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MATHEMATICAL MODEL ANDSTABILITY ANALYSIS FOR GONORRHEA TRANSMISSION DYNAMICS

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ABSTRACT

In this study we, developed and analysed a deterministic model for the transmission dynamics and control of Gonorrhea. Three nonlinear ordinary differential equations were used to describe this spread. The equilibrium points of the model are found and their stabilities are investigated. The basic reproduction number (R_0) was also calculated. The model exhibits two equilibria namely the disease free equilibrium in which all infected compartments are zero and the endemic equilibrium in which all the compartments are greater than zero. It was found that for $(R_0) \leq 1$, the Disease Free Equilibrium is locally asymptotically stable and unstable if otherwise.

Keywords: *Stability, Disease Free Equilibrium, Gonorrhea,Basic reproduction number.*

INTRODUCTION

Gonorrhoea is a sexually transmitted disease (STD) caused by the bacterium Neisseria gonorrhoea that multiplies in the reproductive tract, which includes the cervix, uterus, fallopian tubes and urethra. The exact time when gonorrhoea started cannot be known. Earliest records of the disease are found from 1611. All efforts to eradicate the disease by public health organisations proved abortive. Gonorrhea is the most prevalent sexually transmitted disease (STD) in Nigeria. In fact, in 1963, World Health Organisation (WHO) found Lagos to have the highest gonorrhoea rate in the world. Recent survey report gonorrhoea prevalent to be as high as 28.1%. further studies show a clear association between gonorrhoea in male and female infertility. There is an increase in the prevalence of gonorrhoea among girls, mostly due to sociocultural factor such as the belief that sexual intercourse with a girl who has urethritis cures the condition.

Gonorrhoea is caused by Neisseria bacteria. It is very contagious. The bacteria can enter the body through any opening, such as the mouth, vagina, penis or rectum. Gonorrhoea is mostly spread by sexual intercourse. In men, the infection usually starts as an infection of the urethra. (the urethra is the tube that urine passes through). In women, the bacteria usually first infect the throat or rectum after oral or anal sex. A baby can be infected during childbirth if the mother has gonorrhoea. When the baby passes through the birth canal, the bacteria can get into the baby and infect the baby's eyes. Anyone who is sexually active can get gonorrhoea.

Gonorrhea is being transmitted during vaginal, anal and oral sex (performing or receiving). Yearly gonorrhoea testing is recommended for all sexually active women younger than 25 years, as well as older women with risk factors such as new or multiple sex partner who has a sexually transmitted infection (STD). One can have gonorrhoea without having any obvious symptoms. When you do have symptoms, they usually appear within 2 to 10 days after one is infected. Symptoms in men may have includes:

- Feeling of burning or pain while urinating
- Yellow, green, white or thick discharge from penis
- Urge to urinate often
- Sore throat (after oral sex)
- Pains and enlargement in the scrotum or testicle
- Inflamed anus or rectum (after anal intercourse)

Most women with gonorrhoea shows no symptoms. Some women may have:

- Cloudy, yellow vaginal discharge which may have a bad odour
- Pain or Burning urination
- Pain in the pelvic region
- Irregular bleeding (between periods)
- Vulvitis (swelling of the vulva)
- Stomach pain

The infection, if spread to the rectum, can cause anal itching anal discharge and painful bowel movement. Gonorrhoea, as a result of oral sex, can also occur in the throat, characterized by burning sensation and swollen glands. If a baby get gonorrhoea during childbirth, one or both of the baby's eyes are severely inflamed.

Many illnesses can cause symptoms similar to gonorrhoea. To confirm a diagnosis of gonorrhoea, the health care provider will test a sample of discharge from the urethra of a man's penis or discharge from a woman's cervix. Throat and anal cultures may also be taken if any symptoms suggest a spreading infection. A urine test can also be done to check for the presence of the bacteria responsible for gonorrhoea.

Gonorrhoea is treated with antibiotic medicine, taken by mouth or given as a shot. Many people with gonorrhoea also have chlamydia (another sexually transmitted disease). Because of this, you may be given more than one drug to cure both diseases. The development of antibiotic resistance in Neisseria gonorrhoea is an increasing public health concern. Currently in the United States, gonorrhoea control strategy relies primarily on exertive antibiotic therapy. Consequently, drug resistance is a significant issue requiring careful monitoring, research and scientific development of new treatment regimes. The bacterium has progressively developed resistance to sulfonilamides, penicillin, tetracycline and ciproflaxin which were previously used as effective antibiotic treatments. Currently the CDC STD treatment guidelines recommend dual therapy, which includes a cephalosporin antibiotic, typically ceftriaxone and either azithromycin or doxycycline to treat the majority of gonococcal infections present among adults and adolescents.

