.536646 |
| 15 | YTT15 | $2.09 \mathrm{E}-06$ | -6.81E-05 | $2.92 \mathrm{E}-05$ | -6.40E-05 | $2.21 \mathrm{E}-05$ | -7.11E-05 | 7.4455 E-05 | 0.236805 | 2.868241 |
| 16 | YTT16 | $2.56 \mathrm{E}-06$ | $1.83 \mathrm{E}-05$ | $3.71 \mathrm{E}-05$ | -5.43E-05 | $2.99 \mathrm{E}-05$ | -6.15E-05 | 6.838 E-05 | 0.452714 | 2.130441 |
| 17 | YTT17 | -4.44E-06 | 0.00011 | $5.19 \mathrm{E}-05$ | -2.71E-05 | $4.48 \mathrm{E}-05$ | -3.43E-05 | 5.642 E-05 | 0.986544 | 0.649148 |
| 18 | YTT18 | -1.53E-05 | -4.11E-6 | $4.62 \mathrm{E}-05$ | -5.38E-06 | $3.90 \mathrm{E}-05$ | -1.25E-05 | $2.337 \mathrm{E}-05$ | 0.760024 | 0.086324 |
| 19 | YTT19 | -1.17E-05 | $6.57 \mathrm{E}-05$ | $3.13 \mathrm{E}-05$ | -0.00012 | $2.42 \mathrm{E}-05$ | -0.00013 | $1.3223 \mathrm{E}-04$ | 0.297002 | 9.061258 |
| 20 | YTT20 | -1.61E-05 | 0.00012 | $3.67 \mathrm{E}-05$ | -0.00013 | $2.95 \mathrm{E}-05$ | -0.00014 | $1.43 \mathrm{E}-04$ | 0.466251 | 1.066337 |