Dual therapy is recommended so as to hinder the potential emergence of gonococcal cephalosporin resistance. Surveillance of antimicrobial resistance in Neisseria gonorrhoea in the United States is focus through the Gonococcal Isolate Surveillance Project (GISP). GISP was formed in 1986 to monitor trends in antimicrobial susceptibilities of strains of Neisseria gonorrhoea so as to establish a rational basis for the selection. Using a least condom every time during foreplay and sexual intercourse reduces the chance of contracting gonorrhoea. Even if you don't have symptoms but have had unprotected sex (without a condom), it is advisable to see your local health provider or clinic to be checked for gonorrhoea and other STD.

The history of the discovery of the neisseria gonorrhea infection can be traced to Edinburg in the year 1792, where the surgeon Benjamin Bell

clearly differentiated it from syphilis infection (Benedek, T. G., 2005). Gonorrhea causes a lot of complications in the infected persons, and a prolong infection can lead to severe eye infections, infertility in both men and woman, ectopic pregnancy, spontaneous abortion, still births and eventually death if untreated (Riley, S., 2010). The bacteria targets the cells of the mucous membranes, including the surfaces of the Urethra, vagina cervix, anus, rectum, the lining of the eyelid and the throat (Schiffert Health Center, 2011). Behavioral change is very important in the control of the spread of diseases and as such, can be involved in the interpretation of the diseases outbreak data and estimation of decreases transmission rates (Prabhakararao, G., 2013).
Ibrahim I.A. and Sulaiman U. (2018) studied a mathematical model for the Dynamics of Neisseria Gonorrhea Disease with Natural Immunity and Treatment Effects. In their work, they found out that an increase in treatment rate and boosting the natural immunity of individuals will help in reducing the appearance of new cases of Neisseria gonorrhea infection in the population.

In a related study, Nana-Kyere, Glory Kofi Hogar, Eric Seth N. Marmah, Justice K. (2016) studied a Qualitative Analysis of Neisseria Gonorrhhea Dieases with Treatment Effect, their studies reveal that gonorrhea is deadly when not properly treated by the infected person, due to its complication effects on the host.

In 2013, Prabhakararao developed a mathematical model for the transmission dynamics of gonorrhea infection in Anantapur District, Andhra-Pradesh-India. In developing the model, they partitioned the population into susceptible, infectious and removed compartments and assumed recovery from infection confers permanent immunity. The result of their study showed that the epidemic does not die out, but ultimately approach a steady state with reference to its severity among the population of Anantapur District.

ALSO, Gonorrhea causes a lot of complications in the infected persons, and a prolong infection can lead to severe eye infections, infertility in both men and woman, ectopic pregnancy, spontaneous abortion, still births and eventually death if untreated (Riley, S., 2010).

In their work, (Amalia, G. & Loukas, Z., 2015) they developed and investigated a deterministic epidemic model for the spread of gonorrhea. They showed that the discrete time dynamical system exhibits far more complex dynamics than its continuous analogues. They also carried out stability analysis of their model and established that there are phenomena of Fold Flip bifurcations. The result of their study revealed that male Latex condom use stabilizes the chaotic vibrations of the system to a point where the number of infected individuals remains stable and is significantly small or zero, leading to the control of the disease.

Some studies looked at the impact of treatment on the dynamics of neisseria gonorrhea infection. Amongst them is the work by (Sacrifice, N. K., *et al.*, 2016), where they developed a mathematical model for the dynamics of gonorrhea with treatment effect and studied the model dynamics to understand the epidemic phenomenon and recommended strategies for its control. The result of their study showed that an increase in treatment rate has significant effect on the infective.

Considering the works of the afore-mention authors, the study at hand is an improvement on the cited models above in that it includes;

1. The infected and recovered classes
2. Loss of immunity after recovered.
3. The recovered individuals moves to the susceptible class after another effective contact with the infected person again.

MATERIALS AND METHODS

Model Formulation

A mathematical model for the transmission dynamics and control of Gonorrhea was developed, improving on the existing models as explained in the literature review (birth rate not equal to death removal rate), and standard incidence.