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| 21 | YTT21 | -1.58E-05 | 0.000264 | $4.55 \mathrm{E}-05$ | -0.00012 | $3.84 \mathrm{E}-05$ | -0.00013 | 1.3555 E-04 | 0.821701 | 9.233006 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22 | YTT22 | -1.38E-05 | $5.46 \mathrm{E}-05$ | $4.70 \mathrm{E}-05$ | -3.20E-05 | 3.98E-05 | -3.91E-05 | $5.607 \mathrm{E}-05$ | 0.809365 | 0.851019 |
| 23 | YTT23 | -1.05E-05 | $7.04 \mathrm{E}-05$ | $6.56 \mathrm{E}-05$ | -6.19E-05 | $5.84 \mathrm{E}-05$ | -6.91E-05 | $9.047 \mathrm{E}-05$ | 1.865367 | 2.618017 |
| 24 | YTT24 | -3.03E-06 | 0.000137 | $8.24 \mathrm{E}-05$ | -0.00011 | $7.52 \mathrm{E}-05$ | -0.00012 | 1.4161 E-04 | 3.129756 | 7.668884 |
| 25 | YTT25 | -1.31E-05 | $5.91 \mathrm{E}-05$ | $4.91 \mathrm{E}-05$ | -1.38E-05 | $4.20 \mathrm{E}-05$ | -2.09E-05 | 4.691 E-05 | 0.899676 | 0.243445 |
| 26 | YTT26 | -8.72E-06 | $2.74 \mathrm{E}-05$ | $8.54 \mathrm{E}-05$ | -0.0001 | $7.82 \mathrm{E}-05$ | -0.00011 | 1.3496 E-04 | 3.326455 | 6.232521 |
| 27 | YTT27 | $2.15 \mathrm{E}-05$ | -0.00021 | 0.000121 | -9.17E-05 | 0.000114 | $-9.89 \mathrm{E}-05$ | $1.509 \mathrm{E}-04$ | 7.224628 | 5.070011 |
| 28 | YTT28 | $1.46 \mathrm{E}-06$ | 0.000112 | $5.63 \mathrm{E}-05$ | $1.86 \mathrm{E}-05$ | $4.92 \mathrm{E}-05$ | $1.15 \mathrm{E}-05$ | $5.0526 \mathrm{E}-05$ | 1.190501 | 0.07579 |
| 29 | YTT29 | -4.55E-06 | 0.000126 | $6.08 \mathrm{E}-05$ | -8.46E-08 | $5.36 \mathrm{E}-05$ | -7.24E-06 | $5.408 \mathrm{E}-05$ | 1.420155 | 0.028761 |
| 30 | YTT30 | -2.97E-05 | 0.000272 | $5.85 \mathrm{E}-05$ | -6.64E-05 | $5.14 \mathrm{E}-05$ | -7.36E-05 | 8.97714 E-05 | 1.386828 | 2.965948 |
| 31 | YTT31 | -2.36E-05 | 0.000111 | $5.09 \mathrm{E}-05$ | -4.55E-05 | $4.38 \mathrm{E}-05$ | $-5.26 \mathrm{E}-05$ | 5.661 E-05 | 0.975635 | 1.507281 |
| 32 | YTT32 | -2.77E-05 | 0.000331 | $5.21 \mathrm{E}-05$ | -6.92E-05 | $4.49 \mathrm{E}-05$ | -7.63E-05 | 8.853 E-05 | 1.042566 | 3.237843 |
| 33 | YTT33 | $1.75 \mathrm{E}-05$ | -0.00018 | 0.000116 | 0.000711 | 0.000109 | 0.000704 | $1.297 \mathrm{E}-04$ | 6.784533 | 2.442919 |
| 34 | YTT34 | $3.85 \mathrm{E}-06$ | 0.000319 | $8.89 \mathrm{E}-05$ | -6.24E-05 | $8.17 \mathrm{E}-05$ | -6.96E-05 | $10.732 \mathrm{E}-05$ | 3.947721 | 2.469739 |
| 35 | YTT35 | $-2.59 \mathrm{E}-06$ | 0.000591 | $7.40 \mathrm{E}-05$ | -0.00016 | $6.68 \mathrm{E}-05$ | -0.00017 | $1.833 \mathrm{E}-04$ | 2.844835 | 1.500624 |
| 36 | YTT36 | $1.79 \mathrm{E}-05$ | 0.000657 | 0.000109 | -9.24E-05 | 0.000102 | -9.96E-05 | $1.4256 \mathrm{E}-04$ | 6.39048 | 4.741311 |
| 37 | YTT37 | -1.38E-05 | -0.00113 | 0.000103 | -5.31E-05 | $9.58 \mathrm{E}-05$ | -6.02E-05 | $11.314 \mathrm{E}-05$ | 5.015102 | 1.936708 |
| 38 | YTT38 | -3.25E-05 | 0.000115 | $7.96 \mathrm{E}-05$ | -0.0001 | $7.24 \mathrm{E}-05$ | -0.00011 | $7.240 \mathrm{E}-05$ | 2.885911 | 6.487339 |
| 39 | YTT39 | -8.87E-06 | 0.001186 | $9.72 \mathrm{E}-05$ | $-2.49 \mathrm{E}-05$ | $9.00 \mathrm{E}-05$ | -3.20E-05 | $9.5519 \mathrm{E}-05$ | 4.402552 | 0.566615 |
| 40 | YTT40 | -4.37E-05 | 0.002614 | $7.67 \mathrm{E}-05$ | -2.66E-05 | $6.95 \mathrm{E}-05$ | $-3.38 \mathrm{E}-05$ | 7.728 E-05 | 2.663538 | 0.634454 |
| 41 | YTT41 | -0.00013 | -0.00189 | -3.99E-06 | -7.27E-05 | -1.12E-05 | $-7.99 \mathrm{E}-05$ | 8.068 E-05 | 0.068964 | 3.45412 |
| 42 | YTT42 | -0.00025 | 0.00043 | 0.000201 | -0.00036 | 0.000194 | -0.00037 | 4.1778 E-05 | 2.121703 | 7.28967 |
| 43 | YTT43 | -0.0002 | -0.00133 | -9.28E-05 | $1.78 \mathrm{E}-05$ | -1.00E-04 | $1.06 \mathrm{E}-05$ | 1.00498 E-04 | 5.378697 | 0.062547 |
| 44 | YTT44 | -0.00226 | 0.016168 | -0.00214 | $9.41 \mathrm{E}-06$ | -0.00215 | $2.25 \mathrm{E}-06$ | 2.1500 E-03 | 2.257432 | 0.002949 |
| 45 | YTT45 | -8.78E-05 | -0.00057 | $4.08 \mathrm{E}-05$ | -3.18E-05 | $3.37 \mathrm{E}-05$ | -3.90E-05 | $5.154 \mathrm{E}-05$ | 0.621957 | 0.823719 |