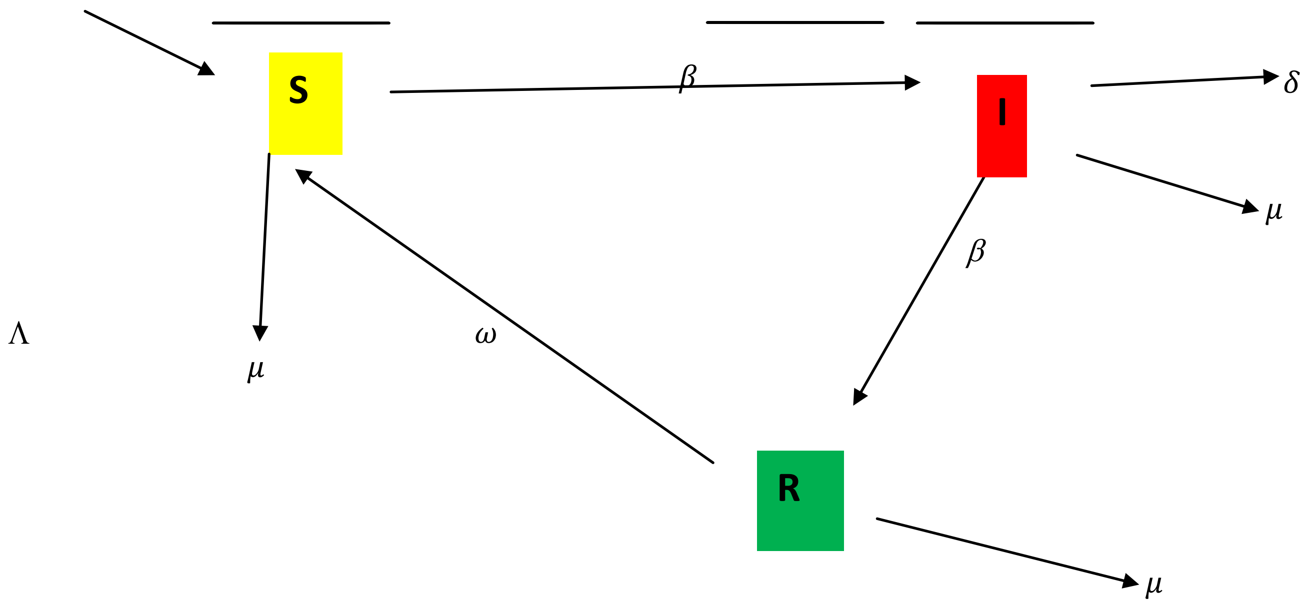

Fig 1.1 Schematic diagram of gonorrhea dynamics model

$$\left.\begin{aligned}\frac{dS}{dt} &= \lambda - \frac{\beta SI}{N} + \omega R + \mu S \\ \frac{dI}{dt} &= \frac{\beta SI}{N} - (\gamma + \delta + \mu) I \\ \frac{dR}{dt} &= \gamma I - (\omega + \mu) R\end{aligned}\right\} \qquad (3.1)$$

where

$$N(t) = S(t) + I(t) + R(t) \qquad (3.2)$$

So that

$$\frac{dN(t)}{dt} = \Lambda - \delta I (S + I + R)\mu \qquad (3.3)$$

in the biological feasible region:

$$\Omega = \left\{ \begin{pmatrix} S \\ I \\ R \end{pmatrix} \in R_{+}^{3} \middle| \begin{aligned} &S \geq 0 \\ &I \geq 0 \\ &R \geq 0 \\ &S + I + R \end{aligned} \right\} \qquad (3.4)$$

Setting

$$\left.\begin{aligned} k_1 &= \gamma + \delta + \mu \\ k_2 &= \omega + \mu \end{aligned}\right\} \qquad (3.5)$$

System (3.1) becomes

$$\left.\begin{aligned}\frac{dS}{dt} &= \Lambda - \frac{\beta SI}{N} + \omega R - \mu S \\ \frac{dI}{dt} &= \frac{\beta SI}{N} k_1 I \\ \frac{dR}{dt} &= \gamma I - k_2 R\end{aligned}\right\} \qquad (3.6)$$

It can be seen that all solutions of the system (3.1) *starting in* Ω which can be shown to be positively invariant.