## Table 4.4: The Stable and Unstable Point Detection

|  |  | Displacement Vector(d2) |  | Stable and Unstable Point (Single Point Displacement) Using LAS |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Single Point Displacement$\begin{aligned} & \text { PT- }=\left[\left(\mathrm{d} p^{\prime} \cdot \operatorname{inv}(\mathrm{Qdp}) \bullet \mathrm{dp}\right) /\right. \\ & \left.\left(2^{\bullet} \mathrm{pv}\right)\right] \end{aligned}$ | $\begin{aligned} & \mathrm{PT}<\mathrm{Fi}(0.05,2, \mathrm{df}) \\ & \mathrm{PT}<1.390 \end{aligned}$ |  |
| S/N | CONTROL <br> POINT <br> NAME |  |  | d2(X) | d2(Y) | PTp (X) | PTP(Y) | (X) | (Y) |
| 1 | YTT1 | 4.25E-08 | -3.88E-05 | 0.000164 | 0.794432 | Stable | Stable |
| 2 | YTT2 | 1.54E-05 | 4.46E-05 | 0.186669 | 1.069351 | Stable | Stable |
| 3 | YTT3 | -2.79E-5 | 1.52E-05 | 0.456962 | 0.080737 | Stable | Stable |
| 4 | YTT4 | 3.96E-06 | 0.000879 | 0.007146 | 4.246152 | Stable | Moved |
| 5 | YTT5 | 7.20E-06 | -8.58E-05 | 0.024044 | 4.110841 | Stable | Moved |
| 6 | YTT6 | 1.57E-05 | -0.00011 | 0.111269 | 7.19099 | Stable | Moved |
| 7 | YTT7 | -1.94E-5 | $2.36 \mathrm{E}-05$ | 0.16821 | 0.31783 | Stable | Stable |
| 8 | YTT8 | 1.48E-05 | -7.48E-05 | 0.103594 | 3.158588 | Stable | Moved |
| 9 | YTT9 | $1.83 \mathrm{E}-05$ | -9.22E-05 | 0.157434 | 4.870269 | Stable | Moved |
| 10 | YTT10 | 2.46E-05 | -0.00011 | 0.293329 | 7.62088 | Stable | Moved |
| 11 | YTT11 | $3.37 \mathrm{E}-05$ | -9.07E-05 | 0.565388 | 4.660503 | Stable | Moved |
| 12 | YTT12 | 3.26E-05 | -4.84E-05 | 0.521227 | 1.323015 | Stable | Stable |
| 13 | YTT13 | 4.09E-06 | 4.35E-05 | 0.010728 | 0.872408 | Stable | Stable |
| 14 | YTT14 | $2.35 \mathrm{E}-05$ | 0.000899 | 0.300271 | 4.536646 | Stable | Moved |
| 15 | YTT15 | 2.21E-05 | -7.11E-05 | 0.236805 | 2.868241 | Stable | Moved |
| 16 | YTT16 | $2.99 \mathrm{E}-05$ | -6.15E-05 | 0.452714 | 2.130441 | Stable | Moved |
| 17 | YTT17 | 4.48E-05 | -3.43E-05 | 0.986544 | 0.649148 | Stable | Stable |
| 18 | YTT18 | 3.90E-05 | -1.25E-05 | 0.760024 | 0.086324 | Stable | Stable |
| 19 | YTT19 | 2.42E-05 | -0.00013 | 0.297002 | 9.061258 | Stable | Moved |
| 20 | YTT20 | $2.95 \mathrm{E}-05$ | -0.00014 | 0.466251 | 1.066337 | Stable | Moved |
| 21 | YTT21 | 3.84E-05 | -0.00013 | 0.821701 | 9.233006 | Stable | Moved |
| 22 | YTT22 | 3.98E-05 | -3.91E-05 | 0.809365 | 0.851019 | Stable | Stable |
| 23 | YTT23 | $5.84 \mathrm{E}-05$ | -6.91E-05 | 1.865367 | 2.618017 | Moved | Moved |
| 24 | YTT24 | 7.52E-05 | -0.00012 | 3.129756 | 7.668884 | Moved | Moved |
| 25 | YTT25 | 4.20E-05 | -2.09E-05 | 0.899676 | 0.243445 | Stable | Stable |
| 26 | YTT26 | 7.82E-05 | -0.00011 | 3.326455 | 6.232521 | Moved | Moved |
| 27 | YTT27 | 0.000114 | -9.89E-05 | 7.224628 | 5.070011 | Moved | Moved |
| 28 | YTT28 | 4.92E-05 | 1.15E-05 | 1.190501 | 0.07579 | Stable | Stable |
| 29 | YTT29 | 5.36E-05 | $-7.24 \mathrm{E}-06$ | 1.420155 | 0.028761 | Moved | Stable |
| 30 | YTT30 | $5.14 \mathrm{E}-05$ | -7.36E-05 | 1.386828 | 2.965948 | Stable | Moved |


| 31 | YTT31 | $4.38 \mathrm{E}-05$ | $-5.26 \mathrm{E}-05$ | 0.975635 | 1.507281 | Stable | Moved |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 32 | YTT32 | $4.49 \mathrm{E}-05$ | $-7.63 \mathrm{E}-05$ | 1.042566 | 3.237843 | Stable | Moved |
| 33 | YTT33 | 0.00010 | 0.000704 | 6.784533 | 2.442919 | Moved | Moved |
| 34 | YTT34 | $8.17 \mathrm{E}-05$ | $-6.96 \mathrm{E}-05$ | 3.947721 | 2.469739 | Moved | Moved |
| 35 | YTT35 | $6.68 \mathrm{E}-05$ | -0.00017 | 2.844835 | 1.500624 | Moved | Moved |
| 36 | YTT36 | 0.000102 | $-9.96 \mathrm{E}-05$ | 6.39048 | 4.741311 | Moved | Moved |
| 37 | YTT37 | $9.58 \mathrm{E}-05$ | $-6.02 \mathrm{E}-05$ | 5.015102 | 1.936708 | Moved | Moved |
| 38 | YTT38 | $7.24 \mathrm{E}-05$ | -0.00011 | 2.885911 | 6.487339 | Moved | Moved |
| 39 | YTT39 | $9.00 \mathrm{E}-05$ | $-3.20 \mathrm{E}-05$ | 4.402552 | 0.566615 | Moved | Stable |
| 40 | YTT40 | $6.95 \mathrm{E}-05$ | $-3.38 \mathrm{E}-05$ | 2.663538 | 0.634454 | Moved | Stable |
| 41 | YTT41 | $-1.12 \mathrm{E}-5$ | $-7.99 \mathrm{E}-05$ | 0.068964 | 3.45412 | Stable | Moved |
| 42 | YTT42 | 0.000194 | -0.00037 | 2.121703 | 7.28967 | Moved | Moved |
| 43 | YTT43 | $-1.00 \mathrm{E}-4$ | $1.06 \mathrm{E}-05$ | 5.378697 | 0.062547 | Moved | Stable |
| 44 | YTT44 | -0.00215 | $2.25 \mathrm{E}-06$ | 2.257432 | 0.002949 | Moved | Stable |
| 45 | YTT45 | $3.37 \mathrm{E}-05$ | $-3.90 \mathrm{E}-05$ | 0.621957 | 0.823719 | Stable | Stable |

## ANALYSIS OF RESULTS

After the presentation of results, the results were analysed as shown in the sub session below.

## Trend and Deformation Analysis of the Displacements Using LAS Method

After the Least Square Estimation (LSE) of the data of the network, the compatibility of the two epochs data was tested with the variance ratio and compatibility test passed. The computed variance ratio of the campaigns is lesser than the Fdistribution critical value for the specified confidence level. The critical value for the 0.05 (95\%) significance level chosen for the Fisher's distribution ( F ) is 1.390 . The test statistic ( T ), which is the ratio of the variances (the larger divided by the small passed. The test on the variance ratio passes at 0.05 significance level (i.e., $1.02088467792425<1.390$ ) of the Fisher's critical value, thus indicating the compatibility between the two epochs and permits further analysis to be carried out for deformation detection and analysis. For the 1D network, the critical value the $0.05(95 \%)$ significance level chosen for the Fisher's distribution (F) is 1.550 and it also passes the compatibility test.