Disease free Equilibrium (DFE)

$$\frac{ds}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$
(3.7)

i.e

$$\Lambda - \frac{\beta SI}{N} + \omega R - \mu S = 0$$
(3.8)

$$\frac{\beta SI}{N} - (\gamma + \delta + \mu) I = 0$$
(3.9)

$$\gamma I - (\omega + \mu) R = 0$$
(3.10)

Let

$$k_1 = (\gamma + \delta + \mu)$$
(3.11)

$$k_2 = (\omega + \mu)$$
(3.12)

yields;

$$\Lambda - \frac{\beta SI}{N} + \omega R - \mu S = 0$$
(3.13)

$$\frac{\beta SI}{N}+k_1 I=0$$
(3.14)

$$\gamma I-(\omega+\mu)R=0$$
(3.15)

From (3.14) yields

$$I\left(\frac{\beta S}{N}-k_1\right)=0$$
(3.16)

$$I=0$$
(3.17)

$$\frac{\beta S}{N}-k_1=0$$
(3.18)

$I=0,\ R=0$ in (3.15) gives

$$\Lambda N-\mu S=0$$
(3.19)

i.e

$$S=\frac{\Lambda}{\mu}$$
(3.20)

The DFE of the system (3.1) exist at the point

$$\begin{pmatrix} S \\ I \\ R \end{pmatrix}=\begin{pmatrix} \frac{\Lambda}{\mu} \\ 0 \\ 0 \end{pmatrix}$$
(3.21)

Basic Reproduction number (R_0)

One of the most important concern in the analysis of epidemiological models is the determination of the asymptotic behaviour of their solution which is usually based on the stability of the associated equilibria. These models typically consist of disease free equilibrium and at least one endemic equilibrium. The local stability is determined based on a threshold parameter known as basic reproduction number R_0 this represents the average number of secondary cases generated by an infected individuals if introduced into a susceptible population with no immunity to the disease in the absence of interventions to control the infection. If $R_0 < 1$, then on average, an infected individual produces less than one newly infected individual over the course of its infection period. In this case, the infection may die out in the long run. Conversely, if $R_0 > 1$, each infected individual produces, on average more than one new infection, the infection will be able to spread in a population. A large value of (R_0) may indicate the possibility of a major epidemic. Similarly, the reproduction number (R_0) represents the average number of secondary cases generated by an infected individual if introduced into a susceptible population where control strategies are used.

A better widely accepted and used method for finding (R_0) that reflect its biological meaning is the next generation operator approach described by Diekmann and Heesterbeek (2000) and subsequently analysed by Van de Driessche and Watmough (2002). Using this technique we obtained the reproduction number, (R_0) of the system (1) which is the spectral radius (ρ) of the next generation matrix, (K), $R_0 = \rho\left(FV^{-1}\right)$.

Then,

$$F = \frac{\beta \alpha s I}{N}$$

(3.22)

$$V = k_1 I$$

(3.23)

i.e

$$(\gamma + \delta + \mu) I$$

(3.24)

$$V = k_1 I$$

(3.25)

$$FV^{-1} = \frac{\beta S}{N}\left(\frac{1}{k_1}\right) = \frac{\beta S^0}{Nk_1}$$

(3.26)

$$S^0 = \frac{kN}{\mu}$$

(3.27)

$$R_0 = \frac{\beta S}{k_1}$$

(3.28)

Definition of Parameters

λ – Recruitment rate of human

β – Effective contact rate between the susceptible and the infected individuals

γ – Rate of infected individuals recovered from the Gonorrhea disease

ω = The rate of Recovered Individual becomes susceptible from gonorrhea

μ – Natural death rate of individuals

S – Susceptible individuals

I – Infected individuals

R – Recovered Individuals from gonorrhea as a result of treatment.

N – Total population of individuals

δ – Death by infection

CONCLUSION

This SIR model was developed for the transmission dynamics and treatment of gonorrhea disease. We assume constant recruitment rate for the disease. From the analysis of the disease free equilibrium we note that if $R_0 < 1$ the DFE is locally asymptotically stable. Also if $R_0 > 1$ in the endemic equilibrium, the state exist i.e an infected person is liable to infect at least one person with the bacteria, which might totally wipe out the whole population if unprotected sex especially with infected multiple person is adopted. Although the nature of gonorrhea by re-infection after treatment can never allow for its total eradication, but it can be reduced to a barely minimum level. And for any reason if $R_0 > 1$ the gonorrhea free equilibrium point is unstable and the endemic equilibrium emerges

Therefore, in order to forestall such tragedy, there should be a means of replenishing the population and provides improved and effective application of control parameters especially to reduce the Gonorrhea prevalence rate. We therefore conclude from our result that a bacteria infected population or community can only be sustained for conditions that makes the origin unstable; of particular importance is the unusual situation where the birth rate is greater than the death rate, with high level of preventive and control measures particularly to reduce the rate of contracting the bacteria.

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