The trends of movements and deformation analysis of the monitoring network was done using the adjusted coordinate differences and the cofactor matrices from both campaigns respectively and by applying the LAS method. The 1D and 2D point coordinates X, Y of each epoch and their cofactor matrices were calculated with two separate network adjustments. The Deformation program calculated displacement in $X$ axis ( $d X$ ), $Y$ axis ( $d Y$ ) and ( $d Z$ ).

The LAS determined the final displacement vector (dp). The data met the convergence criteria after two iterations. The displacement values obtained from the differences of the adjusted coordinates and their transformation by LAS method shows that virtually all the stations have undergone movements' overtime but this however did not result in deformation of all the point to a significant level. The single point displacement test failed for some points thus confirming the existence of deformation for some of the group of selected control points. The summary of the parameters of the deformation detection and analysis for 2D and 1D are shown in Table 4.4 and Table 4.5 respectively. The results is emphasized by the plot of single point displacement vectors ,the stable and unstable points and the relative absolute error ellipse of the 45 stations in the network as represented in Figures 4 .1, 4.2, and 4.3.

Table 4.4: Summary of some Key Parameters of the Deformation Detection and Analysis (2D)

| KEY PARAMETERS |  |
| :--- | :--- |
|  | LAS |
| No of Iteration | 2 |
| Fisher's Distribution Critical Value fo <br> 95\% Confidence Level (F) | 1.390 |
| Calculated Variance Rati <br> (T-rho1/rho2) | 1.02088467792425 |
| The Compatibility Test Passed (T<F) | $1.02088467792425<$ |
|  | $1.390)$ |
| Pooled Variance Factors | $7.77532847804672 e^{-}$ |
|  | 08 |
| Combined Degree of Freedom | 99 |

Table 4.5: Summary of some Key Parameters of the Deformation Detection and Analysis (1D)

| KEY PARAMETERS | SINGLE POINT DISPLACEMENT |
| :--- | :--- |
| No of Iteration | 2 |
| Fisher's Distribution Critical Value fo. <br> 95\% Confidence Level (F) | 1.550 |
| Calculated Variance Rati <br> (T=rho1/rho2) | 1.327053753 |
| The Compatibility Test Passed (T<F) | $1.327053753<1.550)$ |
| Pooled Variance Factors | 0.0958933293890637 |
| Combined Degree of Freedom | 62 |

Figure 4.1: Displacement Vector Pattern after S-Transformation using LAS


Figure 4.2: Displacement Vector Magnitude of the Stations using LAS
 using Coordinate Differences from Different Observational Campaigns


Figure 4.3: Relative Absolute Error Ellipse of the 45 Stations in the Lagos State Secondary Control Network

## CONCLUSIONS

This study has presented successfully the deformation study of a geodetic monitoring network using two epochs data. The major focus has been on the identification of stable and unstable points in the network. The following conclusions are drawn from the study;
> The two epoch data were adjusted by the least square adjustment technique and passed the compatibility test and are therefore compatible.
$>$ The displacement vector obtained from the differences of the adjusted coordinates shows that virtually all the points have undergone movements overtime but this has not however resulted in deformation within the chosen significant level of $95 \%$ confidence limit.
> The single point displacement test failed for some stations thus confirming the existence of deformation for some points. This shows that the Least Absolute Sum (LAS) has the capacity to determine stable and unstable reference points in a geodetic network. The determination of deformation status of reference points is very useful and can be applied for monitoring deformation trends in Dam Sites, Exploration areas, Tunnels and engineering structures.

## RECOMMENDATIONS

Based on the work done in this study, the following points are hereby recommended:

- Using data from more than two epochs will dramatically enhance the detection of any possible change in a deformation detection and analysis study.
- As a future work, other robust and non-robust methods (e.g., Fredericton Approach, Danish Method, Total Least Square, Multi parameter

Transformation, and Congruency testing methods) could be applied for the deformation detection and analysis. Furthermore dynamic model of deformation detection and prediction using the Kalman filtering methods for the velocity and acceleration determination of deformable body should be examined.

- The Survey body in this country (Nigeria), should wakeup to determine how stable her platform is, in order to avert future hazards and disaster by carrying out observations on our network of controls regularly with advanced Differential Global Positioning System (DGPS) with reference to the continuously Operating reference stations (CORS) networks.


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